Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 2/10/21.
Duration: 9.30 a.m.–12.00 noon
Maximum Marks: 100

1. Let $\boldsymbol{T} \in \mathrm{Lin}$. Given that (35)

$$\boldsymbol{T}(\boldsymbol{u} \otimes \boldsymbol{u}) + (\boldsymbol{u} \otimes \boldsymbol{u})\boldsymbol{T}^T = \boldsymbol{0} \quad \forall \boldsymbol{u},$$

determine the most general form of $\boldsymbol{T}$ for the above relation to hold.

2. Let the underlying vector space be two-dimensional, and $\boldsymbol{W}$ be a skew-symmetric tensor in this space. The usual properties of $e^{\boldsymbol{W}}$ that we have derived in the three-dimensional setting may no longer hold, and you may have to carefully use only those relations that are valid. So if you use a particular relation, justify its validity. (30)

   (a) Determine if $e^{\boldsymbol{W}} \in \mathrm{Orth}^+$. If it is orthogonal, determine if it has an axis ($\boldsymbol{Rv} = \boldsymbol{v}$).

   (b) Find the principal invariants of $e^{\boldsymbol{W}}$ in terms of α if $\boldsymbol{W} = \alpha(\boldsymbol{e}_1 \otimes \boldsymbol{e}_2 - \boldsymbol{e}_2 \otimes \boldsymbol{e}_1)$, where $\{\boldsymbol{e}_1, \boldsymbol{e}_2\}$ are the canonical basis vectors for $\Re^2$.

   (c) Find an explicit expression for $e^{\boldsymbol{W}}$ (with a finite number of terms) using the representation for $\boldsymbol{W}$ given in the above sub-part, and use it to find $De^{\boldsymbol{W}}(\boldsymbol{W})[\boldsymbol{U}]$.

3. Using the relations at the end for rigid motion (or a method of your choice), solve the following problem. A rigid disk of radius R rotates with a constant angular speed ω as shown in Fig. 1. Find the angular velocity and angular acceleration of the rigid rod of length L at the configuration shown in the figure, both of which point in a direction perpendicular to the plane of the setup. The entire motion may be treated as occurring within a single plane. (35)

Some relevant formulae

$$\boldsymbol{W} = |\boldsymbol{w}|\,(\boldsymbol{r} \otimes \boldsymbol{q} - \boldsymbol{q} \otimes \boldsymbol{r}), \quad (\boldsymbol{w}/\,|\boldsymbol{w}|\,, \boldsymbol{q}, \boldsymbol{r} \text{ orthonormal}),$$
$$e^{\boldsymbol{T}} = \boldsymbol{I} + \boldsymbol{T} + \frac{1}{2!}\boldsymbol{T}^2 + \cdots,$$

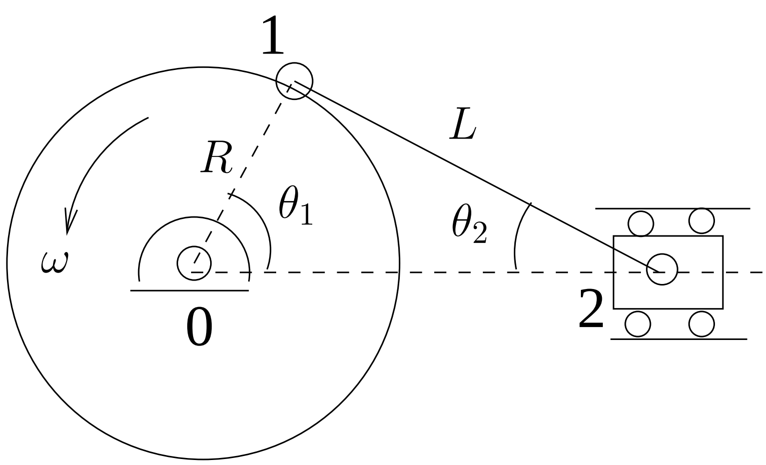

Figure 1: Slider-crank mechanism.

$$
\begin{gathered}
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots , \\
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots , \\
\boldsymbol{v}(\boldsymbol{x}_1, t) = \boldsymbol{v}(\boldsymbol{x}_2, t) + \boldsymbol{w}(t) \times (\boldsymbol{x}_1 - \boldsymbol{x}_2), \\
\boldsymbol{a}(\boldsymbol{x}_1, t) = \boldsymbol{a}(\boldsymbol{x}_2, t) + \dot{\boldsymbol{w}}(t) \times (\boldsymbol{x}_1 - \boldsymbol{x}_2) + \boldsymbol{w}(t) \times [\boldsymbol{w}(t) \times (\boldsymbol{x}_1 - \boldsymbol{x}_2)].
\end{gathered}
$$