# Indian Institute of Science, Bangalore ME 243: Midsemester Test 

Date: 14/10/23.
Duration: 9.30 a.m. -11.30 a.m.
Maximum Marks: 100

1. If $\boldsymbol{T}: \boldsymbol{W}^{2}=0$ for every skew-symmetric tensor $\boldsymbol{W}$, then find the most general form for $\boldsymbol{T}$.
2. Let $\boldsymbol{T} \in \operatorname{Lin}$, be such that $\boldsymbol{T}^{2}=-\alpha \boldsymbol{T}$, where $\alpha>0$.
(a) Find an expression for $\alpha$ in terms of $\boldsymbol{T}$ (Hint: Is Lin an inner product space).
(b) Find an explicit expression for $e^{\boldsymbol{T}}$ in terms of $\boldsymbol{T}$ which has a finite number of terms (Hint: $e^{\alpha}=\sum_{n=0}^{\infty} \alpha^{n} / n!$ )
(c) Using the above information, find $D e^{\boldsymbol{T}}(\boldsymbol{T})[\boldsymbol{U}]$.
3. A circular plate of inner radius $a$ and outer radius $b$ rotates about the $\boldsymbol{e}_{3}$ axis with a constant angular speed of $\omega_{1}$, and simultaneously rotates about the inner periphery with a constant angular speed $\omega_{2}$ as shown in Fig. 1. The cross section remains a straight line at all times as shown by the dotted line in the figure.
(a) Determine the mapping $\boldsymbol{\chi}(\boldsymbol{X}, t)$ for this motion, and the deformation gradient $\boldsymbol{F}(\boldsymbol{X}, t)$, with respect to the fixed coordinate system $\left\{\boldsymbol{e}_{1}, \boldsymbol{e}_{2}, \boldsymbol{e}_{3}\right\}$.
(b) Find the Lagrangian strain $\boldsymbol{E}$ (you need not actually carry out matrix multiplications; just present the relevant expression).
(c) Find the Lagrangian velocity $\tilde{\boldsymbol{v}}(\boldsymbol{X}, t)$.

## Some relevant formulae

$$
\begin{gathered}
\boldsymbol{W}=|\boldsymbol{w}|(\boldsymbol{r} \otimes \boldsymbol{q}-\boldsymbol{q} \otimes \boldsymbol{r}), \quad(\boldsymbol{w} /|\boldsymbol{w}|, \boldsymbol{q}, \boldsymbol{r} \text { orthonormal }) \\
W_{i j}=-\epsilon_{i j k} w_{k}, \\
w_{i}=-\frac{1}{2} \epsilon_{i j k} W_{j k},
\end{gathered}
$$



Figure 1: Rotating plate.

