Indian Institute of Science, Bangalore ME 243: Midsemester Test

Date: 14/10/23. Duration: 9.30 a.m.–11.30 a.m. Maximum Marks: 100

1. If $T: W^2 = 0$ for every skew-symmetric tensor W, then find the most general form for T. (35)

(30)

- 2. Let $T \in \text{Lin}$, be such that $T^2 = -\alpha T$, where $\alpha > 0$.
 - (a) Find an expression for α in terms of T (Hint: Is Lin an inner product space).
 - (b) Find an explicit expression for e^T in terms of T which has a finite number of terms (Hint: $e^{\alpha} = \sum_{n=0}^{\infty} \alpha^n / n!$)
 - (c) Using the above information, find $De^{T}(T)[U]$.
- 3. A circular plate of inner radius a and outer radius b rotates about the e_3 axis with a (35) constant angular speed of ω_1 , and simultaneously rotates about the inner periphery with a constant angular speed ω_2 as shown in Fig. 1. The cross section remains a straight line at all times as shown by the dotted line in the figure.
 - (a) Determine the mapping $\boldsymbol{\chi}(\boldsymbol{X},t)$ for this motion, and the deformation gradient $\boldsymbol{F}(\boldsymbol{X},t)$, with respect to the fixed coordinate system $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$.
 - (b) Find the Lagrangian strain E (you need not actually carry out matrix multiplications; just present the relevant expression).
 - (c) Find the Lagrangian velocity $\tilde{\boldsymbol{v}}(\boldsymbol{X}, t)$.

Some relevant formulae

$$oldsymbol{W} = |oldsymbol{w}| (oldsymbol{r} \otimes oldsymbol{q} - oldsymbol{q} \otimes oldsymbol{r}), \quad (oldsymbol{w}/|oldsymbol{w}|, oldsymbol{q}, oldsymbol{r} ext{ orthonormal}), \ W_{ij} = -\epsilon_{ijk} w_k, \ w_i = -rac{1}{2} \epsilon_{ijk} W_{jk},$$





Figure 1: Rotating plate.