

Indian Institute of Science, Bangalore

ME 243: Midsemester Test

Date: 19/9/26.

Duration: 9.00 a.m.–11.30 a.m.

Maximum Marks: 100

1. Let $E_0 \equiv \text{Orth}^+ \cap \text{Sym} - \{\mathbf{I}\}$. Derive the most general form of $\mathbf{R} \in E_0$. Using this result, (35)
if
 - (a) $\mathbf{TR} = \mathbf{RT}$ for all $\mathbf{R} \in E_0$, find the most general form of $\mathbf{T}$.
 - (b) $\mathbf{T} : \mathbf{R} = 0$ for all $\mathbf{R} \in E_0$, find the most general form of $\mathbf{T}$.
2. If $\mathbf{A}, \mathbf{B} \in \text{Lin}$, determine, using indicial notation or otherwise, if the three eigenvalues of (35)
 $\mathbf{AB}$ and $\mathbf{BA}$ are the same.
 - (a) Let $\mathbf{AB}$ and $\mathbf{BA}$ both be symmetric tensors. Does there exist $\mathbf{Q} \in \text{Orth}^+$ such that
 $\mathbf{AB} = \mathbf{Q}(\mathbf{BA})\mathbf{Q}^T$, and if it does, find at least one such $\mathbf{Q}$.
 - (b) Let $\mathbf{W}_1, \mathbf{W}_2 \in \text{Skw}$ such that $\mathbf{W}_1 : \mathbf{W}_1 = \mathbf{W}_2 : \mathbf{W}_2$. Does there there exist
 $\mathbf{Q} \in \text{Orth}^+$ such that $\mathbf{W}_2 = \mathbf{Q}\mathbf{W}_1\mathbf{Q}^T$, and if it does, find at least one such $\mathbf{Q}$.
3. If $\mathbf{S} = \sum_{i=1}^3 \lambda_i \mathbf{e}_i^* \otimes \mathbf{e}_i^* \in \text{Psym}$, then $\log \mathbf{S} = \sum_{i=1}^3 (\log \lambda_i) \mathbf{e}_i^* \otimes \mathbf{e}_i^*$. Determine if the (30)
following proposed measures of strain vanish if and only if the motion is rigid.
 - (a) $\mathbf{E}_1(\mathbf{F}) = \log \mathbf{U}$, where $\mathbf{U} = \sqrt{\mathbf{F}^T \mathbf{F}}$.
 - (b) $\mathbf{E}_2(\mathbf{F}) = \mathbf{I} - \text{cof } \mathbf{V}$, where $\mathbf{V} = \sqrt{\mathbf{F} \mathbf{F}^T}$. Also determine $D\mathbf{E}_2(\mathbf{I})[\mathbf{U}]$ where the
perturbation is on $\mathbf{F}$, and the evaluation is at $\mathbf{F} = \mathbf{I}$.

Some relevant formulae

$$\begin{aligned}
 \det(\mathbf{T} + \mathbf{U}) &= \det \mathbf{T} + \text{cof } \mathbf{T} : \mathbf{U} + \text{cof } \mathbf{U} : \mathbf{T} + \det \mathbf{U}, \\
 \mathbf{W} &= |\mathbf{w}| (\mathbf{r} \otimes \mathbf{q} - \mathbf{q} \otimes \mathbf{r}), \quad (\mathbf{w}/|\mathbf{w}|, \mathbf{q}, \mathbf{r} \text{ orthonormal}), \\
 (\text{cof } \mathbf{T})^T \mathbf{T} &= (\det \mathbf{T}) \mathbf{I}, \\
 W_{ij} &= -\epsilon_{ijk} w_k, \\
 w_i &= -\frac{1}{2} \epsilon_{ijk} W_{jk}, \\
 I_2(\mathbf{T}) &= \frac{1}{2} [(\text{tr } \mathbf{T})^2 - \text{tr } (\mathbf{T}^2)], \\
 (\text{cof } \mathbf{T})_{ij} &= \frac{1}{2} \epsilon_{imn} \epsilon_{jpq} T_{mp} T_{nq},
 \end{aligned}$$