Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 23/9/04.
Duration: 3.30 p.m.–5.00 p.m.
Maximum Marks: 100

1. Show that $(\boldsymbol{u}, \boldsymbol{T}\boldsymbol{u}) = 0$ for all $\boldsymbol{u} \in V$ if and only if $\boldsymbol{T} \in \text{Skw}$. (25)

2. If $\boldsymbol{w}$ is the axial vector of $\boldsymbol{W} \in \text{Skw}$, find the axial vector of $\boldsymbol{Q}\boldsymbol{W}\boldsymbol{Q}^t$, where $\boldsymbol{Q} \in \text{Orth}^+$. (15)

3. Do the following: (30)

   (a) Derive an expression for $(\boldsymbol{u} \times \boldsymbol{v}) \times \boldsymbol{w}$.

   (b) In the relation
   $$\textbf{cof}\,\boldsymbol{T}(\boldsymbol{u} \times \boldsymbol{v}) := \boldsymbol{T}\boldsymbol{u} \times \boldsymbol{T}\boldsymbol{v} \quad \forall \boldsymbol{u}, \boldsymbol{v} \in V.$$
   let $\boldsymbol{u} = \boldsymbol{e}_q \times \boldsymbol{e}_j$ and $\boldsymbol{v} = \boldsymbol{e}_q$. Simplify, and find an expression for $(\textbf{cof}\,\boldsymbol{T})_{ij}$.

   (c) Use this expression to find an expression for $\phi(\boldsymbol{T}) = (\textbf{cof}\,\boldsymbol{T}) : (\textbf{cof}\,\boldsymbol{T})$ in terms of $\boldsymbol{T}$ and $\boldsymbol{T}^t$.

   (d) Find $\partial\phi/\partial\boldsymbol{T}$ using this expression.

4. Let (30)
   $$\mathit{\Gamma}_{ijk} := \frac{1}{2}\left(\frac{\partial C_{jk}}{\partial X_i} + \frac{\partial C_{ki}}{\partial X_j} - \frac{\partial C_{ij}}{\partial X_k}\right),$$
   where $\boldsymbol{C} = \boldsymbol{F}^t\boldsymbol{F}$. Note that $\mathit{\Gamma}_{ijk} = \mathit{\Gamma}_{jik}$.

   (a) Write the indicial notation expression for the components of $\boldsymbol{C}$ in terms of components of $\boldsymbol{\chi}$, and use it to find the first term on the right hand side of the above expression. Next permute the indices i, j, k to find the second term, and then permute them yet again to find the third one. Using these three expressions, evaluate the entire right hand side of the above equation.

   (b) Use this expression for $\mathit{\Gamma}_{ijk}$ and show that
   $$\frac{\partial \mathit{\Gamma}_{jli}}{\partial X_k} - \frac{\partial \mathit{\Gamma}_{jki}}{\partial X_l} + C^{-1}_{pq}\left(\mathit{\Gamma}_{jkp}\mathit{\Gamma}_{ilq} - \mathit{\Gamma}_{jlp}\mathit{\Gamma}_{ikq}\right) = 0, \qquad 1 \le i, j, k, l \le 3.$$

Some relevant formulae

$$\boldsymbol{W} = |\boldsymbol{w}|\,(\boldsymbol{r} \otimes \boldsymbol{q} - \boldsymbol{q} \otimes \boldsymbol{r}), \quad (\boldsymbol{w}/|\boldsymbol{w}|, \boldsymbol{q}, \boldsymbol{r} \text{ orthonormal})$$
$$\boldsymbol{T}\boldsymbol{e}_p = T_{mp}\boldsymbol{e}_m,$$
$$\epsilon_{ijk} = \boldsymbol{e}_i \cdot (\boldsymbol{e}_j \times \boldsymbol{e}_k),$$
$$\epsilon_{ijk}\epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}.$$