ME257: Assignment 1

Due: 1/2/2016

Note: Use first principles (the 'delta-operator' method) to derive the appropriate equations. DO NOT use the Euler-Lagrange equations derived in class directly.

1. Find the function $y = f(x)$ that extremizes the functional

$$\int_{x_1}^{x_2} [3(y')^2 + 4x]\, dx,$$

   given that $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, 3)$.

2. The tapered bar shown has a Young's modulus E, and is loaded by an axial body force $\boldsymbol{b} = b(x)\boldsymbol{e}_x$. It is fixed at the left end and free at the other end. Assume the displacements to be a function of x alone, and that $u_y = u_z = 0$, i.e., $\boldsymbol{u} = u(x)\boldsymbol{e}_x$. The area at any point along the bar is given by

$$A(x) = A_1\left(1 - \frac{x}{L}\right) + A_2\frac{x}{L}.$$

   Using the expressions for strains and stresses, find an expression for the potential energy of this system. Then, using the first variation of the potential energy, find the governing differential equation using the appropriate boundary conditions at $x = 0$ and $x = L$.

3. Consider a flexible membrane simply supported at the boundary lying in the xy plane, and with a tensile force T per unit length which is everywhere constant in the membrane. If a normal pressure distribution $q(x, y)$ is applied to cause a small deflection, $w(x, y)$ in the z direction, the potential energy is given by

$$\Pi = \frac{T}{2}\int_A \left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] dA - \int_A qw\, dA.$$

   Find out the governing differential equation using the first variation of the above functional and the given boundary condition.

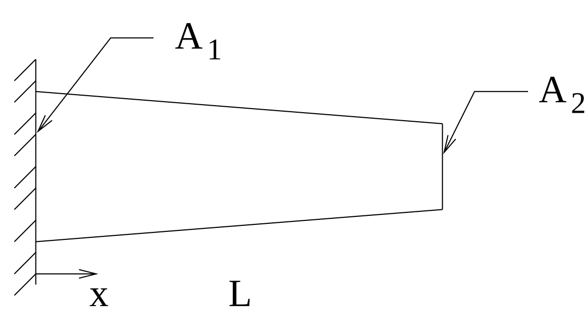

Figure 1:

4. Consider the following differential equation on the domain $x_1 \leq x \leq x_2$:

$$\frac{d^2}{dx^2}\left(a\frac{d^2u}{dx^2}\right) - \frac{d}{dx}\left(b\frac{du}{dx}\right) + cu = f,$$

where a, b, c and f are known functions of x, and $a(x), b(x), c(x) \geq 0\,\forall x$. Develop the variational formulation for the above differential equation in the form:
Find u such that

$$a(u, v) = L(v) \quad \forall v$$

Identify the appropriate essential and natural boundary conditions. Show that $a(.,.)$ is symmetric and positive definite. Formulate the minimization problem corresponding to the variational formulation above for boundary conditions of your choice.

5. Consider the following differential equation on the domain $[1, 3]$:

$$x^3\frac{d^4y}{dx^4} + 6x^2\frac{d^3y}{dx^3} + 6x\frac{d^2y}{dx^2} - 10x = 0.$$

The boundary conditions are $y = y' = 0$ at $x = 1$ and $x = 3$. Find the variational formulation (V) and the minimization problem (M) for this differential equation. Use either form (V) or (M) to find a one parameter approximate solution using the Rayleigh-Ritz method. Use the differential form (D) to find a one-parameter approximate solution using the Galerkin method. Do the two approximate solutions match? Compare the approximate solutions for y and y' with the exact solution obtained by writing the differential equation as

$$\frac{d^2}{dx^2}\left(x^3\frac{d^2y}{dx^2}\right) = 10x.$$

(You may use MATLAB to find the integration constants and plot your results. Plot your results for y and y' separately.)