Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 21/2/2001.
Duration: 10.00 a.m.–11.30 a.m.
Maximum Marks: 100

1. In the Timoshenko beam theory, the potential energy function for certain boundary conditions is given by (50)

$$\Pi = \int_0^L \left[\frac{1}{2} EI \left(\frac{d\beta}{dx} \right)^2 + \frac{1}{2} kGA \left(\frac{dw}{dx} - \beta \right)^2 - qw \right] dx - M_0 \beta|_{x=L} - V_0 w|_{x=L} ,$$

   where the Young modulus E, moment of inertia I, area of cross-section A, correction factor k, shear modulus G, and load intensity q are all given functions of x, and $w(x)$ and $\beta(x)$ are the transverse displacement and rotation of the plane perpendicular to the mid-surface which are to be determined. It is given that the boundary conditions at $x = 0$ are of essential type while those at $x = L$ are of the natural type. Setting the first variation of Π to zero, find the coupled set of governing equations for w and β (*do not attempt to uncouple or to solve the equations*). State the appropriate boundary conditions at $x = 0$ and $x = L$.

2. The equation of heat conduction through a fin of length $L = 0.05$ m is given by (50)

$$\frac{d^2\theta}{dx^2} - \gamma^2 \theta = 0, \quad 0 < x < L.$$

   where $\gamma = 20\ \mathrm{m}^{-1}$. The boundary conditions are given to be

$$\theta(0) = 300\,^{\circ}\mathrm{C}; \quad \left. \frac{d\theta}{dx} \right|_{x=L} = 0.$$

   Develop the variational formulation corresponding to the above governing equation and boundary conditions, and use it to develop the finite element formulation. Using (i) two linear elements of equal length (ii) one quadratic element with midnode at the center, find the values of θ at $x = L/2$ and $x = L$, and the value of the gradient $\frac{d\theta}{dx}$ at $x = 0$. Compare your solution at these points with the analytical solution

$$\theta = \theta(0) \frac{\cosh \gamma (L - x)}{\cosh \gamma L}.$$

In formulating the stiffness matrices, you may directly use the formulae given below.

Some Relevant Formulae

For a linear 1-D element of length l_e:

$$\int_0^{l_e} \boldsymbol{N}^t \boldsymbol{N} \, dx = \frac{l_e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$

$$\int_0^{l_e} \frac{d\boldsymbol{N}^t}{dx} \frac{d\boldsymbol{N}}{dx} \, dx = \frac{1}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

For a quadratic 1-D element of length l_e with midnode at the center:

$$\int_0^{l_e} \boldsymbol{N}^t \boldsymbol{N} \, dx = \frac{l_e}{30} \begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix},$$

$$\int_0^{l_e} \frac{d\boldsymbol{N}^t}{dx} \frac{d\boldsymbol{N}}{dx} \, dx = \frac{1}{3l_e} \begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}.$$