Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 24/2/2016.
Duration: 12.00 noon.–1.30 p.m.
Maximum Marks: 100

1. Let Ω denote the domain, and let $\Gamma \equiv \Gamma_t \cup \Gamma_u$ denote the boundary of Ω. Let (60)

$$\boldsymbol{\epsilon} = \frac{1}{2}\left[\boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^T\right]. \tag{1}$$

   A functional is defined as

$$\Pi = \int_\Omega \mu \boldsymbol{\epsilon}(\boldsymbol{u}) : \boldsymbol{\epsilon}(\boldsymbol{u})\, d\Omega - \int_\Omega (\boldsymbol{\nabla} \cdot \boldsymbol{u}) p\, d\Omega - \int_{\Gamma_t} \boldsymbol{u} \cdot \bar{\boldsymbol{t}}\, d\Gamma - \int_\Omega \boldsymbol{u} \cdot (\rho \boldsymbol{b})\, d\Omega,$$

   where μ is a positive constant, $\bar{\boldsymbol{t}}$ is a prescribed function on Γ_t and $\rho\boldsymbol{b}$ is a prescribed function on Ω. The scalar field variable p and the vector-valued field variable $\boldsymbol{u}$ are both functions of the position vector $\boldsymbol{x}$. Assume that $\boldsymbol{u} = \bar{\boldsymbol{u}}$ on Γ_u. By setting the first variation of Π to zero, find the strong form of the governing equations on Ω and the boundary conditions on Γ_t for the field variables $\boldsymbol{u}$ and p. The equations should be tensorial and should be in terms of $\boldsymbol{u}$ and p (e.g., by substituting for $\boldsymbol{\epsilon}$ using Eqn. (1)).

2. A three-bar truss sits on a 30° incline as shown in Fig. 1. All members have the same Young modulus and cross sectional area E and A. Member 1-2 is vertical, member 2-3 is horizontal and member 1-3 is parallel to the incline. The lengths are as shown. All joints are frictionless pin-joints. Node 1 is fixed, while node 3 is on roller supports as shown. (40)

   (a) If the element stiffness matrix in a local coordinate system aligned along a truss member is given by

$$\boldsymbol{K}' = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},$$

   then derive the expression for the element stiffness matrix of a truss member inclined at an angle θ with respect to the global x–y axes. *Do not* carry out any matrix multiplications that arise. Just state the relevant matrices. This applies to all the following parts as well.

   (b) State the element stiffness matrices for the three truss elements with respect to the global x–y system. Indicate using numbers above and at the side of each element matrix where you would assemble them in the global stiffness matrix.

   (c) Similarly form the global load vector.

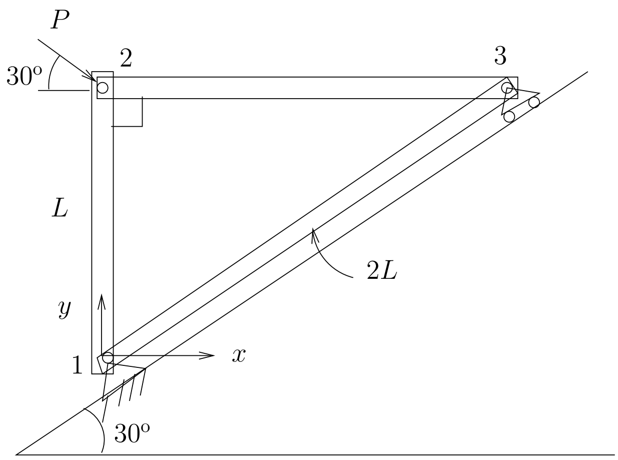

Figure 1: Three-bar truss.

(d) Incorporate the boundary conditions using a Lagrange multiplier technique. *Explain* how you get the relevant matrices (i.e., do not directly use formulae from the notes). *Do not* enforce the boundary conditions at node 1 using Lagrange multipliers, and do not attempt to solve the global set of equations.

(e) Explain how you will recover the stresses in a typical element, say, the inclined member 1–3.