Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 26/2/2017.
Duration: 2.30 p.m.–4.00 p.m.
Maximum Marks: 100

1. Let Ω denote a two-dimensional domain, and let $\boldsymbol{u} = (u_x, u_y)$ be a vector-valued field variable defined on Ω. A functional corresponding to $\boldsymbol{u} = \boldsymbol{0}$ on the entire boundary is defined as (60)

$$\Pi = \frac{1}{2}\int_\Omega \left\{ \alpha \left[\left(\frac{\partial u_x}{\partial x}\right)^2 + \left(\frac{\partial u_x}{\partial y}\right)^2 + \left(\frac{\partial u_y}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial y}\right)^2 \right] + \beta \left[\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right]^2 + \gamma(u_x^2 + u_y^2) \right\} d\Omega,$$

   (a) Convert the above functional to tensorial form.

   (b) By using the first variation of the above functional in tensorial form, find the variational form, and then, subsequently, the strong form of the governing equations and the *allowable* boundary conditions.

   (c) For this part, assume $\alpha = 0$, $\beta = \gamma = 1$. The square domain of unit dimension shown in Fig. 1 is subjected to the essential boundary conditions $\boldsymbol{u} = \boldsymbol{0}$ on the edges $x = 0$ and $y = 0$, and the values of the allowable natural boundary condition that you have found in the previous part are $(1, 0)$ and $(0, 2)$ on the edges $x = 1$ and $y = 1$, respectively. Modify the variational formulation that you have derived to account for the natural boundary condition in this problem, and find a one-parameter Rayleigh–Ritz approximation for $\boldsymbol{u}$.

2. The structure shown in Fig. 2 is subjected to a vertical force F as shown. All members have the same Young modulus E, cross sectional area A, and length L. Assume the horizontal member to be rigid. Using the given finite element matrices (no need to derive them, but provide justifications for how you combine the stiffness matrices and so on), find the deflection under the load F, and the stress in each of the members. (40)

Some Relevant Formulae

For a linear 1-D element of length L:

$$\boldsymbol{K} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \boldsymbol{B} = \frac{1}{L}\begin{bmatrix} -1 & 1 \end{bmatrix}.$$

For a beam element of length L:

$$\boldsymbol{K} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}.$$

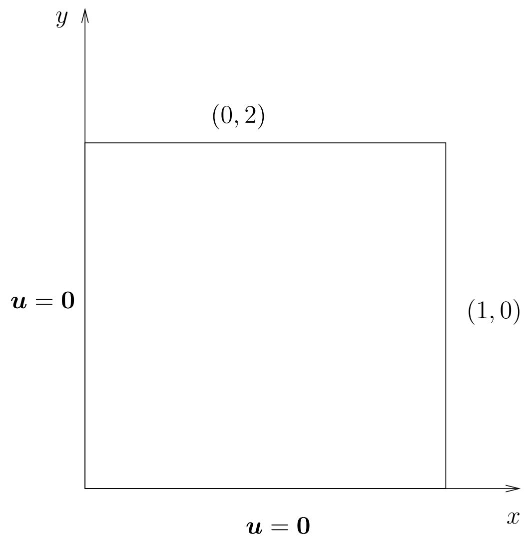

Figure 1: A square domain of unit dimension.

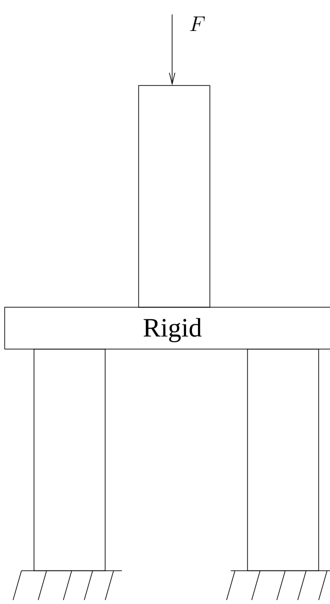

Figure 2: Structure subjected to vertical force F.