Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 23/2/2019.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. Let Ω denote the domain, and let $\Gamma \equiv \Gamma_t \cup \Gamma_u$ denote the boundary of Ω. A functional is defined as (40)

$$\Pi = \int_{\Omega} \left[\boldsymbol{\nabla} \boldsymbol{u} : \boldsymbol{T} - \rho \boldsymbol{u} \cdot \boldsymbol{b} - \frac{1}{2} \boldsymbol{T} : \mathsf{C}^{-1} \boldsymbol{T} \right] d\Omega - \int_{\Gamma_t} \boldsymbol{u} \cdot \bar{\boldsymbol{t}} \, d\Gamma.$$

where $\boldsymbol{T}(\boldsymbol{x})$ is a second-order tensorial field (not necessarily symmetric), $\boldsymbol{u}(\boldsymbol{x})$ is a vector-valued field, C is a fourth-order tensor with major symmetry ($\mathsf{C}_{ijkl} = \mathsf{C}_{klij}$) but not necessarily minor symmetries, and with inverse defined in a natural way ($\boldsymbol{A} = \mathsf{C}\boldsymbol{B}$ implies $\boldsymbol{B} = \mathsf{C}^{-1}\boldsymbol{A}$ with C^{-1} also having major symmetry), $\bar{\boldsymbol{t}}$ is a prescribed function on Γ_t, and $\rho \boldsymbol{b}$ is a prescribed function on Ω. Assume that $\boldsymbol{u} = \bar{\boldsymbol{u}}$ on Γ_u. By setting the first variation of Π to zero, find the strong form of the governing equations on Ω and the boundary conditions on Γ_t for the field variables $\boldsymbol{u}(\boldsymbol{x})$ and $\boldsymbol{T}(\boldsymbol{x})$. The equations should be tensorial and should be in terms of $\boldsymbol{u}(\boldsymbol{x})$ and $\boldsymbol{T}(\boldsymbol{x})$, and no relation between $\boldsymbol{u}(\boldsymbol{x})$ and $\boldsymbol{T}(\boldsymbol{x})$ should be assumed at the outset. *Justify* all steps rigorously.

2. The one-dimensional variational formulation for the displacement $u(x)$ in a bar of uniform cross-section with area A, length L, coefficient of thermal expansion α, and Young modulus E, which is subjected to a constant temperature change T_Δ in the absence of body and surface forces, is given by (60)

$$\int_0^L \frac{dv}{dx} \tau(x) \, A \, dx = 0 \quad \forall v.$$

Using the relation $\tau = E(du/dx - \alpha \Delta T)$, develop an isoparametric finite element formulation for linear elements, i.e., develop the element stiffness matrix and load vector for a two-node bar element using the relations $u = N_1(\xi)u_1 + N_2(\xi)u_2$ and $x = N_1(\xi)x_1 + N_2(\xi)x_2$. Using two linear elements of equal length solve the problem shown in Fig. 1 where two bars of equal length L are separated by a given distance Δ in the reference configuration with the left end of the left bar fixed at $x = 0$, and the right end of the second bar connected to a wall via a spring of spring constant k, which is undeformed in the reference configuration. The entire left bar is heated by an amount T_Δ which causes its right end to come into contact with the left end of the second bar and push it to the right. Write the finite element matrix equations for finding the displacements $\hat{u}_1$–$\hat{u}_4$. Impose any constraints on the displacements other than for $\hat{u}_1$ using a Lagrange multiplier technique (so that the Lagrange multiplier is also part of your matrix equations; do not directly use the results from the notes but show how you obtain your equations from first principles). Do not attempt to solve the equations but indicate how you incorporate boundary conditions in your finite element equations so that you are able to solve for $\hat{u}_1$–$\hat{u}_4$.

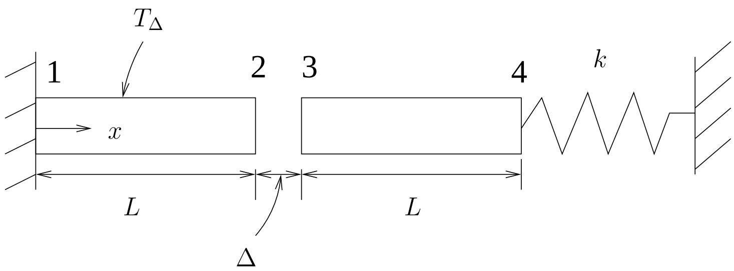

Figure 1: Problem 2: Initial setup.