Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 26/2/2000.
Duration: 10.00 p.m.–11.30 p.m.
Maximum Marks: 100

1. Formulate the variational formulation (V) in the form $a(u,v) = L(v)\ \forall v$ for the following set of equations (40)

$$-\frac{\partial}{\partial x}\left(A_{11}\frac{\partial u}{\partial x} + A_{12}\frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial y}\left(A_{21}\frac{\partial u}{\partial x} + A_{22}\frac{\partial u}{\partial y}\right) + f = 0 \text{ in } \Omega,$$

$$u = u_0 \text{ on } \Gamma_1,$$

$$\left(A_{11}\frac{\partial u}{\partial x} + A_{12}\frac{\partial u}{\partial y}\right) n_x + \left(A_{21}\frac{\partial u}{\partial x} + A_{22}\frac{\partial u}{\partial y}\right) n_y = t_0 \text{ on } \Gamma_2,$$

where (n_x, n_y) are the components of the normal to Γ, A_{ij} and f are given functions of position (x, y) in Ω, and u_0 and t_0 are known functions on Γ_1 and Γ_2 ($\Gamma_1 \cup \Gamma_2 = \Gamma$). It is also given that $A_{12} = A_{21}$.

Is $a(.,.)$ symmetric and positive-definite (justify)? Formulate the minimization problem (M) (if possible) corresponding to (V).

2. The equation for unidirectional, steady, fully developed flow in a cylindrical pipe of radius R is given by (60)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + \frac{8Q}{\pi R^4} = 0,$$

where Q is the (constant) given flow rate, and u is the axial velocity. Note that u is the only nonzero component of velocity, and that u is a function of the radial distance, r, alone, i.e., $u = u(r)$. Assuming the 'no-slip' boundary condition at the cylinder wall, i.e., $u = 0$ at $r = R$, find the exact solution to the above differential equation. Next, develop the variational formulation corresponding to the above equation, and formulate the finite element formulation based on it, for one quadratic element. Using one quadratic element with its first node at $r = 0$, the second at $r = R/2$ and the third at $r = R$, find the velocity, u, at the nodes. Compare your finite element solution with the exact solution. [The shape functions are given by $N_1(\xi) = 0.5(\xi^2 - \xi)$, $N_2(\xi) = (1 - \xi^2)$, $N_3(\xi) = 0.5(\xi^2 + \xi)$].