Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 6/3/99.
Duration: 4.00 p.m.–5.30 p.m.
Maximum Marks: 100

1. Formulate the variational formulation (V) in the form $a(u, v) = L(v)\ \forall v$ for the following heat conduction problem on a square domain: (40)

$$k\boldsymbol{\nabla}^2 T + Q = 0,$$
$$T = 0 \text{ on sides } x = 1 \text{ and } y = 1,$$
$$\boldsymbol{n} \cdot \boldsymbol{\nabla} T = 0 \text{ on sides } x = 0 \text{ and } y = 0.$$

   Is $a(.,.)$ symmetric and positive-definite (justify)? Using a Rayleigh-Ritz approximation of the form

$$T = c(1 - x^2)(1 - y^2),$$

   find c.

2. A load P acts at the end $x = L$ of a bar member which is fixed at one end and connected to a spring at the other as shown in the figure. The spring is undeformed before the application of the load P. Formulate the expression for the potential energy of this one-dimensional system. Derive the variational formulation, and then the strong form of the governing equations and boundary conditions from this potential energy expression. Using a linear-element model with one element derive the stiffness matrix and load vectors from the variational formulation, and solve for the displacement at the end $x = L$. (60)