# Indian Institute of Science, Bangalore

# ME 257: Midsemester Test

**Date:** 23/2/2002.
**Duration:** 9.30 a.m.–11.00 a.m.
**Maximum Marks:** 100

1. The potential function in a heat transfer problem is given by (40)

$$\Pi = \int_{\Omega} \left[ \left( \boldsymbol{\nabla} T + \frac{1}{2k}\boldsymbol{q} \right) \cdot \boldsymbol{q} + QT \right] d\Omega + \int_{\Gamma_q} \bar{q} T \, d\Gamma,$$

where $T$ is the temperature, $\boldsymbol{q}$ is the heat flux, $k$ is the thermal conductivity, $Q$ is the heat generated per unit volume, and $\bar{q}$ is the externally applied heat flux on $\Gamma_q$. The temperature is prescribed to be $\bar{T}$ on $\Gamma_T$. Setting the first variation of $\Pi$ to zero, find the coupled set of governing equations for $T$ and $\boldsymbol{q}$ (*do not attempt to uncouple or to solve the equations*), and the appropriate boundary condition on $\boldsymbol{q}$ on $\Gamma_q$. Justify all your steps carefully. (Hint: Remember that $T$ and $\boldsymbol{q}$ are to be treated as *independent* variables, and no relation between them should be assumed at the outset).

2. The governing equation for the displacement in a bar of uniform cross-section with area $A$, length $L$, coefficient of thermal expansion $\alpha$, and Young modulus $E$, which is subjected to a temperature change $\Delta T$, is given by (60)

$$\frac{d}{dx}\left[ EA \left( \frac{du}{dx} - \alpha \Delta T \right) \right] = 0, \quad 0 < x < L.$$

Develop the variational formulation corresponding to the above governing equation and the boundary conditions $u(0) = u(L) = 0$, and use it to develop an isoparametric finite element formulation. Using one quadratic element with midnode at the center, and the values $E = 2 \times 10^7\,\text{N/cm}^2$, $A = 10\,\text{cm}^2$, $\alpha = 2 \times 10^{-5}/^\circ\text{C}$, $L = 20\,\text{cm}$ and $\Delta T = -40 - 8x\,^\circ\text{C}$, find the values of the reactions at $x = 0$ and $x = L$, and the stress distribution in the bar. Compare your solution with the analytical solution. For formulating the stiffness matrix, you may directly use the formulae given below; for formulating the load vector, use natural coordinates.

### Some Relevant Formulae

For a quadratic 1-D element of length $l_e$ with midnode at the center:

$$N_1 = -\frac{1}{2}\xi(1-\xi); \quad N_2 = 1 - \xi^2; \quad \frac{1}{2}\xi(1+\xi).$$

$$\int_0^{l_e} \frac{d\boldsymbol{N}^t}{dx}\frac{d\boldsymbol{N}}{dx}\,dx = \frac{1}{3l_e}\begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}.$$