Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 25/2/2010.
Duration: 11.30 a.m.–1.00 p.m.
Maximum Marks: 100

1. The governing equation and boundary condition for the torsion warping function are given by (60)

$$\boldsymbol{\nabla}^2\psi \equiv \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0, \qquad \text{in } A,$$
$$(\boldsymbol{\nabla}\psi)\cdot\boldsymbol{n} = yn_x - xn_y \quad \text{on } C.$$

   Starting with the statement $\int_A \phi \boldsymbol{\nabla}^2\psi \, dA = 0$ (where ϕ is the variation of ψ), formulate the variational formulation for the above problem. Is the operator $a(.,.)$ symmetric and positive definite? If it is, state the potential energy functional.

   Use the above variational formulation to find the Rayleigh-Ritz solution for the torsion of an elliptical bar whose equation of contour is given by $x^2/a^2 + y^2/b^2 = 1$, by assuming $\psi = c_0 xy$ and determining c_0. (Hint: Use the transformation $x = ar\cos\theta$, $y = br\sin\theta$, $dA = abr\, dr d\theta$ and $\int_0^{2\pi} \sin^2\theta \, d\theta = \int_0^{2\pi} \cos^2\theta \, d\theta = \pi$ to evaluate the integrals).

2. The one-dimensional variational formulation for the displacement $u(x)$ in a bar of uniform cross-section with area A, length L, coefficient of thermal expansion α, and Young modulus E, which is subjected to a temperature change ΔT of the form $a_0 + a_1 x$ (where a_0 and a_1 are given, and x measured from the left end of the bar) in the absence of body and surface forces, is given by (40)

$$\int_0^L \frac{dv}{dx}\tau(x)\, A\, dx = 0.$$

   Using the relation $\tau = E(du/dx - \alpha\Delta T)$, and the boundary conditions $u(0) = u(L) = 0$, develop an isoparametric finite element formulation for linear elements, i.e., develop the element stiffness matrix and load vector for a two-node bar element using the relations $u = N_1(\xi)u_1 + N_2(\xi)u_2$ and $x = N_1(\xi)x_1 + N_2(\xi)x_2$. Using two linear elements of equal length (i.e., three equispaced nodes), and the values $E = 2\times 10^7\,\text{N/cm}^2$, $A = 10\,\text{cm}^2$, $\alpha = 2\times 10^{-5}/^\circ\text{C}$, $L = 20\,\text{cm}$ and $\Delta T = 40 + 8x\,^\circ\text{C}$, find the displacement at the center, the values of the reactions at $x = 0$ and $x = L$, and the value of the stress at $x = 0$.