Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 20/2/2012.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. The governing equation and boundary condition for the torsion conjugate function are given by (60)

$$\boldsymbol{\nabla}^2 g \equiv \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 0, \quad \text{in } A,$$
$$g = \frac{1}{2}(x^2 + y^2) \quad \text{on } C.$$

Let g_δ be the variation of g. Using this notation, formulate the variational statement for the above problem. Do *not* check for symmetry, V-ellipticity etc.

Use the above variational formulation to find a one-parameter Rayleigh-Ritz solution for the torsion of a

(a) circular bar of radius a. Is your approximate solution also an exact solution?

(b) rectangular bar with cross-section shown in Fig. 1. You can express the final solution for the constant in terms of integrals (which you need not evaluate).

2. The governing equation for the steady, full-developed flow of a Newtonian fluid down an inclined plane (see Fig. 2) is (40)

$$-\mu \frac{d^2 w}{dx^2} = \rho g \sin\beta,$$

where w is the z-component of velocity, μ is the viscosity, ρ is the density, and g is the gravitational acceleration. The boundary conditions associated with this problem are that the shear stress is zero at $x = 0$, and that the velocity is zero at $x = L$:

$$\mu \left.\frac{dw}{dx}\right|_{x=0} = 0; \quad w(L) = 0.$$

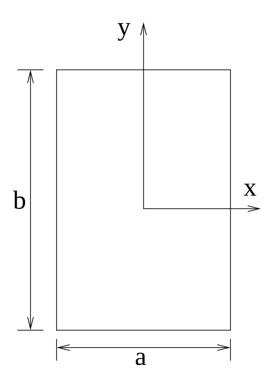

Figure 1: Rectangular bar undergoing torsion.

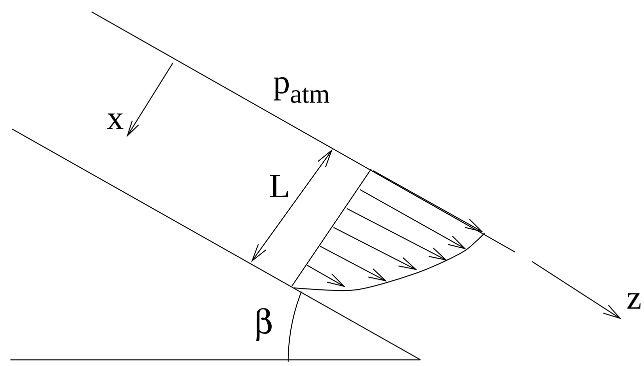

Figure 2: Flow down an incline.

(a) Find the exact solution for w using the governing differential equation and boundary conditions.

(b) Formulate the variational problem, and a finite element method based on this formulation.

(c) Using one quadratic element (with midnode at the center), find the finite element solution for w, and compare with the exact solution at $x = 0$, $x = L/2$ and $x = L$. Comment on the agreement or otherwise of the finite element solution with the exact solution on the entire domain $[0, L]$.

Some Relevant Formulae

For a quadratic 1-D element of length L with midnode at the center:

$$N_1(\xi) = 0.5(\xi^2 - \xi),\ N_2(\xi) = (1 - \xi^2),\ N_3(\xi) = 0.5(\xi^2 + \xi),\ dx = \frac{L}{2} d\xi,$$

$$\int_0^L \boldsymbol{N}^T \boldsymbol{N}\, dx = \frac{L}{30} \begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix},$$

$$\int_0^L \frac{d\boldsymbol{N}^T}{dx} \frac{d\boldsymbol{N}}{dx}\, dx = \frac{1}{3L} \begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}.$$