Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 23/2/2013.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. We saw in ME242 that certain systems are governed by the biharmonic equation on a two-dimensional domain. Given $q(x, y)$, consider the biharmonic equation (60)

$$\boldsymbol{\nabla}^4 w = q(x, y) \quad \text{on } \Omega, \tag{1}$$

where, for example,

$$\boldsymbol{\nabla}^4 w \equiv \boldsymbol{\nabla}^2(\boldsymbol{\nabla}^2 w) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right),$$

in a Cartesian coordinate system. With w_δ denoting the variation of w, carry out the variational formulation for the governing equation in Eqn. (1) on the domain Ω. Determine the *allowable* essential and natural boundary conditions on the boundary C of Ω. Note that Ω is *not necessarily* rectangular in shape. Your final answers for the variational formulation and boundary conditions must be in tensorial form.

Assuming the boundary conditions prescribed on the entire boundary to be of the natural type (choose any nonzero value such as $\bar{f}$ or $\bar{g}$ for these conditions that you wish), formulate the potential energy functional (if such a functional exists) in tensorial form.

Now consider the rectangular domain shown in Fig. 1, which has essential boundary conditions prescribed to be zero over the entire boundary. Use the above variational formulation that you have developed to find a one-parameter Rayleigh-Ritz solution for w. You can express the final solution for the constant in terms of integrals (which you need not evaluate).

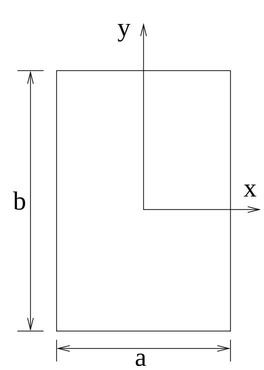

Figure 1: Rectangular domain with prescribed essential boundary conditions on its edges.

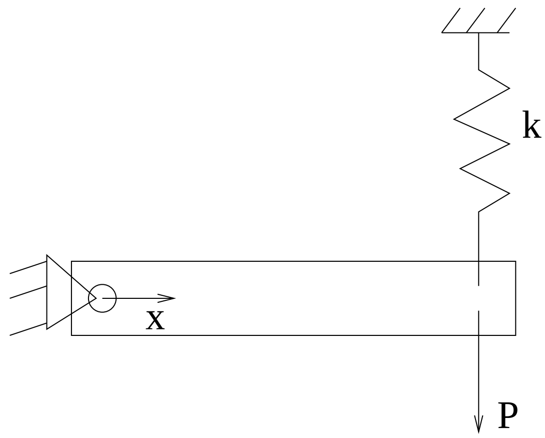

Figure 2:

2. Write the potential energy for the setup shown in Fig. 2, where the member is supported by a pin joint at the left, and a spring of spring constant k at the right. Assume the Young modulus to be E, cross sectional area to be A, moment of inertia to be I, length to be L, and $d\Omega = A\,dx$ (1D approximation). (40)

   (a) Develop the finite element equations either by developing the variational formulation from the potential energy expression and then discretizing, or by minimizing the discretized version of the potential energy.

   (b) Find the horizontal and vertical displacement at the point of application of the load P. The answer must be derived using the finite element equations alone. *Answers based on 'intuition', even if correct, are not admissible.* Use minimum number of finite elements while discretizing. You may directly use the results at the end of the question paper without writing expressions for the shape functions.

Some Relevant Formulae

For a linear 1-D element of length L:

$$\boldsymbol{K} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

For a beam element of length L:

$$\tau_{xx} = -\frac{My}{I} = E\epsilon_{xx}.$$

$$\boldsymbol{K} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$