# Indian Institute of Science, Bangalore

# ME 257: Midsemester Test

**Date:** 24/2/2014.
**Duration:** 11.15 a.m.–1.00 p.m.
**Maximum Marks:** 100

1. The compatibility relations are given by (60)

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{\epsilon}) = \boldsymbol{0},$$

where

$$[\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{\epsilon})]_{ij} = \epsilon_{imn}\epsilon_{jst}\frac{\partial^2 \epsilon_{nt}}{\partial x_m \partial x_s}. \tag{1}$$

Substituting the constitutive relation $\boldsymbol{\epsilon} = (1+\nu)\boldsymbol{\tau} - \nu(\mathrm{tr}\,\boldsymbol{\tau})\boldsymbol{I}$ (taking $E = 1$) into the above relation, write the compatibility relations in terms of $\boldsymbol{\tau}$. You may use indicial notation, but the final result should be in tensorial form.

Next, develop the variational formulation for the stress compatibility relation that you have derived. Denote the variation of $\boldsymbol{\tau}$ by $\boldsymbol{\tau}_\delta$, and assume both $\boldsymbol{\tau}$ and $\boldsymbol{\tau}_\delta$ to be symmetric. Assume $\boldsymbol{\tau}_\delta(\boldsymbol{\nabla} \times \boldsymbol{\tau}) = \boldsymbol{0}$ on the boundary $\Gamma$. Your final variational formulation should be expressed in tensorial form. Use

$$(\boldsymbol{\nabla} \times \boldsymbol{\tau})_{ij} = \epsilon_{irs}\frac{\partial \tau_{js}}{\partial x_r}, \tag{2}$$

to derive the variational formulation. (Hint: The final variational form may or may not be symmetric).

2. A thick-walled circular cylinder is subjected to pressure loading on the surface $r = r_1$ (40) as shown in Fig. 1. Assuming $u_r = u_r(r)$, and $u_\theta = u_z = 0$, the governing equation is

$$\frac{d\tau_{rr}}{dr} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0.$$

(a) Develop the variational formulation based on the above equation (Hint: Combine the $\tau_{rr}$ terms under one derivative sign). Incorporate the boundary conditions in this formulation. Take the length of the cylinder along $z$ to be unity.

(b) Using the relations

$$\epsilon_{rr} = \frac{du_r}{dr}; \quad \epsilon_{\theta\theta} = \frac{u_r}{r}, \quad \text{(other strains zero)}$$
$$\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon}) + 2\mu\boldsymbol{\epsilon},$$

write the variational formulation in the form $\boldsymbol{\epsilon}_c^T(\boldsymbol{u}_\delta)\boldsymbol{\tau}_c$, where

$$\boldsymbol{\tau}_c = \begin{bmatrix} \tau_{rr} \\ \tau_{\theta\theta} \end{bmatrix} = \boldsymbol{C}\begin{bmatrix} \epsilon_{rr} \\ \epsilon_{\theta\theta} \end{bmatrix} = \boldsymbol{C}\boldsymbol{\epsilon}_c,$$

and where $\boldsymbol{C}$ is a matrix that you have to find in terms of $\lambda$ and $\mu$.

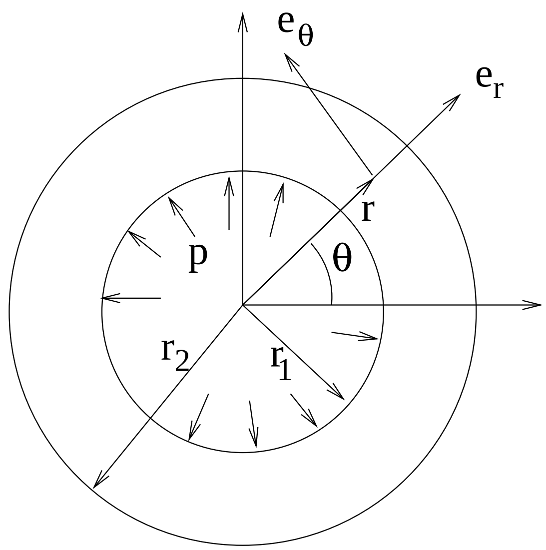


Figure 1:

(c) Using one linear element (two nodes), formulate the stiffness matrix in terms of the strain-displacement matrix $\boldsymbol{B}$ (give the expression for this matrix in natural coordinates), and the load vector for solving the pressure vessel problem. The formulation should be done using natural coordinates. *Do not* evaluate any integrals that arise (just give the mathematical expressions). Will the stiffness matrix be singular? Justify without actually finding the stiffness matrix. (In other words, can you directly solve for the two nodal displacements using $\boldsymbol{K}\hat{\boldsymbol{u}} = \boldsymbol{f}$.)

(d) Find the exact solution for $u_r$ by solving

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(ru_r)}{dr}\right] = 0,$$

and using appropriate boundary conditions.