Indian Institute of Science, Bangalore

ME 257: Midsemester Test

Date: 25/2/2015.
Duration: 12.00 noon.–1.30 p.m.
Maximum Marks: 100

1. The governing equation for a symmetric second-order tensor $\boldsymbol{\epsilon}$ is given by (60)

$$\boldsymbol{\nabla}^2\boldsymbol{\epsilon} + \boldsymbol{\nabla}[\boldsymbol{\nabla}(\operatorname{tr}\boldsymbol{\epsilon})] - \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{\epsilon}) - [\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{\epsilon})]^T = \mathbf{0},$$

   where $\boldsymbol{\nabla}^2$ is to be interpreted as '$\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}$'. Develop the variational formulation for the above governing equation. Denote the variation of $\boldsymbol{\epsilon}$ by $\boldsymbol{\epsilon}_\delta$. You may leave the boundary terms as they are without incorporating any boundary conditions. The final result should be in tensorial form. Denote the tensorial form of $\partial\epsilon_{ij}/\partial x_k$ by $\boldsymbol{\nabla}\boldsymbol{\epsilon}$. The final variational form may be unsymmetric.

2. The governing equation for the circumferential velocity u_θ through a circular channel of inner and outer radii a and b, assuming that $u_r = u_z = 0$, $u_\theta = u_\theta(r)$, is given by (40)

$$\frac{d^2u_\theta}{dr^2} + \frac{1}{r}\frac{du_\theta}{dr} - \frac{u_\theta}{r^2} = \frac{k}{\nu r},$$

   where k and ν are given constants. The boundary conditions are $u_\theta|_{r=a} = u_\theta|_{r=b} = 0$.

   (a) Develop the variational formulation based on the above equation (Hint: Combine the first two terms under one derivative sign). Incorporate the boundary conditions in this formulation. Take the length of the channel along z to be unity. Your variational formulation should be such that the associated bilinear form $a(u, v)$ is symmetric.

   (b) State the associated potential energy functional (but do not use to formulate the finite element formulation, which should be based on the variational formulation).

   (c) Using one quadratic element (with midnode at the center of the element), formulate the stiffness matrix in terms of the strain-displacement matrix $\boldsymbol{B}$ and shape function matrix $\boldsymbol{N}$ (give the expressions for these matrices in natural coordinates), and the load vector. The formulation should be done using natural coordinates. *Do not* evaluate any integrals that arise (just give the mathematical expressions). State the boundary conditions to be imposed on the nodal variables $\hat{\boldsymbol{u}}$.