ME261: Assignment 1

Due: 26/8/16

1. Using indicial notation, show that

$$\begin{aligned}
(\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) &= (\boldsymbol{a} \cdot \boldsymbol{c})(\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{a} \cdot \boldsymbol{d})(\boldsymbol{b} \cdot \boldsymbol{c}), \\
(\boldsymbol{a} \otimes \boldsymbol{b})^T &= \boldsymbol{b} \otimes \boldsymbol{a}, \\
(\boldsymbol{a} \otimes \boldsymbol{b})(\boldsymbol{c} \otimes \boldsymbol{d}) &= (\boldsymbol{b} \cdot \boldsymbol{c})\boldsymbol{a} \otimes \boldsymbol{d}, \\
(\boldsymbol{a} \otimes \boldsymbol{b}) : (\boldsymbol{u} \otimes \boldsymbol{v}) &= (\boldsymbol{a} \cdot \boldsymbol{u})(\boldsymbol{b} \cdot \boldsymbol{v}), \\
\boldsymbol{T}(\boldsymbol{a} \otimes \boldsymbol{b}) &= (\boldsymbol{T}\boldsymbol{a}) \otimes \boldsymbol{b}, \\
(\boldsymbol{a} \otimes \boldsymbol{b})\boldsymbol{T} &= \boldsymbol{a} \otimes (\boldsymbol{T}^T \boldsymbol{b}), \\
\boldsymbol{T} : (\boldsymbol{a} \otimes \boldsymbol{b}) &= \boldsymbol{a} \cdot \boldsymbol{T}\boldsymbol{b}.
\end{aligned}$$

2. Show using indicial notation that

$$\boldsymbol{R} : (\boldsymbol{S}\boldsymbol{T}) = (\boldsymbol{S}^T \boldsymbol{R}) : \boldsymbol{T} = (\boldsymbol{R}\boldsymbol{T}^T) : \boldsymbol{S} = (\boldsymbol{T}\boldsymbol{R}^T) : \boldsymbol{S}^T,$$

3. If $\boldsymbol{S}$ and $\boldsymbol{W}$ are symmetric and antisymmetric tensors, respectively, and $\boldsymbol{T}$ is an arbitrary second-order tensor prove that

$$\begin{aligned}
\boldsymbol{S} : \boldsymbol{T} = \boldsymbol{S} : \boldsymbol{T}^T &= \boldsymbol{S} : \left[\frac{1}{2}(\boldsymbol{T}^T + \boldsymbol{T})\right] \\
\boldsymbol{W} : \boldsymbol{T} = -\boldsymbol{W} : \boldsymbol{T}^T &= \boldsymbol{W} : \left[\frac{1}{2}(\boldsymbol{T} - \boldsymbol{T}^T)\right] \\
\boldsymbol{S} : \boldsymbol{W} &= 0.
\end{aligned}$$

4. Prove using indicial notation that

$$\begin{aligned}
\mathbf{cof}\,\boldsymbol{T}(\boldsymbol{u} \times \boldsymbol{v}) &= (\boldsymbol{T}\boldsymbol{u}) \times (\boldsymbol{T}\boldsymbol{v}), \\
(\mathbf{cof}\,\boldsymbol{T})^T &= \frac{1}{2}\left[(\mathrm{tr}\,\boldsymbol{T})^2 - \mathrm{tr}\,(\boldsymbol{T}^2)\right]\boldsymbol{I} - (\mathrm{tr}\,\boldsymbol{T})\boldsymbol{T} + \boldsymbol{T}^2, \\
\mathbf{cof}\,(\boldsymbol{R}\boldsymbol{S}) &= (\mathbf{cof}\,\boldsymbol{R})(\mathbf{cof}\,\boldsymbol{S}).
\end{aligned}$$

5. Prove using indicial notation that

$$\begin{aligned}
\det \boldsymbol{T} &= \det \boldsymbol{T}^T, \\
\det(\boldsymbol{R}\boldsymbol{S}) &= (\det \boldsymbol{R})(\det \boldsymbol{S}), \\
\det(\boldsymbol{R} + \boldsymbol{S}) &= \det \boldsymbol{R} + \mathbf{cof}\,\boldsymbol{R} : \boldsymbol{S} + \boldsymbol{R} : \mathbf{cof}\,\boldsymbol{S} + \det \boldsymbol{S}, \\
\epsilon_{pqr}(\det \boldsymbol{T}) &= \epsilon_{ijk} T_{ip} T_{jq} T_{kr} = \epsilon_{ijk} T_{pi} T_{qj} T_{rk}.
\end{aligned}$$