Indian Institute of Science
ME 261: Midsemester Test (Retest)

Date: 5/10/16.
Duration: 11.00 a.m.–12.30 a.m.
Maximum Marks: 100

1. Using indicial notation find an expression for $\mathbf{cof}\,\boldsymbol{W}$ in terms of $\boldsymbol{w}$. Using this expression find the eigenvalues of $\mathbf{cof}\,\boldsymbol{W}$ in terms of $\boldsymbol{w}$. (35)

2. Let (30)
$$\boldsymbol{R} = \boldsymbol{e}\otimes\boldsymbol{e} + \cos\alpha(\boldsymbol{I} - \boldsymbol{e}\otimes\boldsymbol{e}) + \sin\alpha(\boldsymbol{r}\otimes\boldsymbol{q} - \boldsymbol{q}\otimes\boldsymbol{r}),$$
where $\{\boldsymbol{e}, \boldsymbol{q}, \boldsymbol{r}\}$ forms an orthonormal basis. Determine if $\boldsymbol{R}$ is a rotation, and if it is, find its axis. Also find $(\boldsymbol{r}\cdot\boldsymbol{R}\boldsymbol{r})$ in terms of α.

3. Let $\boldsymbol{x}$ denote the position vector and t denote time, and let $\phi(\boldsymbol{x}\cdot\boldsymbol{x} - ct)$ be a scalar-valued function of $(\boldsymbol{x}, t)$ with c being a constant. Find the value of the constant k such that the following equation is satisfied (35)
$$\boldsymbol{\nabla}^2\phi + k\frac{\partial\phi}{\partial t} = \frac{4(\boldsymbol{x}\cdot\boldsymbol{x})}{c^2}\frac{\partial^2\phi}{\partial t^2}.$$

Some relevant formulae

$$\begin{aligned}
w_i &= -\frac{1}{2}\epsilon_{ijk}W_{jk},\\
W_{ij} &= -\epsilon_{ijk}w_k,\\
(\mathbf{cof}\,\boldsymbol{T})_{ij} &= \frac{1}{2}\epsilon_{imn}\epsilon_{jpq}T_{mp}T_{nq},\\
I_2(\boldsymbol{T}) &= \frac{1}{2}\left[(\operatorname{tr}\boldsymbol{T})^2 - \operatorname{tr}\boldsymbol{T}^2\right],\\
\det\boldsymbol{T} &= \epsilon_{ijk}T_{i1}T_{j2}T_{k3} = \epsilon_{ijk}T_{1i}T_{2j}T_{3k}
\end{aligned}$$