# Indian Institute of Science
# ME 261: Midsemester Test

**Date:** 6/11/16.
**Duration:** 2.30 p.m.–4.00 p.m.
**Maximum Marks:** 100

1. Our goal is to *derive* the Wronskian $W(x)$ for the Bessel equation (40)

$$x^2y'' + xy' + (x^2 - n^2)y = 0, \tag{1}$$

where $n$ is a constant. Let $\{y_1, y_2\}$ be two fundamental solutions of Eqn. (1).

   (a) Write an expression for $W(x)$ in terms of $\{y_1, y_2\}$ and their derivatives.

   (b) Using this expression find $W'(x)$ in terms of $\{y_1, y_2\}$ and their derivatives.

   (c) Using the fact that $\{y_1, y_2\}$ are fundamental solutions of Eqn. (1), simplify the above expression for $W'(x)$.

   (d) Solve the resulting ODE for $W(x)$ subject to the condition $W(1) = 2/\pi$.

   (e) *Given* one solution $y_1(x)$, find the other solution $y_2(x)$ in terms of $y_1(x)$ using the expression for $W(x)$ that you have derived above.

2. We are interested in solving the ODE (60)

$$y'' + \left(4x - \frac{1}{x}\right)y' + 4x^2y = f(x), \quad (x > 0). \tag{2}$$

It is given that the above ODE can be transformed to one with constant coefficients (you *need not* prove this).

   (a) Find the transformation $\xi = g(x)$ (with the condition $g(1) = 1$ to solve for any constants) that will transform the above ODE to one with constant coefficients. Assuming $f(x) = 0$, first find the complementary solution of the ODE with constant coefficients, and hence the ODE with variable coefficients. *Do not* use the formulae from the notes directly. Deduce everything from basic principles.

   (b) Directly using the formulae

$$u_1 = -\int \frac{fy_2\,dx}{y_1y_2' - y_1'y_2} + c_1,$$
$$u_2 = \int \frac{fy_1\,dx}{y_1y_2' - y_1'y_2} + c_2,$$

   where $y(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$, find the general solution to Eqn. (2).