Indian Institute of Science, Bangalore

UE 204: Endsemester Exam

Date: 26/4/14.
Duration: 2.00 p.m.–5.00 p.m.
Maximum Marks: 100

Note: Throughout the paper, you can neglect the transverse shear contribution to the energy. If no geometrical or material properties are given, assume A, J, I to be the area, polar moment of inertia and area moment of inertia, and E and G to be the Young modulus and shear modulus. You may directly use the formulae at the back.

1. A block in the shape of a rectangular parallelepiped with dimensions $L \times h \times b$ (see Fig. 1, where the front, top and side views are shown) is subjected to pressure p along the top surface. The Young modulus and Poisson ratio are E and ν. The block is free to move along the x direction, and is constrained by frictionless surfaces on the bottom, and the sides $z = \pm b/2$. Assuming a uniform state of stress and strain, find the stresses and strains in the block. Show the stresses on the element shown along the top surface in the side view, and draw the Mohr's circle corresponding to this state of stress. Find the maximum shear stress, and show the element (with proper orientation) on which it acts. (10)

2. A composite rod with a circular cross section of area A is constructed by gluing together a steel rod and an aluminum rod each of length L. The glued joint is weaker either than the steel or the aluminum and fails at a tensile stress of τ_u. The composite bar is fastened to two rigid supports as shown in Fig. 2 in a stress-free state at a uniform temperature T_0. Derive an expression for the temperature T_u at which the glued joint at C will fail in tension. Assume that the temperature remains uniform in the bar. The Young modulus and coefficient of thermal expansion are, respectively, E_s and α_s for steel and E_a and α_a for aluminum. You may use the energy method or otherwise. (15)

3. A beam of circular cross section of radius a bent into the shape of a quarter circle of radius R and lying in the x-y plane is subjected to a load $P\boldsymbol{e}_z$ as shown in Fig. 3. Find M_x, M_y at a section cut at an angle θ from the x-axis (It might help to use the vectorial method.). Using the transformation (25)

$$\begin{bmatrix} M_r \\ M_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix},$$

find M_r and M_θ. (Any other correct method for finding M_r and M_θ is also acceptable.) Use these moments to find the deflection under the load P.

Consider a two-dimensional element on the surface at $(x, y, z) = (0, R, a)$ with $(\pm \boldsymbol{e}_x, \pm \boldsymbol{e}_y)$ as normals to its faces, and $\boldsymbol{e}_z$ normal to the plane in which it lies, and show the stresses acting on this element. You need not draw the Mohr's circle diagram.

4. Consider the setup shown in Fig. 4. The spring has a spring constant k. The vertical bar is joined to the horizontal ones using pin joints. Apart from the lengths, all geometrical and material properties of all members (other than the spring) are identical. The original (undeflected) length of the spring before the application of the load P is L. Find the deflection under the load P. (30)

5. We saw in the test for the problem shown in Fig. 5 that the pressure is given by $p = \rho g z$, where ρ is the density of the fluid and g is the gravitational acceleration. Assume the width of the beam into the paper as unity. Using the relation (20)

$$EI \frac{d^4 v}{dx^4} = q,$$

find the deflection of the cantilever beam shown in Fig. 5 in a direction perpendicular to $\bar{x}$, and as a function of $\bar{x}$. *Do not use any other method for finding the deflection.*

Some relevant formulae

$$\epsilon_{xx} = \frac{1}{E}\left[\tau_{xx} - \nu(\tau_{yy} + \tau_{zz})\right], \qquad \gamma_{xy} = \frac{\tau_{xy}}{G},$$
$$\epsilon_{yy} = \frac{1}{E}\left[\tau_{yy} - \nu(\tau_{zz} + \tau_{xx})\right], \qquad \gamma_{xy} = \frac{\tau_{yz}}{G},$$
$$\epsilon_{zz} = \frac{1}{E}\left[\tau_{zz} - \nu(\tau_{xx} + \tau_{yy})\right], \qquad \gamma_{xy} = \frac{\tau_{xz}}{G}.$$

$$\int_0^{\pi/2} \sin^2\theta \, d\theta = \frac{\pi}{4},$$
$$\int_0^{\pi/2} \cos^2\theta \, d\theta = \frac{\pi}{4},$$
$$\int_0^{\pi/2} (1 - \cos\theta)^2 \, d\theta = \frac{3\pi}{4} - 2,$$
$$\int_0^{\pi/2} (1 - \sin\theta)^2 \, d\theta = \frac{3\pi}{4} - 2.$$

$$\Pi^* = \frac{P^2 L}{2EA} + \alpha P L \Delta T - P u,$$

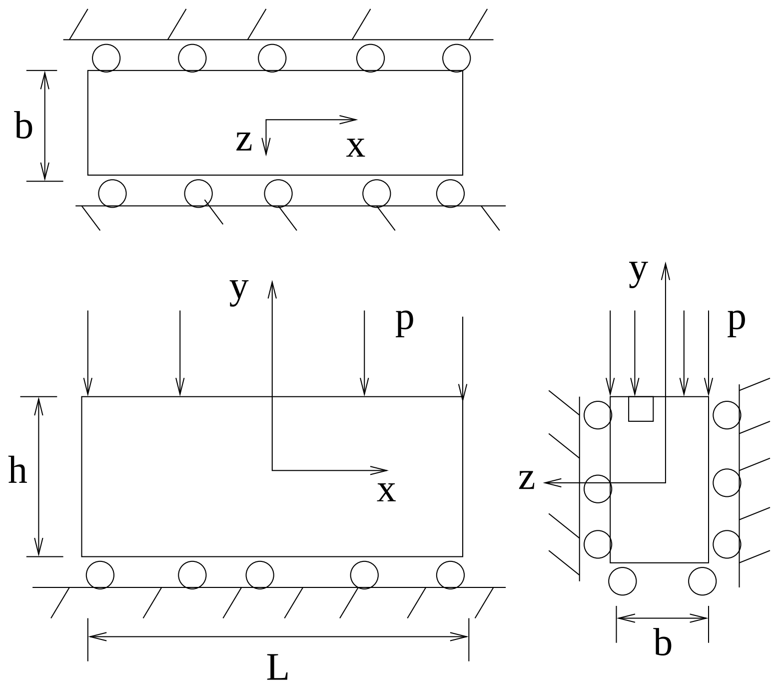

Figure 1: Problem 1.

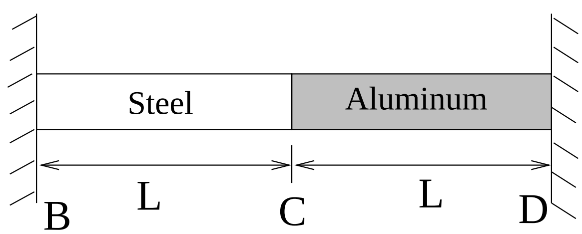

Figure 2: Problem 2.

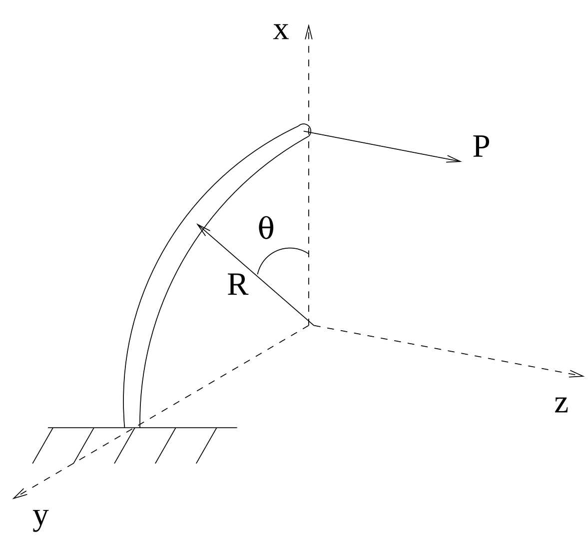

Figure 3: Problem 3.

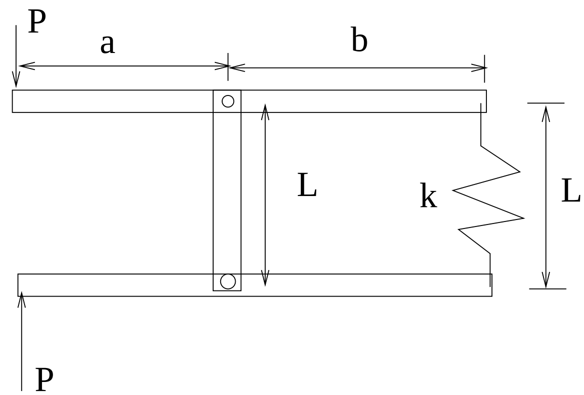

Figure 4: Problem 4.

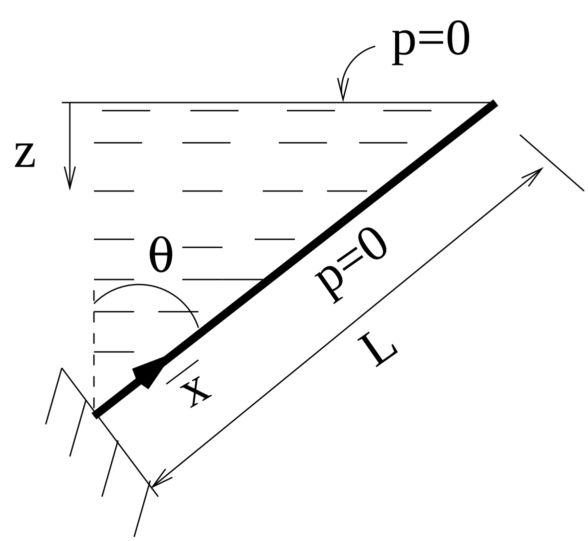

Figure 5: Problem 5.