Indian Institute of Science, Bangalore

UE 204: Endsemester Exam

Date: 24/4/15.
Duration: 9.00 a.m.–12.00 noon
Maximum Marks: 100

Note: Throughout the paper, you can neglect the transverse shear contribution to the energy. If no geometrical or material properties are given, assume A, J, I to be the area, polar moment of inertia and area moment of inertia, and E and G to be the Young modulus and shear modulus. You may directly use the formulae at the back.

1. A thin circular disc of inner and outer radii a and b, is subjected to pressure p on its outer edge $r = b$, and is bonded to a rigid inclusion at its inner edge $r = a$ as shown in Fig. 1. Assuming the displacements and stresses to be given by (15)

$$u_r = \frac{1}{E}\left[-(1+\nu)\frac{A}{r} + 2(1-\nu)Br\log r - B(1+\nu)r + 2C(1-\nu)r\right],$$
$$u_\theta = \frac{4}{E}Br\theta,$$
$$\tau_{rr} = \frac{A}{r^2} + B(1+2\log r) + 2C,$$
$$\tau_{\theta\theta} = -\frac{A}{r^2} + B(3+2\log r) + 2C,$$
$$\tau_{r\theta} = 0,$$

find the constants A, B and C. Take a 2D element at the inner radius $r = a$, and show the stresses on it for $p = 1$, $a = 1$, $b = 2$, $\nu = 0.25$ and $E = 1$. Find the principal stresses using the Mohr's circle diagram.

2. We considered in the test the following problem of a frame that has pin joints at A, B and D, and is subjected to a vertical load P as shown in Fig. 2. Assuming the material and geometric properties (except the length) for the two members to be identical and assuming $b = a$, find the deflection δ under the load P. (25)

3. A beam bent into the shape of a semi circle of radius a lies in the x-y plane, and is subjected to a load $P\boldsymbol{e}_z$ as shown in Fig. 3. The ends $\theta = \pm\pi/2$ of the beam are fixed. The goal is to find the deflection of the beam under the load P. (30)

   (a) With a view towards applying Castigliano's theorem, find the appropriate energy in terms of internal and external forces. This energy should

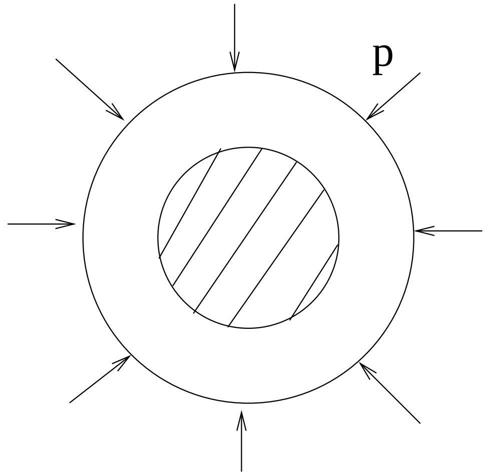

Figure 1: Thin circular disc subjected to pressure on the outer surface, and bonded to a rigid inclusion at its inner surface.

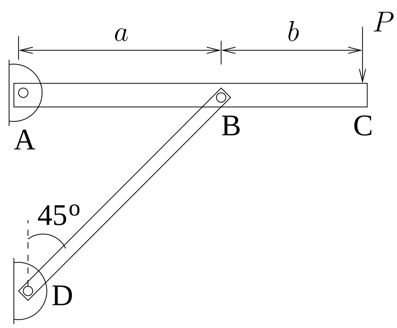

Figure 2: Problem 2.

be explicitly stated (i.e., all integrals or derivatives in the expressions should be evaluated using the formulae at the back). While solving for the unknowns that enter into the energy equation, it may help to express them in matrix form such as $\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$, where $\boldsymbol{u}$ is a vector of unknowns, and then find $\boldsymbol{u} = \boldsymbol{K}^{-1}\boldsymbol{f}$.

(b) Using this energy expression, state the equations for finding the deflection. For example, if Π^* is your energy and F is an internal force, then state an equation such as $\partial \Pi^*/\partial F = 0$ without simplifying this equation. Similarly, state all other equations that on solving will allow one to find the deflection. *You need not solve these equations, but you need to state them.*

4. Consider the setup shown in Fig. 4a, with a load P applied at the center of a double-cantilever beam, and springs of spring constant k at a distance of a from the walls. The springs are undeformed before the application of the load P. Using the 'superposition' method, find the deflection under the load P. *You need not simplify lengthy expressions, but you need to state them.* (30)

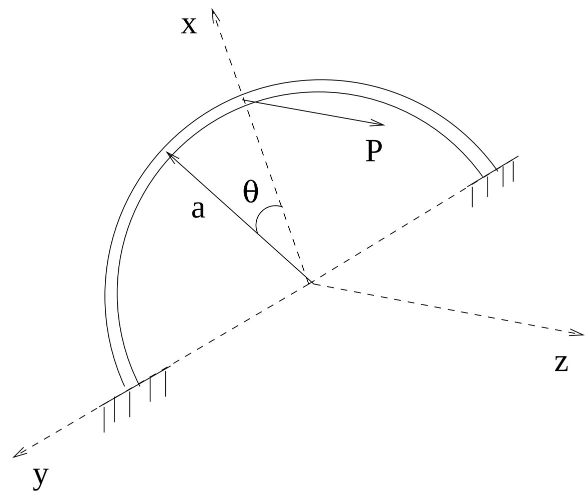

Figure 3: Problem 3.

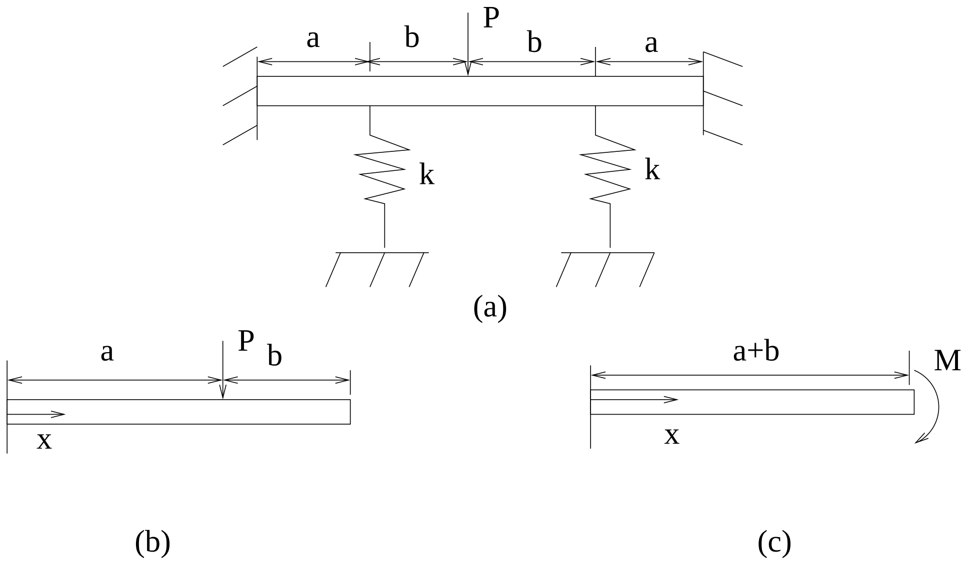

Figure 4: Problem 4.

For applying this method, you may directly use the following formulae for the deflection and the slope:

Cantilever beam of length $a + b$ with a point load P applied at $x = a$ (see Fig. 4b)

$$\begin{aligned} \delta(x) &= \frac{P}{6EI}(3x^2a - x^3), && 0 \leq x \leq a, \\ &= \frac{P}{6EI}(3xa^2 - a^3), && a \leq x \leq a + b, \\ \phi|_{x=a+b} &= \frac{Pa^2}{2EI}. \end{aligned}$$

Cantilever beam of length $a + b$ with a moment M applied at $x = a + b$ (see Fig. 4c)

$$\begin{aligned} \delta(x) &= \frac{Mx^2}{2EI}, \\ \phi|_{x=a+b} &= \frac{M(a+b)}{EI}. \end{aligned}$$

Some relevant formulae

$$\begin{aligned} \int_0^{\pi/2} \sin\theta \, d\theta &= \int_0^{\pi/2} \cos\theta \, d\theta = 1, \\ \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta &= \frac{1}{2}, \\ \int_0^{\pi/2} \sin^2\theta \, d\theta &= \int_0^{\pi/2} \cos^2\theta \, d\theta = \frac{\pi}{4}. \end{aligned}$$