# Indian Institute of Science, Bangalore

# UE 204: Endsemester Exam

**Date:** 25/4/16.
**Duration:** 2.00 p.m.–5.00 p.m.
**Maximum Marks:** 100

Note: Throughout the paper, you can neglect the transverse shear contribution to the energy. If no geometrical or material properties are given, assume $A$, $J$, $I$ to be the area, polar moment of inertia and area moment of inertia, and $E$ and $G$ to be the Young modulus and shear modulus. You may directly use the formulae at the back.

1. A uniform pressure $p$ is applied to the lateral surface of a solid circular cylinder of radius $R$ and length $L$. (15)

   (a) Assuming the stresses to be uniform throughout the domain, the top and bottom surfaces to be traction free, and using symmetry considerations and the equilibrium equations given by

$$
\begin{aligned}
0 &= \frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r}, \\
0 &= \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r}, \\
0 &= \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\tau_{rz}}{r},
\end{aligned}
$$

   find the stresses in the cylinder.

   (b) Using the above solution, state (without proving) the stress and strain component matrices at $(r, \theta, z)$ for the problem shown in Fig. 1 (where $T$ denotes the torque), i.e.,

$$
\begin{bmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{r\theta} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{rz} & \tau_{\theta z} & \tau_{zz} \end{bmatrix}_{(r,\theta,z)} = ?, \qquad \begin{bmatrix} \epsilon_{rr} & \epsilon_{r\theta} & \epsilon_{rz} \\ \epsilon_{r\theta} & \epsilon_{\theta\theta} & \epsilon_{\theta z} \\ \epsilon_{rz} & \epsilon_{\theta z} & \epsilon_{zz} \end{bmatrix}_{(r,\theta,z)} = ?
$$

2. In the setup shown in Fig. 2, find the deflection under the load $P$. All joints may be assumed to be pin-joints. Also assume that all elements such as the beams and the spring are unstressed before the application of the load. (25)

3. A thin wire of cross sectional area $A_0$ is tied to the two ends of a semicircular beam of radius $R$ along its diameter as shown in Fig. 3. Note that the *initial* shape (i.e., before the application of any load) of the beam is semicircular. The cross sectional area of this semicircular beam is $A$. There is no tension (35)

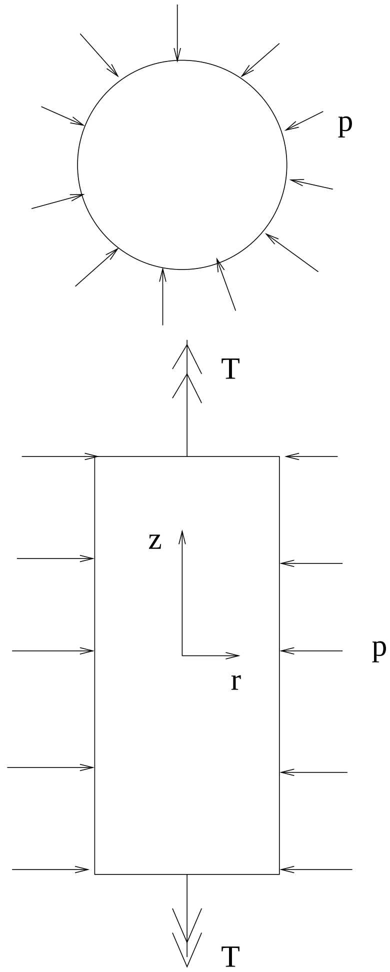


Figure 1: Part (b) of Problem 1.

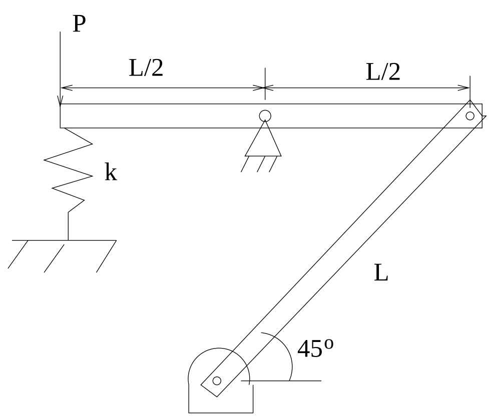


Figure 2: Problem 2.

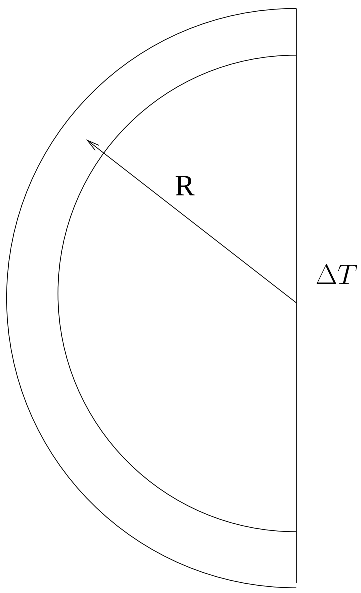


Figure 3: Problem 3.

and no ‘slack’ in the wire after it has been tied. The temperature of the wire (and the wire only) is now reduced by $\Delta T$. Find the stress generated in the wire.

4. A circular bar of radius $R$ is fixed to a wall at one end, and has a beam of length $2L$ welded to it at its center at other end as shown by the dark circle in Figure 4. A moment $M$ is applied at each of the ends of this beam as shown. Assuming that the welded joint acts as a ‘cantilever’ type support at the center, find the deflection and the slope at the ends where the moment $M$ is applied. You may directly use the formulae below. Next consider an element at the top of the circular cross section at the wall (i.e., at point A in the figure). The sides of this element are parallel and perpendicular to the axis of the cylinder. Show the stress state on this element, and then using a Mohr circle diagram, find the maximum and minimum principal stresses at this point. (25)

# Some relevant formulae

$$\Pi^* = \frac{P^2 L}{2EA} + \alpha P L \Delta T - P u,$$

Cantilever beam of length $L$ with a moment $M$ applied at $x = L$:

$$v\big|_{x=L} = \frac{ML^2}{2EI},$$
$$\left.\frac{dv}{dx}\right|_{x=L} = \frac{ML}{EI}.$$

$$\int_0^{\pi/2} \sin^2\theta\, d\theta = \int_0^{\pi/2} \cos^2\theta\, d\theta = \frac{\pi}{4}.$$

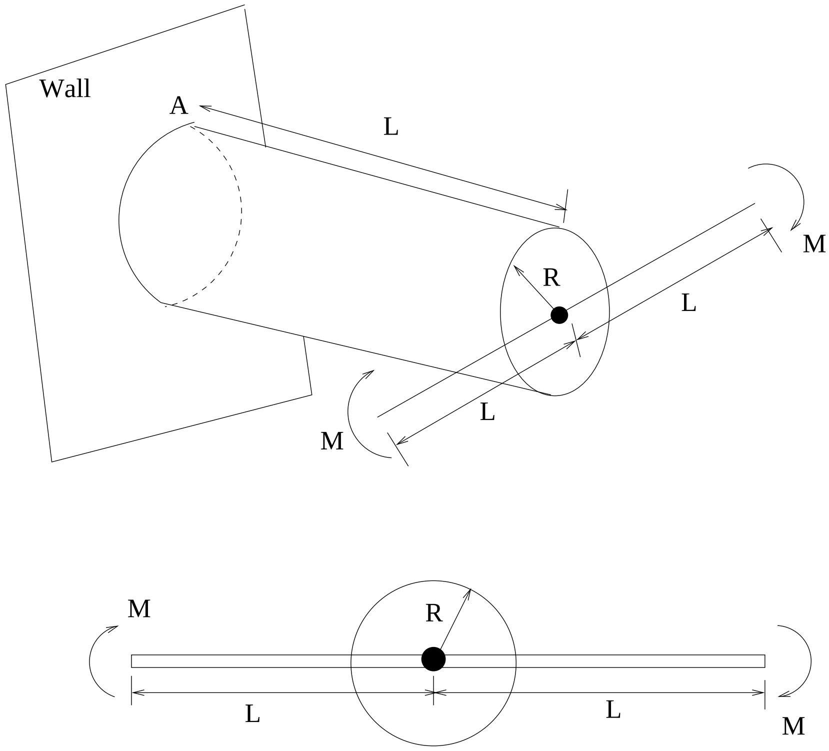


Figure 4: Problem 4.