Indian Institute of Science

UE 204: Midsemester Test

Date: 25/3/16.
Duration: 9.30 a.m.–10.30 a.m.
Maximum Marks: 100

1. A *thick-walled* hollow sphere of inner radius a and outer radius b is subjected to a uniform pressure load p at its inner surface $r = a$ as shown in Fig. 1. The outer surface $r = b$ is in contact with a smooth rigid surface so that it cannot move radially outward. Our goal is to find the displacement and stress distribution in the sphere as a function of the spherical coordinates r-θ-ϕ. For this situation, the equations of equilibrium reduce to

$$\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial r} = 0,$$
$$\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial \theta} = 0,$$
$$\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial \phi} = 0,$$

where

$$\operatorname{tr}\boldsymbol{\epsilon} = \frac{1}{r^2}\frac{\partial(r^2 u_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial\phi}.$$

The stress–displacement relations are given by

$$\tau_{rr} = \lambda(\operatorname{tr}\boldsymbol{\epsilon}) + 2\mu\frac{\partial u_r}{\partial r},$$
$$\tau_{\theta\theta} = \lambda(\operatorname{tr}\boldsymbol{\epsilon}) + \frac{2\mu}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right),$$
$$\tau_{\phi\phi} = \lambda(\operatorname{tr}\boldsymbol{\epsilon}) + \frac{2\mu}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_\phi}{\partial\phi} + u_r + u_\theta\cot\theta\right),$$

with the remaining stresses zero.

Using appropriate symmetry arguments and boundary conditions, find the displacements and stresses as a function of (r, θ, ϕ) and λ and μ.

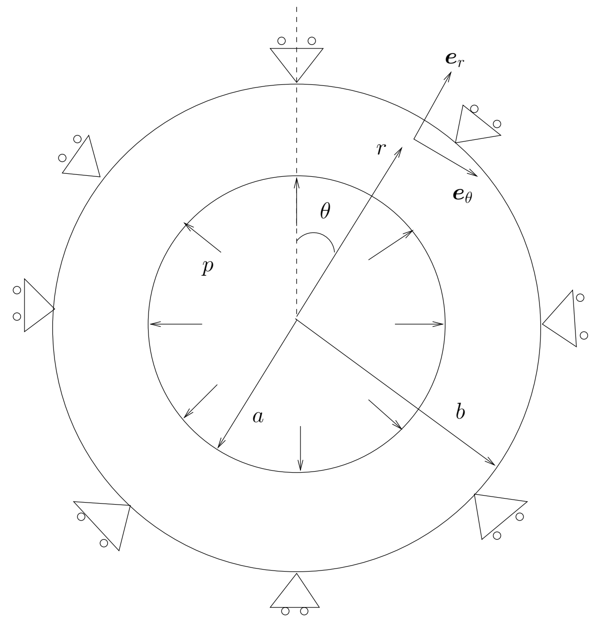

Figure 1: Problem 1.