ME304: Applied Maths for Mechanics
Part II: Asymptotic expansions
Problem Set 1 March 5, 2023

Many of these questions are open-ended. Some might lead to fruitful research problems. As always, they draw on material discussed during our meetings but take things a few notches higher. If you find yourself tied up in knots, please use one of the many references cited during our meetings. They may provide useful clues, even though these problems were made up by us.

Problem 1: Expansions

Determine uniformly valid asymptotic expansions for the following functions:

- $f(x) = \left(\frac{x+\epsilon\exp(-x/\epsilon)}{x+\epsilon+\epsilon x^2}\right)^{1/2}$
- $f(x) = \cosh(\epsilon x) - 1 + \frac{\epsilon(1+x)}{x^2+\epsilon^2}$

Please plot the original function along with the uniformly valid expansion as a function of x for various values of ϵ.

Problem 2: Bessel functions

Find a series solution for the Bessel equation of order ν given by:

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0 \tag{1}$$

using the Frobenius method discussed in class. Following this, find an asymptotic solution for the resulting Bessel function $J_\nu(x)$ as $x \to \infty$. This is done by first changing variables to $z = \sqrt{x}y$ and then seeking a solution of the form

$$z(x) \sim \exp(\omega x) \sum a_n x^{\lambda - n} \tag{2}$$

where ω and λ are unknown constants. Compare this with the solution obtained using Frobenius series and with the asymptotic form of the Bessel function found in handbooks

$$J_\nu(x) \sim \frac{C}{\sqrt{x}} \cos(x + \alpha) \tag{3}$$