Here are the topics for the final term paper for ME304. Please prepare a short report ($\leq$ 5 pages) outlining the primary ideas and method of solution. Do not include figures unless they are your own and generated specifically for the report. This will be followed by an $O(1)$ hour chalk-talk. In each case, try to use the notation we've developed in class.

Topic 1: Motion of vortices in the Ginzburg-Landau equation

The dynamics of vortices evolving under the G-L equation (a reaction-diffusion type PDE) is similar in spirit to our discussion of vortex evolution under potential flow. Please see Pismen and Rubinstein, *Physica D*, 1991 for a detailed discussion of vortex dynamics using matched asymptotic expansions, apart from drawing on our early discussion of complex variables. You can present the entireity of this manuscript.

Topic 2: Dip coating

There is a famous theory for determining the thickness of a liquid film formed during dip coating of a rectangular plate, see Landau and Levich, *Acta Physicochimica URSS*, 1942. The exposition contains a classical (and among one of the early) applications of the method of matched asymptotic expansions and you should be able to comprehend it using ideas we've discussed in class.

Topic 3: Moffatt eddies

The occurrence of eddies or local vortices in Stokes flow within a wedge is similar to the elastic wedge problem we discussed in class. Read through the original paper (Moffatt, *Journal of Fluid Mechanics*, 1964) and present the main results.

Topic 4: Lubrication of rough surfaces

The treatment of hydrodynamic lubrication of rough surfaces requires an application of the multiple-scale method discussed in class, followed by an averaging procedure. This is discussed by Elrod, *Journal of Lubrication Technology*, 1973.

Topic 5: Boundary layer flow

Boundary layer theory was first introduced (as we all perhaps know too well) by Prandtl to describe nearly inviscid flow. Starting from the full Navier-Stokes equations, develop a method to show that the boundary layers you'd usually see described for flow past a flat plate possess the same structure as the ODEs we described in class. Derive the nature of the boundary layer as a result. Any standard text on fluid mechanics should be of use.

Topic 6: Needle business

In the article by Lo, *Journal of Fluid Mechanics*, 1983, you will find a clear description of the method of matched asymptotic expansions as well as some potential pitfalls. Please present the entire paper. Try your best to work out the higher order algebra as well.

Topic 7: Flow past a sphere/cylinder

Higher order approximations to the singular perturbation problem of Stokes flow past a sphere/cylinder are determined in the paper by Proudman and Pearson, *Journal of Fluid Mechanics*, 1956 using matched asymptotic expansions. You can present either the flow past a sphere (Sec. 3) or a cylinder (Sec. 4) along with the rest of the paper.

Topic 8: Lighthill's method

One technique that is often useful is to perform so-called 'coordinate perturbations' , i.e., the introduction of an expansion in the independent variable (Π_1 in our original notation) in terms of an as-yet undetermined independent variable. This was originally described in a general form by Lighthill (building on work by Poincaré) in Lighthill, *Philosophical Magazine*, 1949.

Topic 9: The method of renormalization

A generalization of the Lighthill method is commonly referred to as renormalization and is described in Pritulo, *Journal of Applied Mathematics and Mechanics*, 1963. Present the entire manuscript, the applications to PDEs at the end can be skipped if necessary.

Topic 10: Kramers-Kronig and Bode plots

The relation between causality, KK relations and Bode plots (commonly used in dynamics and vibration) is discussed in the article by Bechhoefer, *American Journal of Physics*, 2011. Please present the entire paper, with special focus on Sec. VI since the initial sections have already been covered in class.

Topic 11: Asymptotics in fracture

Matched asymptotic analysis finds use in problems that frequently arise in fracture mechanics. A succinct presentation is provided by Willis, *International Journal of Fracture*, 1999. Please present all but Sec. 4 of this manuscript (involving the dynamic part).

Topic 12: Complex functions for Hele-Shaw flows

A technique exploiting complex functions and conformal mapping for analyzing Hele-Shaw flows is presented by Howison, *Journal of Fluid Mechanics*, 1986. Since we haven't discussed complex conformal maps explicitly in class, focus on the method and rederive the primary results presented in this paper.

Topic 13: 2D bubble/drop dynamics

A technique somewhat similar to that in the previous topic is used to solve for the dynamics of a bubble/drop due to circulation by Crowdy, *Physics of Fluids*, 1999. This draws on some ideas from our discussion of point vortices in class. Present the entire paper, but restrict attention to the case of 2 simple poles (as the author does).

Topic 14: Taylor scraper

Solve the trivial looking Taylor-scraper problem described in Taylor, 1960. Present the complete solution and think of some special cases where this may be applied.

Topic 15: Contact line dynamics

The dynamics of wetting of a solid surface by a fluid film is described by Voinov, *Fluid Mechanics*, 1977. The main results can be derived using the thin film formulation we discussed in class. You may find the discussion in Sec. 15.2.1 of Eggers and Fontelos to be useful. Derive the results in the latter reference and compare them to the ones in the paper by Voinov.