Static Failure criteria: Part I

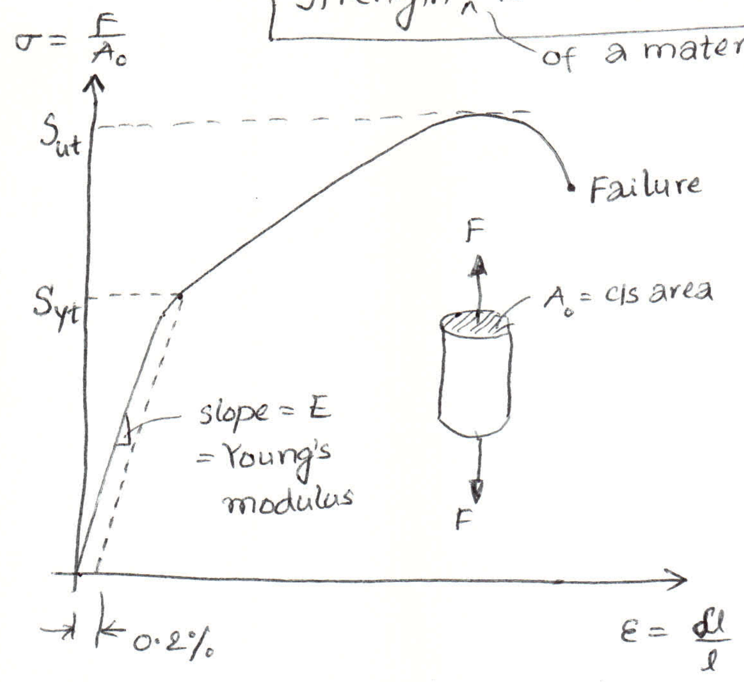
Static loads are loads that do not vary with time or loads that are applied so slowly that none of the dynamic effects (e.g., inertia loads) are significant. Failure of mechanical components under static loads is called static failure.

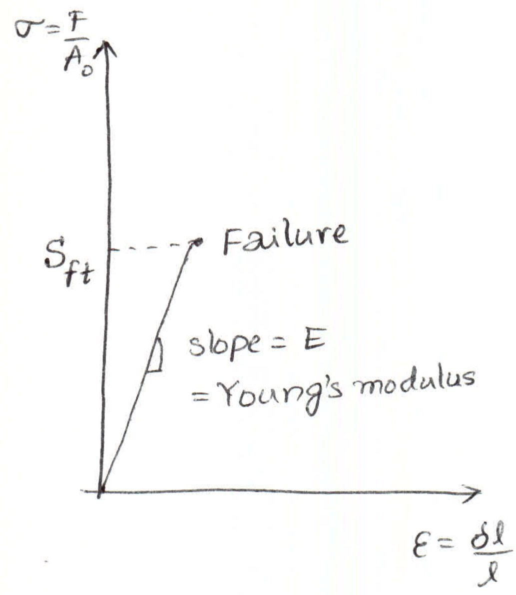
Failure in mechanical components can occur due to many reasons: material failure as in yielding or breaking into pieces; buckling; excessive wear; excessive vibrations, etc. Here, we will concern ourselves with the first type, namely, material failure.

Material failure is directly related to intrinsic property of a material of the component, called "material strength". Most commonly, material strength is measured in the laboratory under controlled conditions. The most common test is tensile loading test to obtain strength of a material. Materials can be broadly classified into <u>ductile</u> and <u>brittle</u> materials. Simply put, ductile materials show excessive local deformation before breaking into pieces whereas brittle materials simply break without showing significant local deformations. For instance, metals are ductile and ceramics are brittle.

Consider typical stress vs. strain curves for ductile and brittle material specimens under tensile loading. Both have a limited range in which they return to original state when unloaded. This is called the elastic range. For brittle materials, the curve ends abruptly at the end of elastic range. See figures.

$\boxed{\text{Strength of a material is stress at which failure occurs}}$





Ductile

Brittle

S_{yt} = yield strength in tension

S_{ut} = ultimate strength in tension

S_{ft} = Fracture strength in tension

Note 1 In compression, ductile material strengths are essentially the same. That is, $S_{yc} = S_{yt}$ and $S_{ut} = S_{uc}$. For brittle materials, S_{fc} is usually 10 to 15 times larger than S_{ft}.

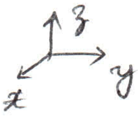
Note 2 The strain at which yielding is said to occur is typically 0.2%, i.e., $\varepsilon = 0.002$.

Note 3 This test also gives, E, the Young's modulus.

The fundamental hypothesis for formulating failure criteria for components is that the component fails at some point inside it when stress there equals the stress at which the laboratory test specimen of the same material fails.

The question then is, which stress at a point in the component or the test specimen?

The tensile test specimen has one stress, namely, normal stress along the axis. A point in a component, in general, can have six stresses, namely, σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx}. As we discussed, the values of these are dependent on the choice of coordinate system x-y-z.



Since coordinate system is chosen in any desired manner, we need a stress that doesn't depend upon the coordinate system. There are several ways to define such a stress. Each leads to a failure criterion. First, for ductile materials, we will discuss three criteria.

1) Maximum normal stress criterion
2) Maximum shear stress criterion
3) (Maximum) von Mises stress criterion

Ductile, static failure criterion I : Max. normal stress

We discussed that principal stress, σ_1, σ_2, and σ_3, are independent of the coordinate system. Note also that we order them as $\sigma_1 > \sigma_2 > \sigma_3$. With this, σ_1 is the maximum normal stress and σ_3 is the minimum normal stress. Minimum stress matters when loading is compressive.

So, we require

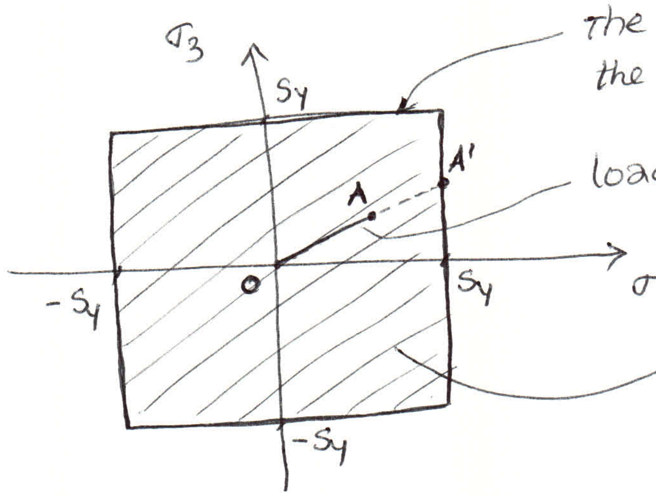
$$\underbrace{\begin{aligned}\sigma_1 &< S_{yt}\\ \sigma_3 &> -S_{yc}\end{aligned}}_{\text{Yielding}} \qquad \underbrace{\begin{aligned}\sigma_1 &< S_{ut}\\ \sigma_3 &> -S_{uc}\end{aligned}}_{\text{Failure}}$$

<u>Note</u> S_{yc} = a positive number indicating compressive strength

$\because$ $S_{yc} = S_{yt}$, we will simply write both as S_y from now onwards.

For ductile materials, we always design to lie below S_y.

We can display this result graphically as follows.



The edges of the square constitutes the failure curve.

load line

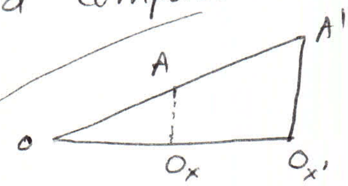
All of this cross-hatched area is <u>safe zone</u> under this criterion.

If point A indicates the values of σ_1 and σ_3 for a critical point inside a component, then the component is safe.

A <u>critical</u> point is the point inside a component at which the stress is maximum.

$$\boxed{\text{Factor of safety} = \frac{OA'}{OA} = \frac{S_y}{\sigma_1}}$$

How safe is it when it is safe?



From similar triangles,

$$\frac{OA'}{OA} = \frac{O_x'}{O_x} = \frac{S_y}{\sigma_1}$$

Ductile, static failure criterion II : Max. shear stress

If you think about how materials fail, it is usually
the shear that causes the failure. Some controlled
experiments indicate that if you apply pure normal
stress that is equal in all direction on a body, the
body can withstand stresses far beyond the strength
for that material. So, using maximum shear stress
makes more sense, and as we will see, we will be
more <u>conservative</u> if we use this criterion over the
previous one.

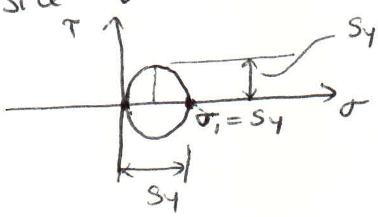
Recall that three extreme shear stresses
exist.

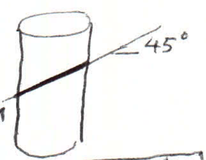
$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} \; ; \quad \tau_{12} = \frac{\sigma_1 - \sigma_2}{2} \; ; \quad \tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

Again, because of our ~~de~~ ordering $\sigma_1 > \sigma_2 > \sigma_3$,
τ_{13} will be the largest of the three. So, let us
concern ourselves with σ_1 and σ_3 plane once again.
(in displaying the safe zone under this criterion).
Before that, we have to answer the question: <u>what is the shear strength?</u> (to compare τ_{13} with).

Let us draw Mohr's circle for the state of stress in
a tensile test specimen.

S_{ys} = max. shear stress at 45° angle
It occurs in this plane





Since, stress at which failure occurs is strength, $S_{ys} = \frac{S_y}{2}$ = shear strength

Therefore, under this criterion, we require

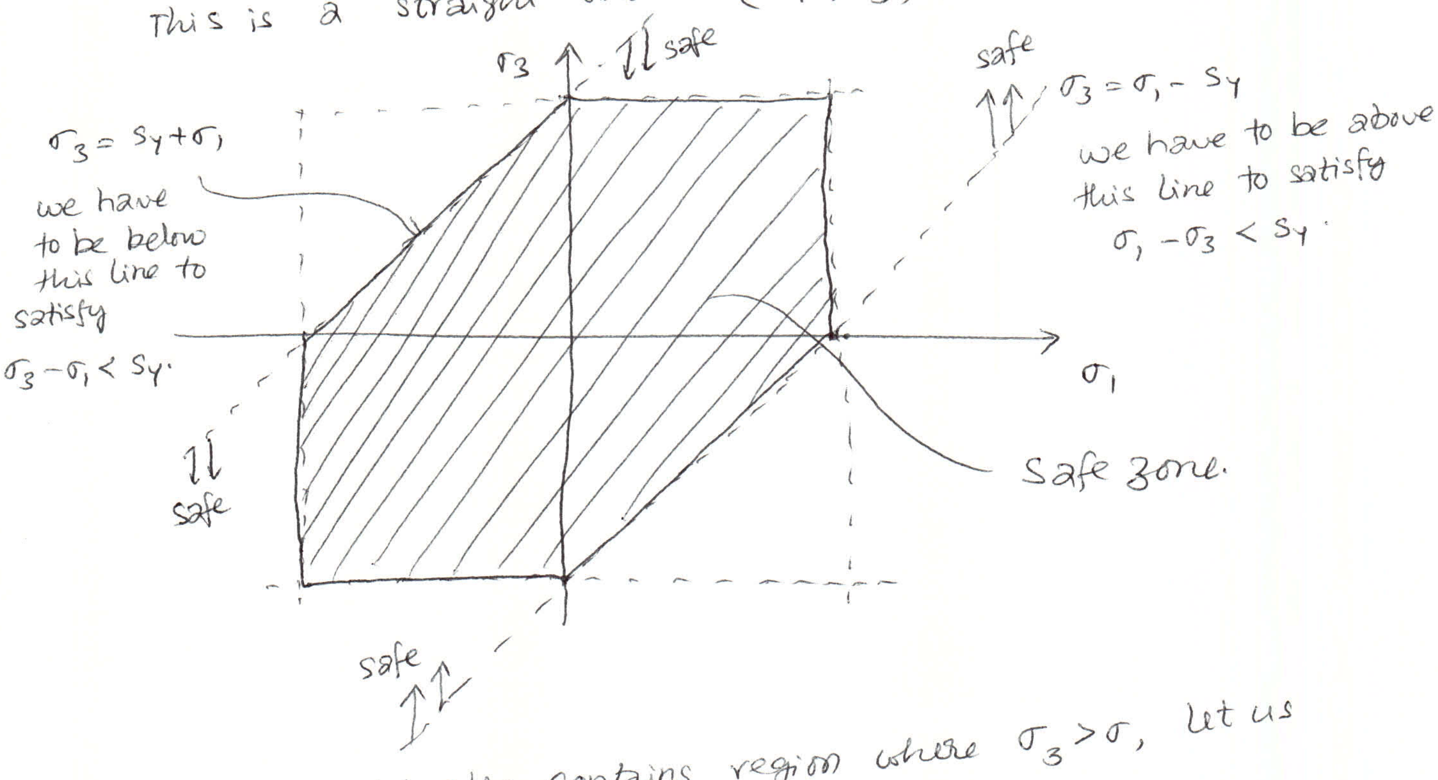
$\tau_{13} < S_{ys}$

$\Rightarrow \dfrac{\sigma_1 - \sigma_3}{2} < \dfrac{S_y}{2} \Rightarrow \sigma_1 - \sigma_3 \leq S_y$ Limiting case

Let us consider the limiting case: $\sigma_1 - \sigma_3 = S_y$

$\Rightarrow \sigma_3 = \sigma_1 - S_y$

This is a straight line in (σ_1, σ_3) plane.



Since the plot also contains region where $\sigma_3 > \sigma_1$, let us also consider, $\dfrac{\sigma_3 - \sigma_1}{2} < \dfrac{S_y}{2} \Rightarrow \sigma_3 - \sigma_1 < S_y$

$\Rightarrow \sigma_3 = S_y + \sigma_1$ (limiting case)

we cannot violate maximum normal stress criterion either.

So, we get the shaded hexagon as the safe zone under this theory. Since its area is smaller than square of the previous criterion, this criterion is more conservative.