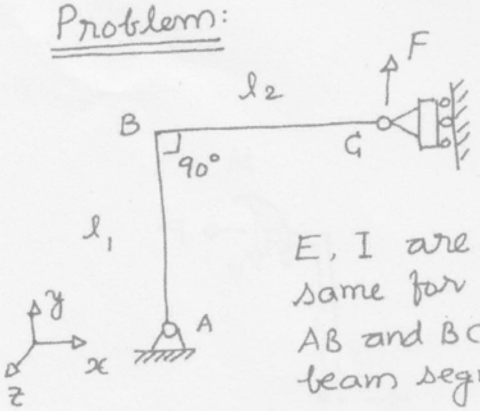
Problem:



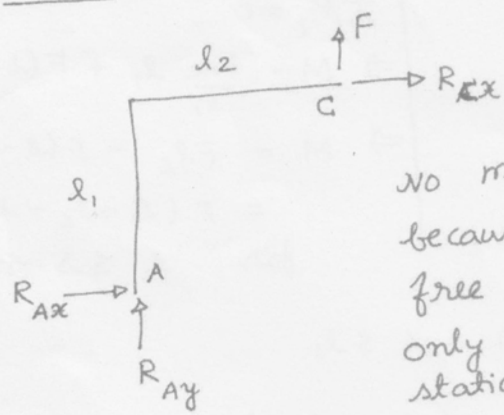
E, I are the same for AB and BC beam segments.

Point A is pinned and cannot move in x and y directions. Point C is pinned but is allowed to move in the y direction but not in the x direction.

Determine the deformation of segments AB and BC due to force F applied at C in the y direction.

Solution:

Step 1: Free-body diagram showing the reaction forces



No moment reactions at A and C because they are pinned and are free to rotate.

only three reaction forces. ∴ It is a statically determinate system.

$$\sum F_x = 0 \Rightarrow R_{Ax} + R_{Cx} = 0 \Rightarrow R_{Ax} = -R_{Cx} = -\frac{Fl_2}{l_1}$$

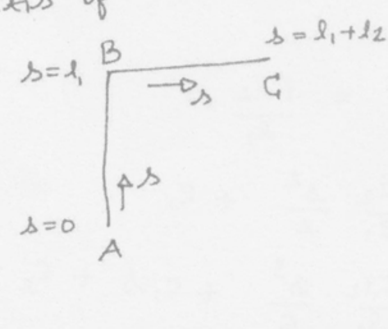
$$\sum F_y = 0 \Rightarrow R_{Ay} + F = 0 \Rightarrow R_{Ay} = -F$$

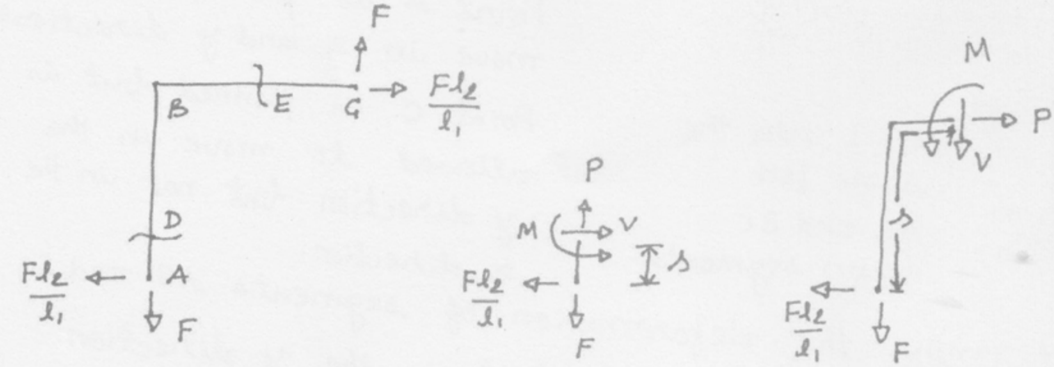
$$\sum M_z = 0 \Rightarrow Fl_2 - R_{Cx} l_1 = 0 \Rightarrow R_{Cx} = \frac{Fl_2}{l_1}$$

(taking moment about z at point A)

Step 2: Bending moment

Let us define a variable s that runs along the axis of the L-beam.





$\Sigma F_x = 0 \Rightarrow P = F$

$\Sigma F_y = 0 \Rightarrow V = \frac{Fl_2}{l_1}$

Taking moments at D,

$\Sigma M_z = 0$

$\Rightarrow M - \frac{Fl_2}{l_1} s = 0$

$\Rightarrow M = \frac{Fl_2}{l_1} s$

for $0 \le s \le l_1$

$\Sigma F_x = 0 \Rightarrow P = \frac{Fl_2}{l_1}$

$\Sigma F_y = 0 \Rightarrow V = -F$

Taking moments at E,

$\Sigma M_z = 0$

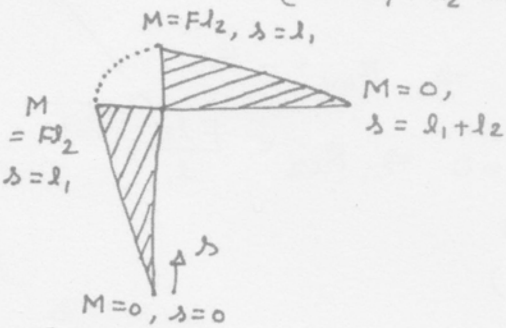
$\Rightarrow M - \frac{Fl_2}{l_1} l_1 + F(s - l_1) = 0$

$\Rightarrow M = Fl_2 - F(s - l_1)$

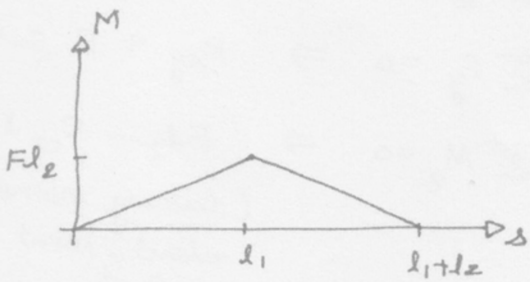
$= F(l_1 + l_2 - s)$

for $l_1 \le s \le l_1 + l_2$

So, $M = \begin{cases} \frac{Fl_2}{l_1} s & 0 \le s \le l_1 \\ F(l_1 + l_2 - s) & l_1 \le s \le l_1 + l_2 \end{cases}$



Bending moment diagram



(L-beam stretched to be straight)

Step 3: Deformation

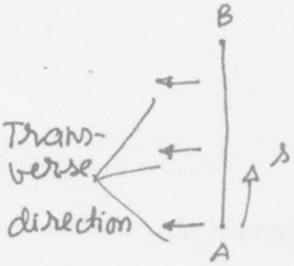
Segment AB: $EI \frac{d^2 w}{ds^2} = M = \frac{Fl_2}{l_1} s$

By integration, $EI \frac{dw}{ds} = \frac{Fl_2}{l_1} \frac{s^2}{2} + C_1$

By integration, $EI\, w = \frac{Fl_2}{l_1} \frac{s^3}{6} + C_1 s + C_2$

$$\Rightarrow \quad w = \frac{1}{EI}\left(\frac{Fl_2}{l_1}\frac{s^3}{6} + c_1 s + c_2 \right) \quad \text{for } 0 \le s \le l_1$$

$w(s)$ is the transverse (i.e., perpendicular) displacement. For segment AB, it is as follows



$$\left. \begin{aligned} w_{s=0} &= 0 \\ w_{s=l_1} &= 0 \end{aligned} \right\} \text{Boundary conditions}$$

$$w_{s=0} = 0 \Rightarrow c_2 = 0$$

$$w_{s=l_1} = 0 \Rightarrow \frac{1}{EI}\left(\frac{Fl_2}{l_1}\frac{l_1^3}{6} + c_1 l_1 \right) = 0$$

$$\Rightarrow c_1 = -\frac{Fl_2}{l_1}\frac{l_1^2}{6} = -\frac{Fl_1 l_2}{6}$$

$$\therefore \; w(s) = \frac{1}{EI}\left(\frac{Fl_2}{l_1}\frac{s^3}{6} - \frac{Fl_1 l_2}{6} s \right) \quad \text{for } 0 \le s \le l_1$$

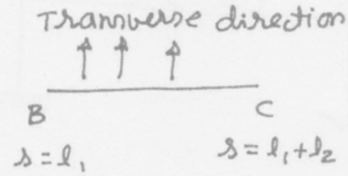
$$\frac{dw}{ds} = \frac{1}{EI}\left(\frac{Fl_2}{l_1}\frac{s^2}{2} - \frac{Fl_1 l_2}{6} \right); \quad \left.\frac{dw}{ds}\right|_{s=l_1} = \frac{Fl_1 l_2}{3EI}$$

Segment BC:

$$EI \frac{d^2 w}{ds^2} = M = F(l_1 + l_2 - s)$$

By integration, $EI \dfrac{dw}{ds} = F(l_1+l_2)s - \dfrac{Fs^2}{2} + c_3$

By integration, $EI\,w = F(l_1+l_2)\dfrac{s^2}{2} - \dfrac{Fs^3}{6} + c_3 s + c_4$



Boundary conditions

$$w_{s=l_1} = 0; \quad \left.\frac{dw}{ds}\right|_{s=l_1} = \frac{Fl_1 l_2}{3EI} \quad \text{Slope continuity!}$$

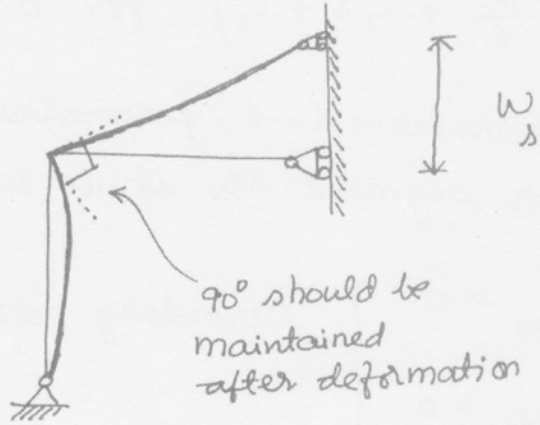
$w_{s=l_1+l_2}$ = (we need to calculate)

$$w_{s=l_1} = 0 \Rightarrow c_4 = 0; \quad \left.\frac{dw}{ds}\right|_{s=l_1} = \frac{Fl_1 l_2}{3EI} \Rightarrow c_3 = -\frac{2Fl_1 l_2}{3} - \frac{Fl_1^2}{2}$$

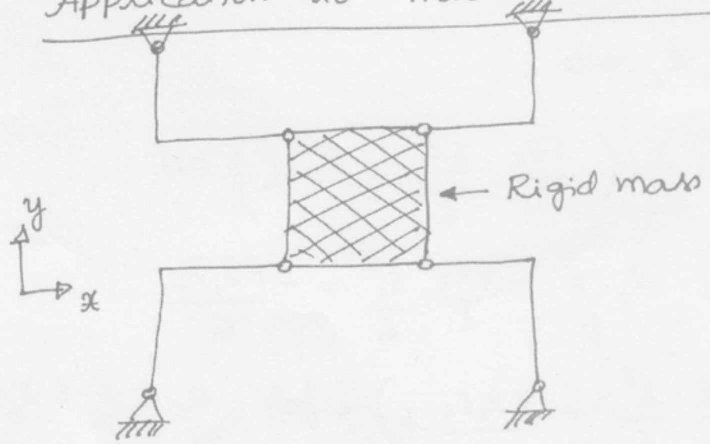
$$\therefore \; w(s) = \frac{1}{EI}\left(F(l_1+l_2)\frac{s^2}{2} - \frac{Fs^3}{6} - \frac{2Fl_1 l_2}{3}s - \frac{Fl_1^2}{2}s \right) \quad \text{for } l_1 \le s \le (l_1+l_2)$$

(compute this)

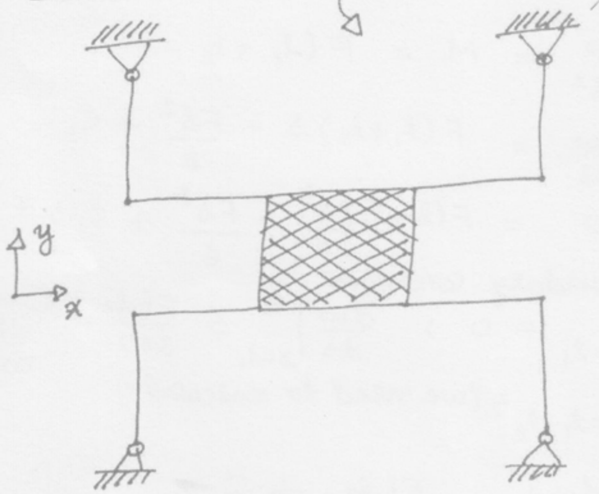
$w_{s=l_1+l_2} =$

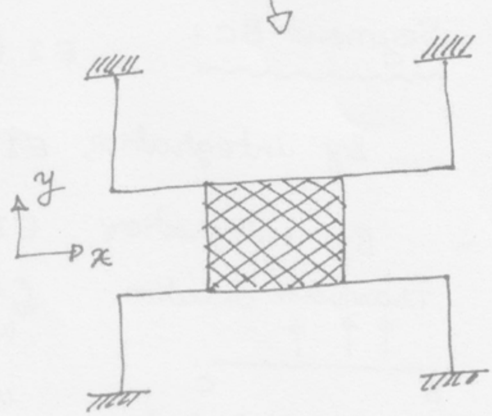


Application to micromechanical suspension



How about this suspension? And, this?





Try to sketch deformed configurations of the three suspensions for force applied in the y-direction on the mass.
And then, for force applied in the x-direction.