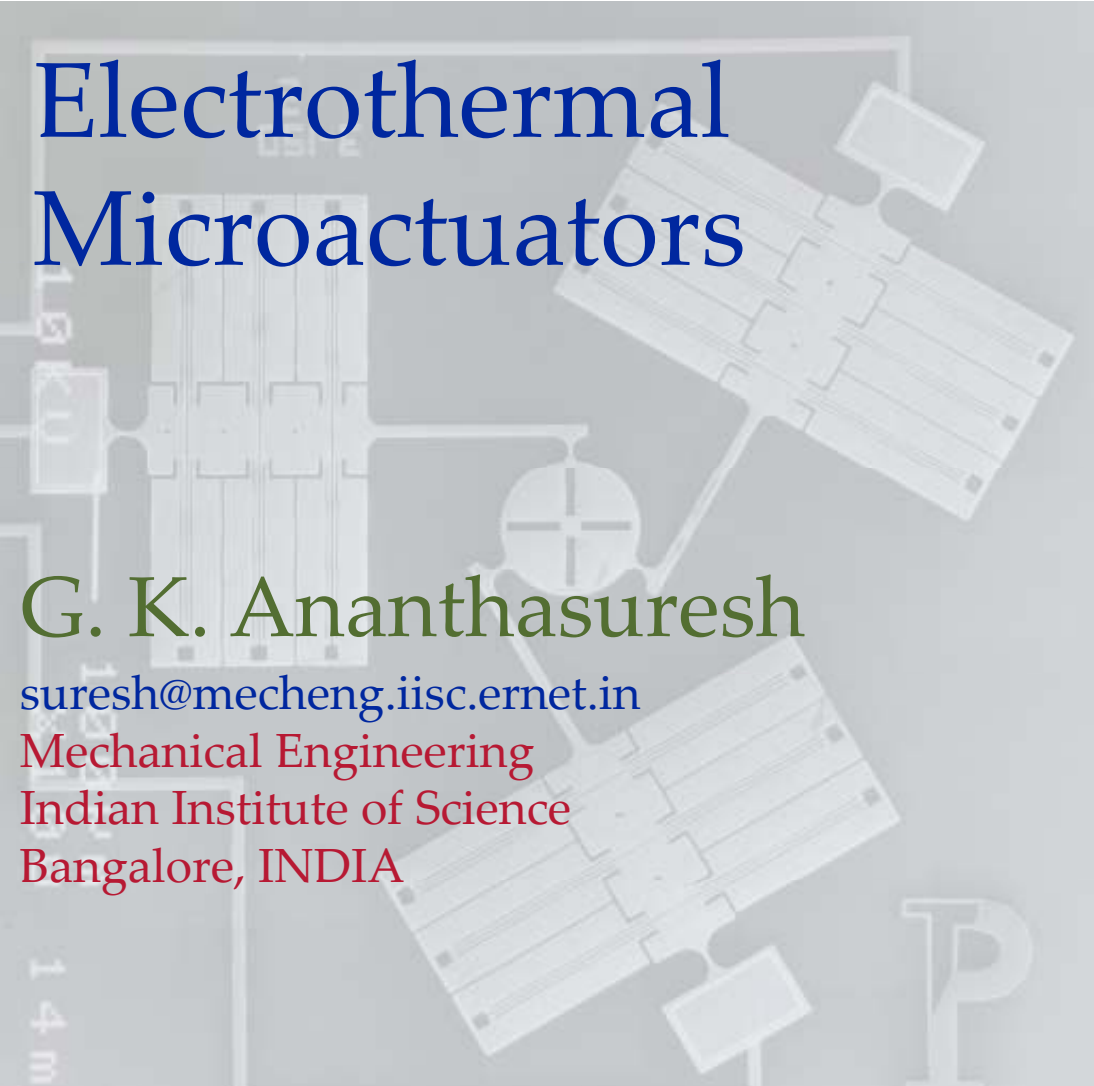


# Electrothermal Microactuators

A grayscale micrograph of electrothermal microactuators. The image shows several rectangular structures with internal patterns, likely made of thin-film materials. A central circular feature with a crosshair is visible. Scale bars are present, with '1.0 μm' on the left and '1.4 μm' at the bottom left. A large 'P' is visible in the bottom right corner.

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November 2014, ME 237/NE 211 course in IISc.

## Effects of heating on mechanical deformation

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$$\alpha = \frac{d\varepsilon}{dT} \quad \text{Temperature coefficient of expansion}$$

$$\varepsilon(T) = \varepsilon(T_0) + \alpha(T - T_0) \quad \text{Uniaxial thermal strain}$$

$$\varepsilon_{mismatch}(T) = (\alpha_f - \alpha_s)(T - T_0)$$

Mismatched thermal strain and stress between a film and a substrate that are bonded to each other.

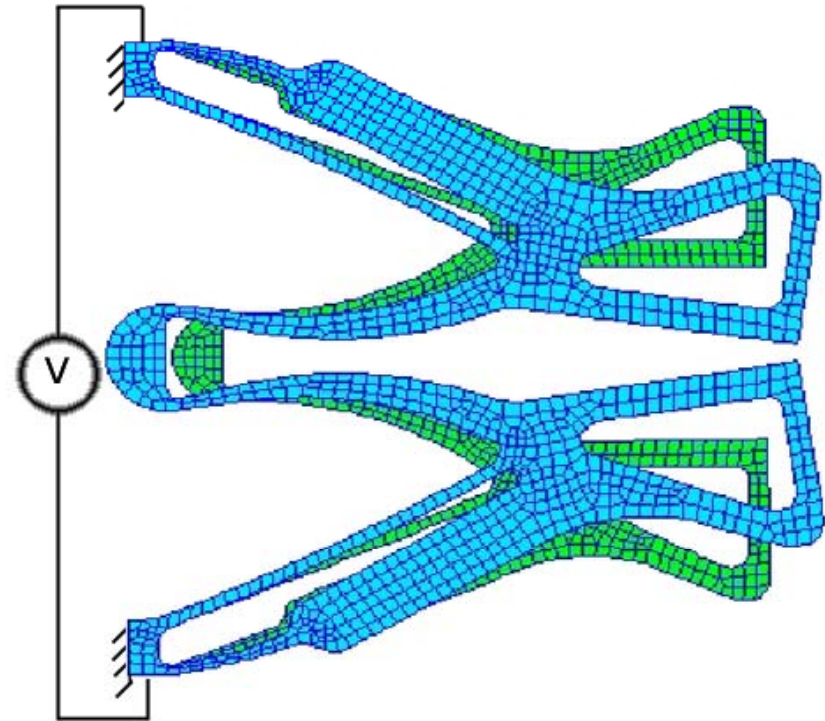
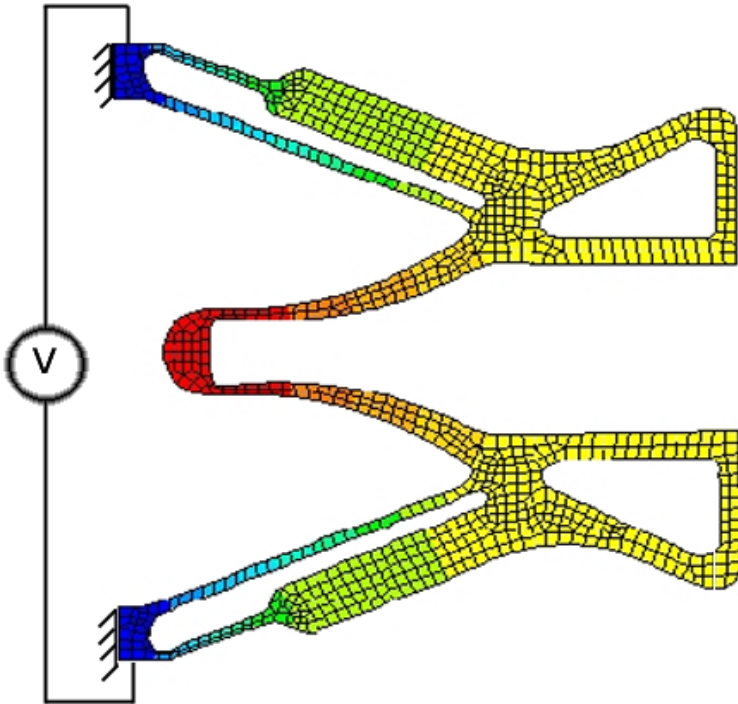
$$\sigma_{mismatch} = \left( \frac{E}{1 - \nu} \right) \varepsilon_{mismatch}$$

$$\varepsilon_z = - \left\{ \alpha_f + 2\nu(\alpha_f - \alpha_s) \right\} (T - T_0)$$

Total strain for a sandwiched film in the thickness (z) direction

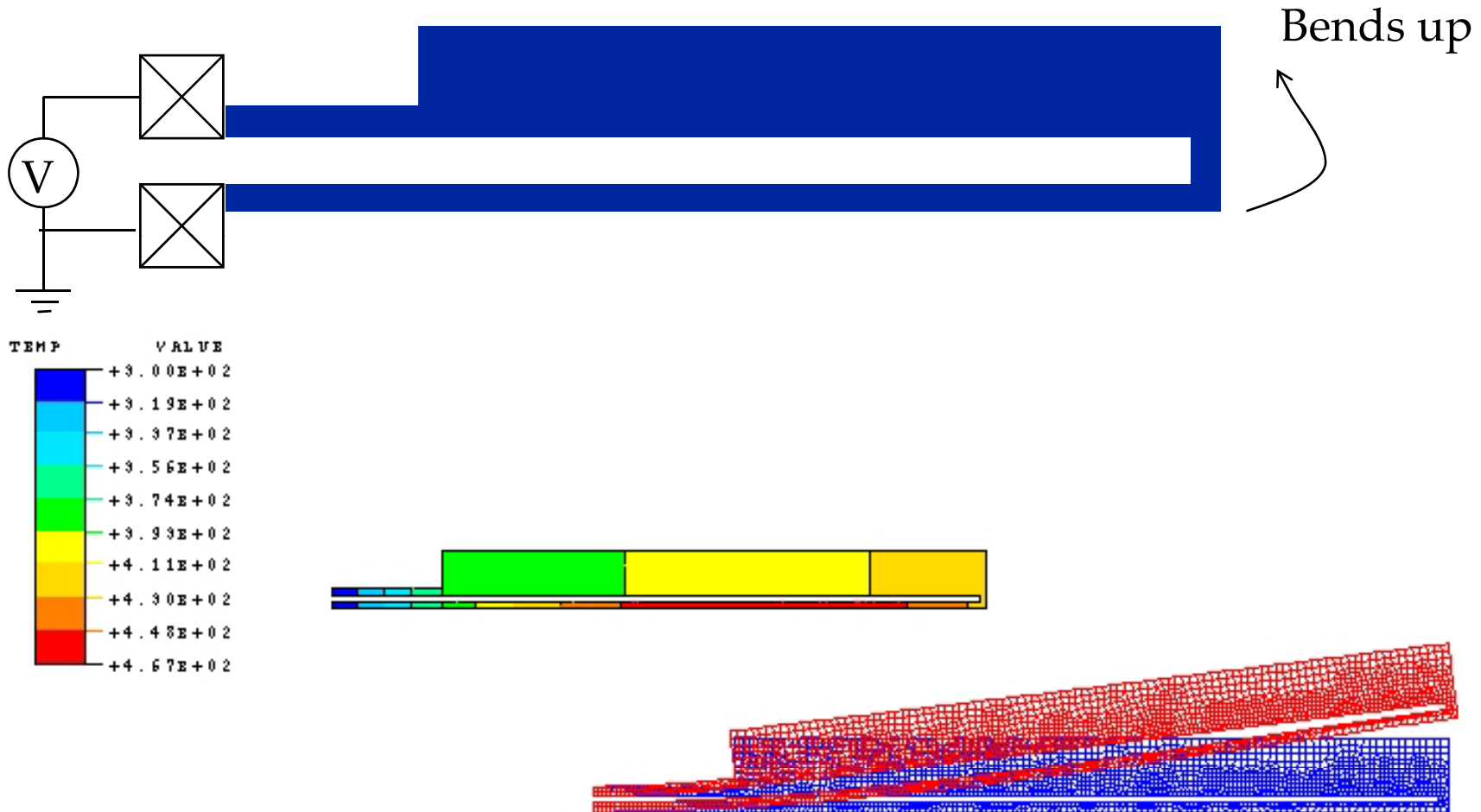
3 *Embedded actuation:*  
Actuator and mechanism are together.

---



# Heatuator: Series connection

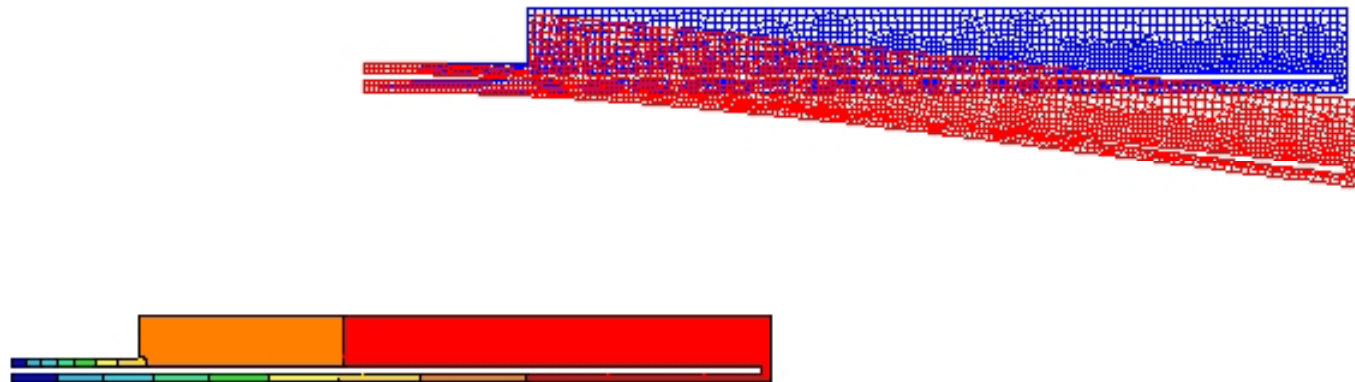
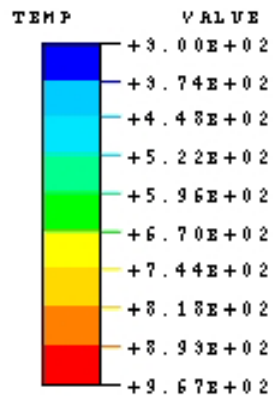
(Guckel et al., 1992; Comtois and Bright, 1996)



# Heatuator: parallel connection

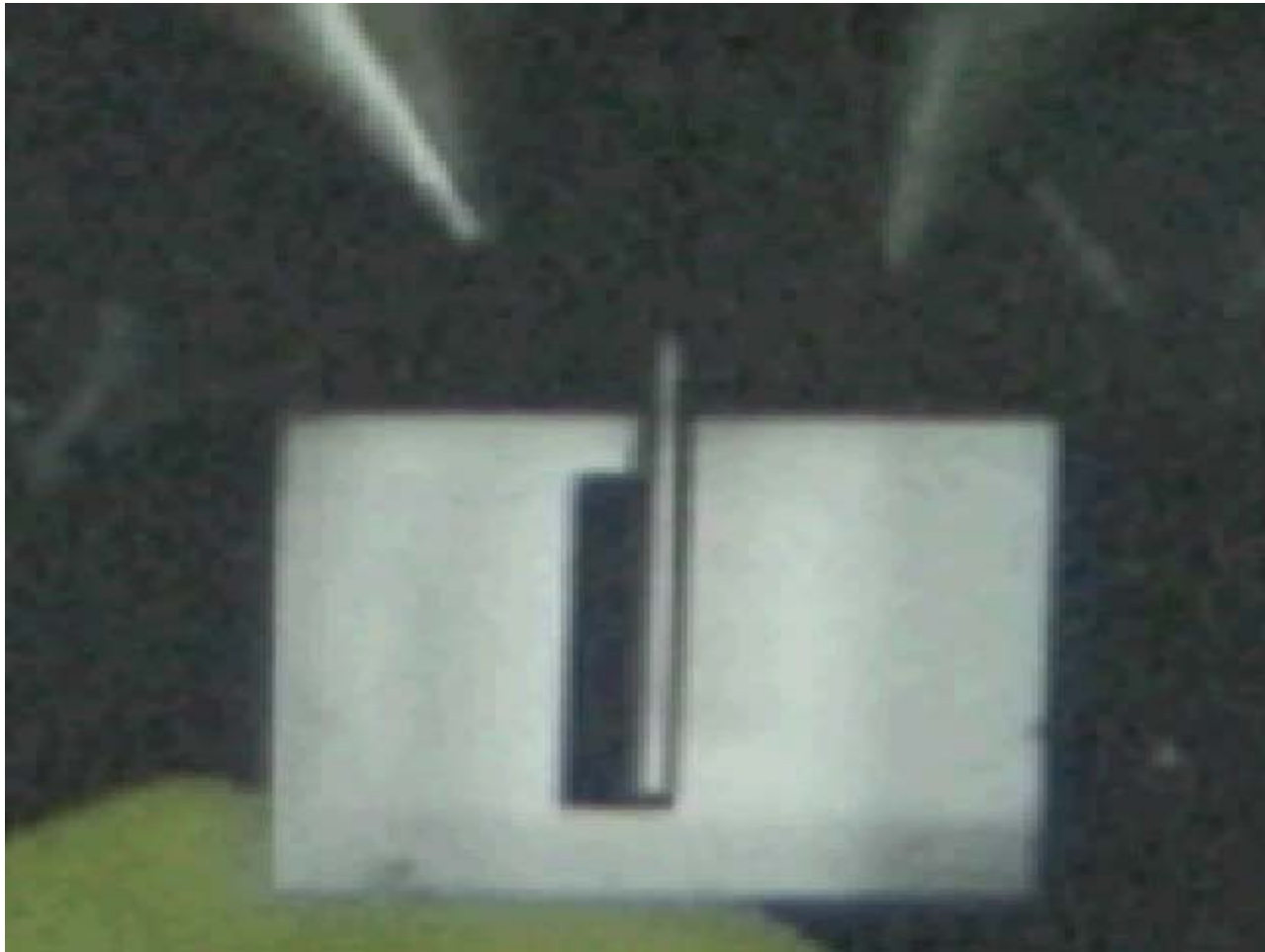
(Moulton and Ananthasuresh, 1997)

*Parallel connection*

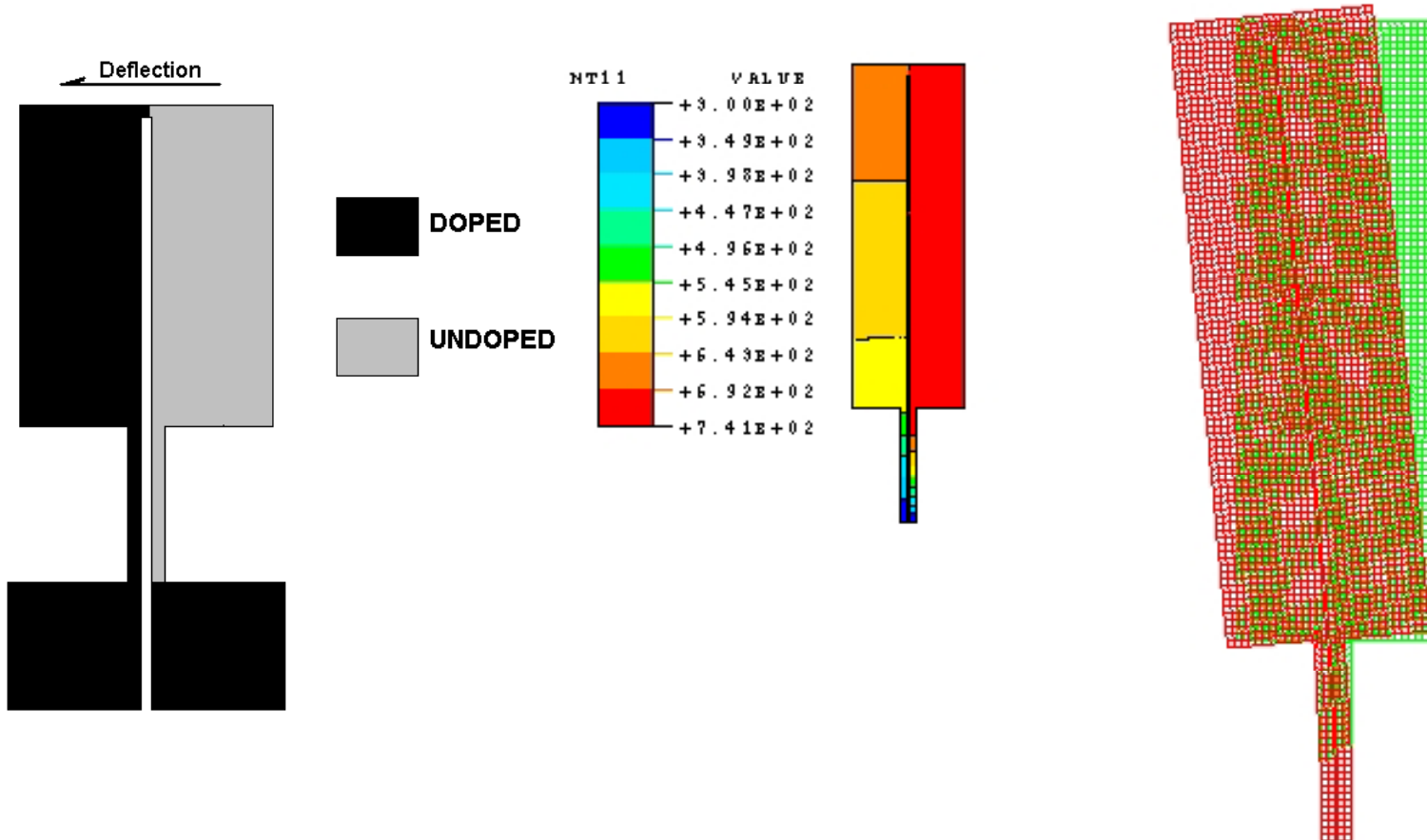


# Heatuator: working

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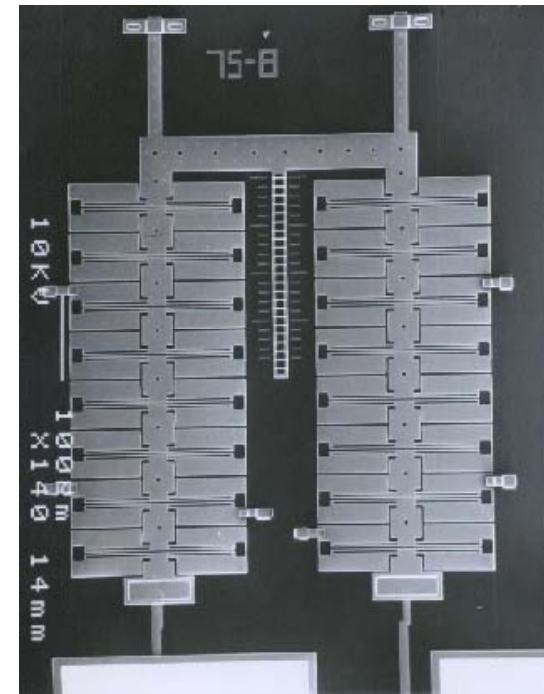
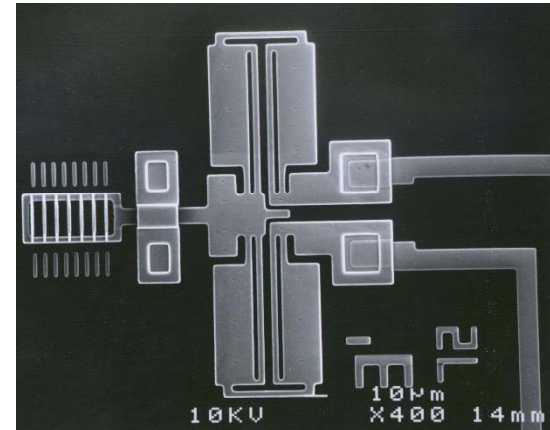
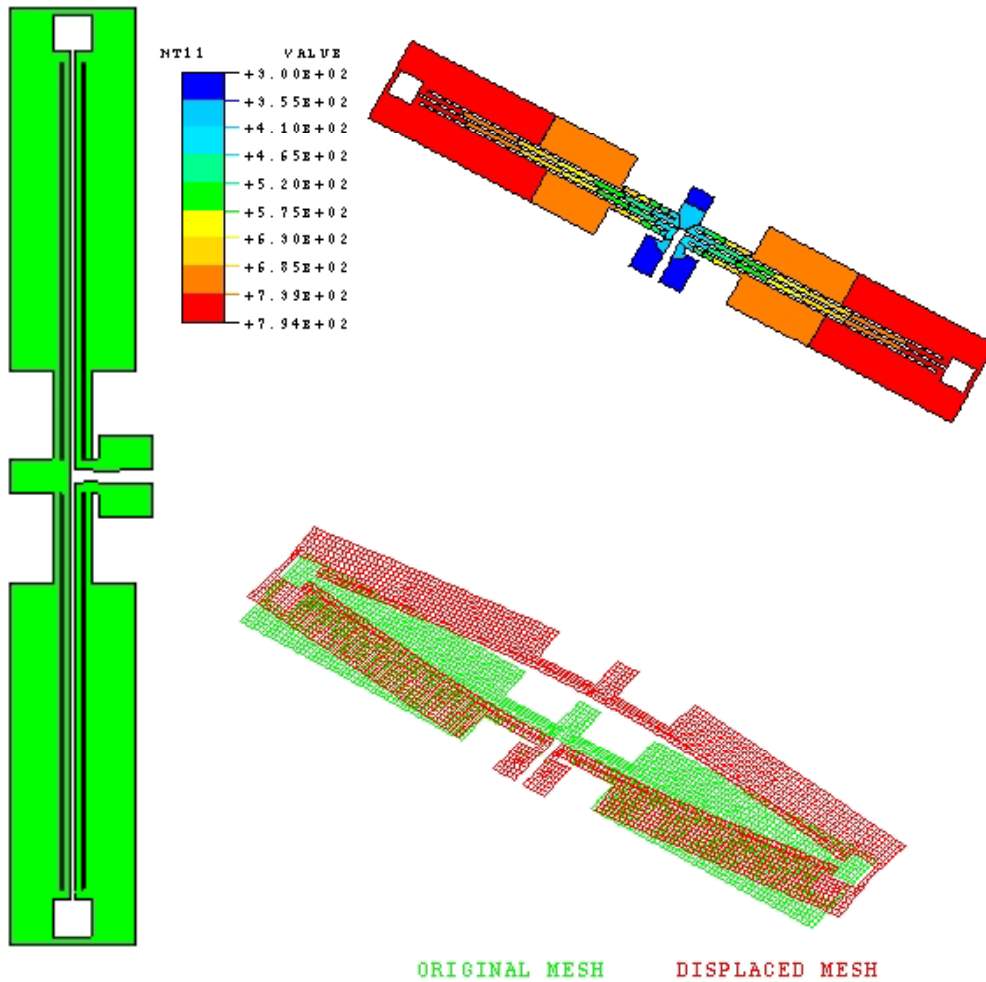


# Heatuator with elective doping (if made with silicon)



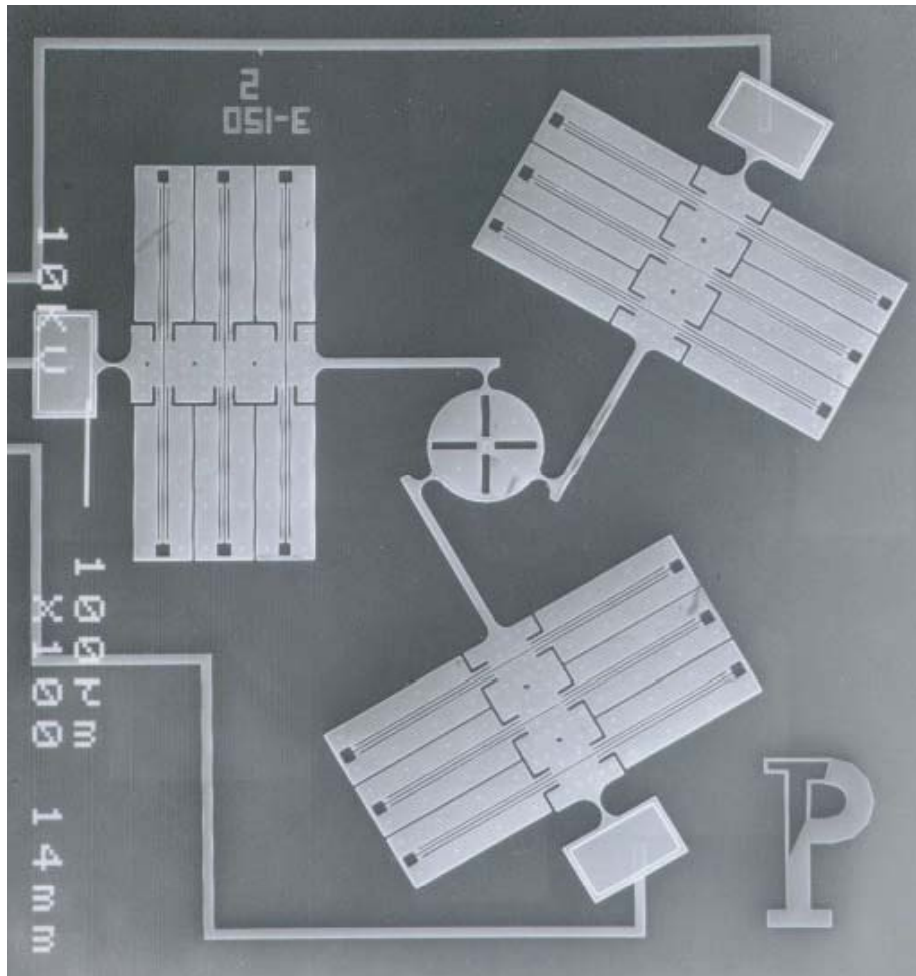


# ETC expansion block



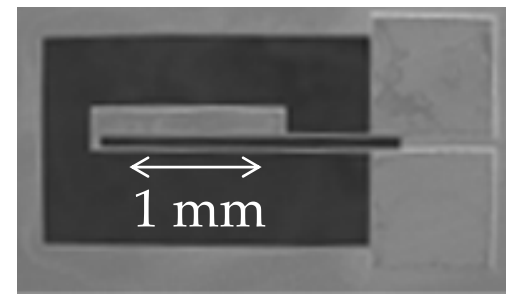
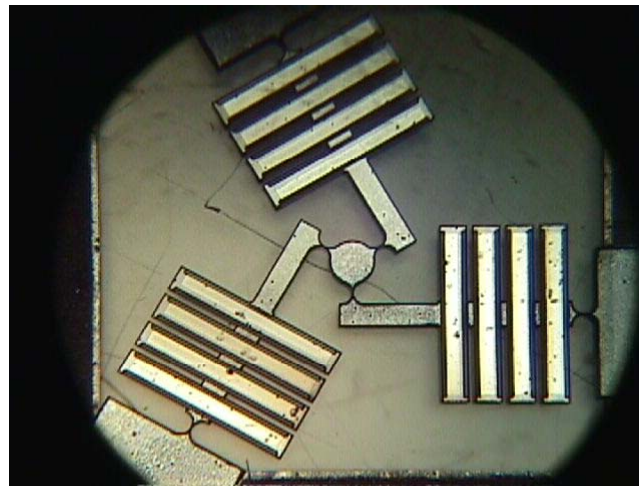
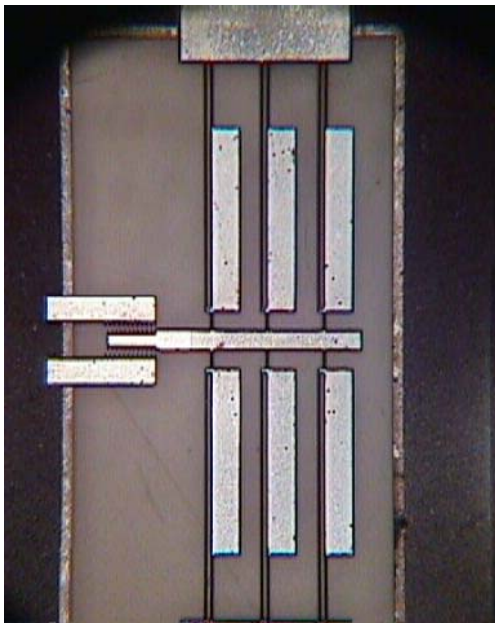
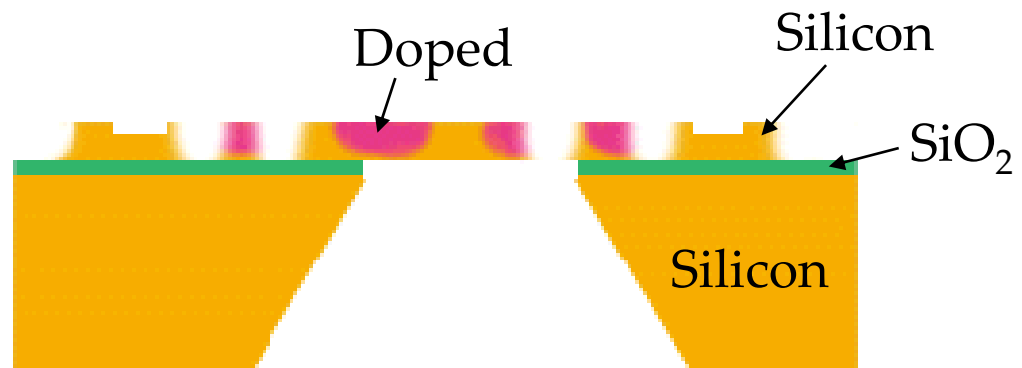


# ETC Parallel micro manipulator

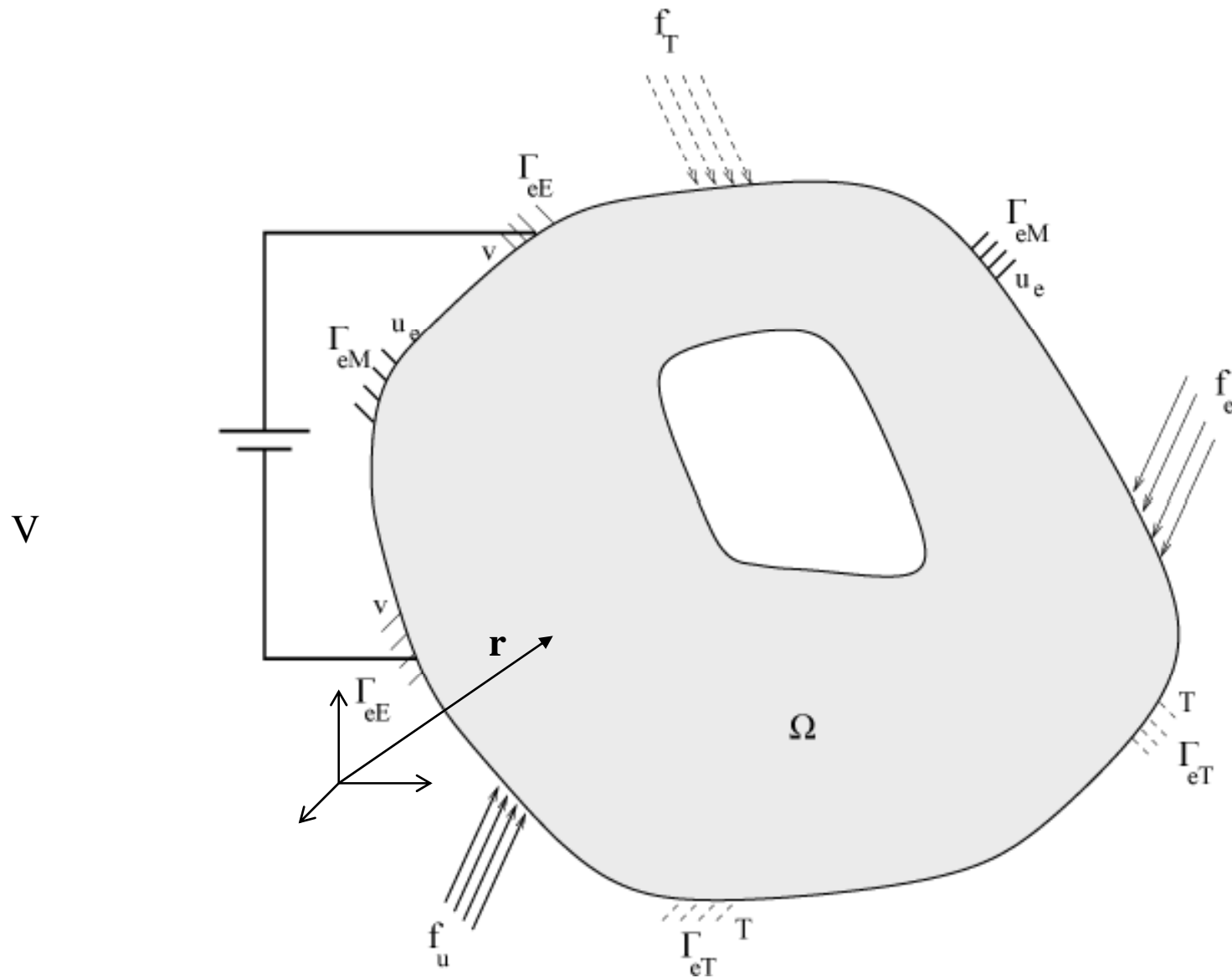


With three degrees of freedom; Made using MUMPs, polysilicon.

# Devices made with PennSOIL



# Modeling



## Governing equations (steady-state)

Electrical Domain

$$\begin{aligned}\nabla \cdot (\tilde{k}_e \nabla v) + i_e &= 0 && \text{in } \Omega \\ v &= v_e && \text{on } \Gamma_{eE} \\ \bar{n} \cdot (\tilde{k}_e \nabla v) &= f_e && \text{on } \Gamma_{nE}\end{aligned}$$

Elastic Domain

$$\begin{aligned}\nabla \cdot \tilde{\sigma} + \bar{F} &= 0 && \text{in } \Omega \\ \tilde{\sigma} &= \tilde{E}[\tilde{\varepsilon} - \alpha(T - T_0)\tilde{I}] && \text{in } \Omega \\ \tilde{\varepsilon} &= \frac{\nabla \bar{u} + (\nabla \bar{u})^T}{2} && \text{in } \Omega \\ \bar{u} &= \bar{u}_e && \text{on } \Gamma_{eM} \\ \tilde{\sigma} \bar{n} &= \bar{f}_u && \text{on } \Gamma_{nM}\end{aligned}$$

Thermal Domain

$$\begin{aligned}\nabla \cdot (\tilde{k}_t \nabla T) + \dot{q}_T &= 0 && \text{in } \Omega \\ \dot{q}_T &= \tilde{k}_e \nabla v \cdot \nabla v && \text{in } \Omega \\ T &= T_e && \text{on } \Gamma_{eT} \\ \bar{n} \cdot (\tilde{k}_t \nabla T) &= f_T && \text{on } \Gamma_{nT}\end{aligned}$$

Inter-domain Coupling

$$\tilde{k}_e(T), \quad q_T(v), \quad \tilde{E}(T), \quad \alpha(T),$$

Nonlinearity

$$\tilde{k}_e(T), \quad \tilde{k}_t(T), \quad f_T(T), \\ \tilde{E}(T), \quad \alpha(T).$$

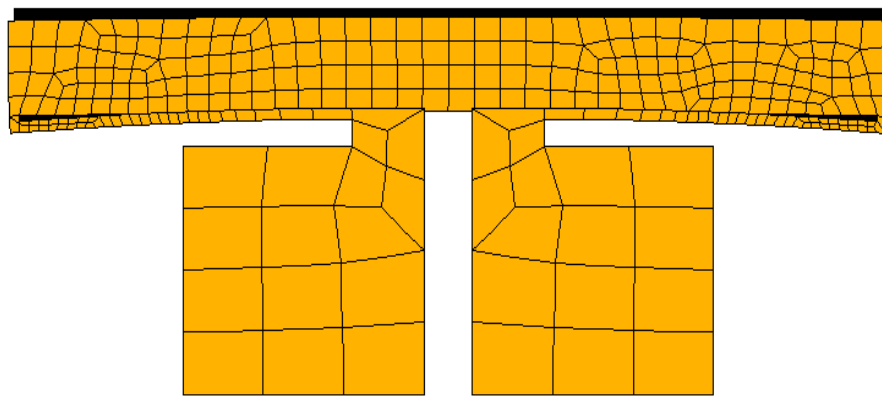
# Thermal modeling

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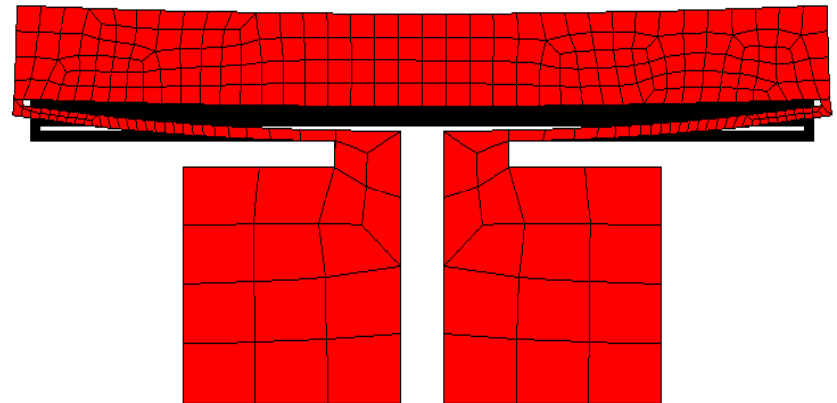
- **Convection**
  - Temperature dependence of heat transfer properties.
  - Size dependence of heat transfer properties.
- **Radiation**
  - View / Shape factors.
  - Radiation heat transfer between parts of the same device at different temperatures.
- **Boundary Conditions**
  - Essential Boundary conditions at the device anchor.
  - Natural Boundary conditions at the device anchor.
- **Conduction through trapped air volume**
  - Conduction between parts of the same device at different temperature with an intervening trapped air volume.
  - Conduction from the underside of the device to the substrate through the air trapped between them.
- **Temperature dependence of thermo-physical Properties**

# Why convection and radiation ?

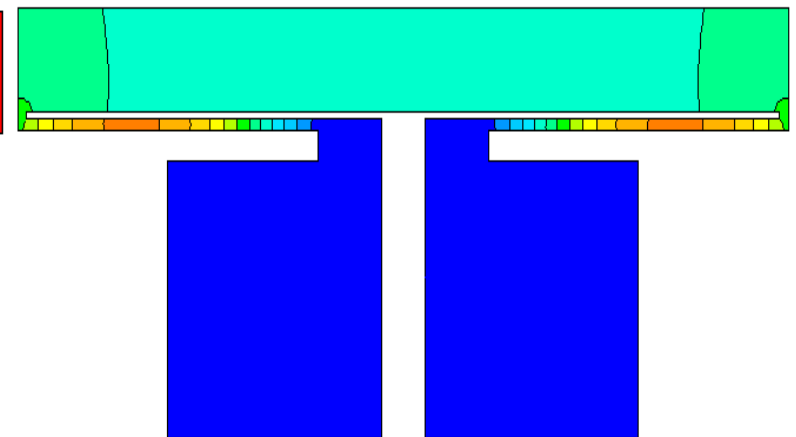
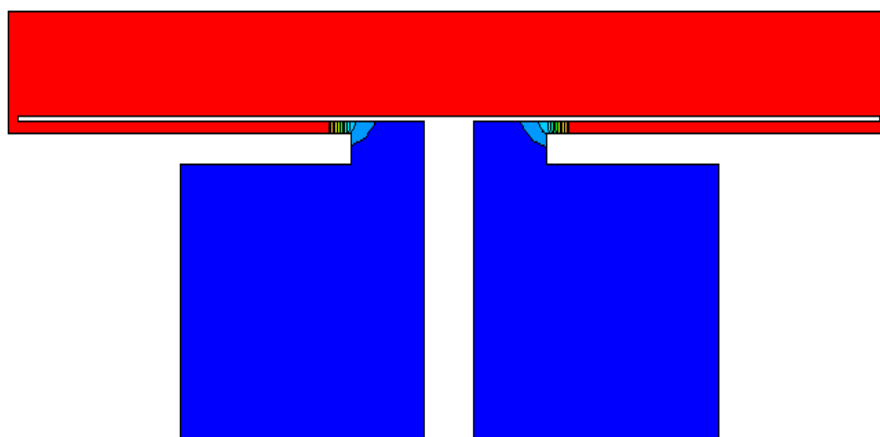
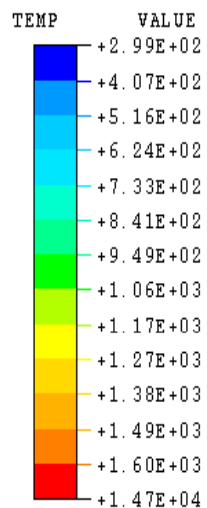
Thermal Expansion Device (TED), Cragun & Howell (1998)



*Without* convection or radiation

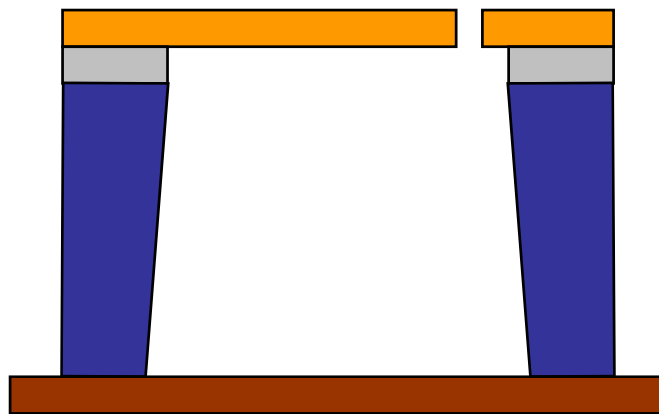







*With* convection and radiation



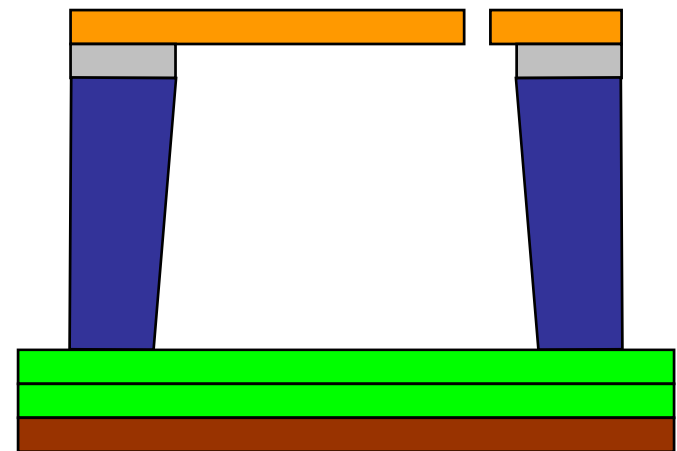
# EBC v/s NBC

*Essential* Boundary C  
*Thermally Grounded*



-  Silicon Device
-  SiO<sub>2</sub>
-  Silicon Handle
-  Glass
-  Ground

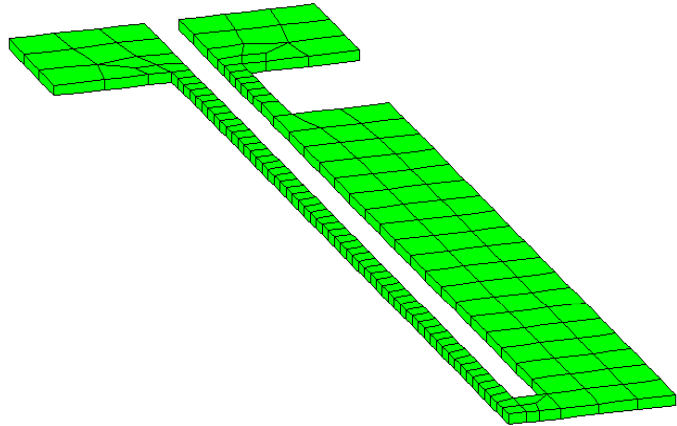
*Natural* Boundary C  
*Not Thermally Grounded*





# The Finite Element model

---



**20** node, **3-D Continuum** elements in ABAQUS

**Fully Coupled** Electro-Thermal Analysis

**Sequentially Coupled** Thermo-Elastic Analysis

With temperature dependent material properties and heat transfer coefficients.

# Thermal Boundary Conditions and Scaling : Case Studies

## ➤ Same *Maximum Temperature at Steady State*

➤ EBC + Meso

➤ NBC + Meso

Made using PennSOIL

➤ EBC + Micro

➤ NBC + Micro

Made using MUMPs

## ➤ Same *Power Input*

➤ EBC + Meso

➤ NBC + Meso

➤ EBC + Micro

➤ NBC + Micro

## • Same *Applied Voltage*

– EBC + Meso

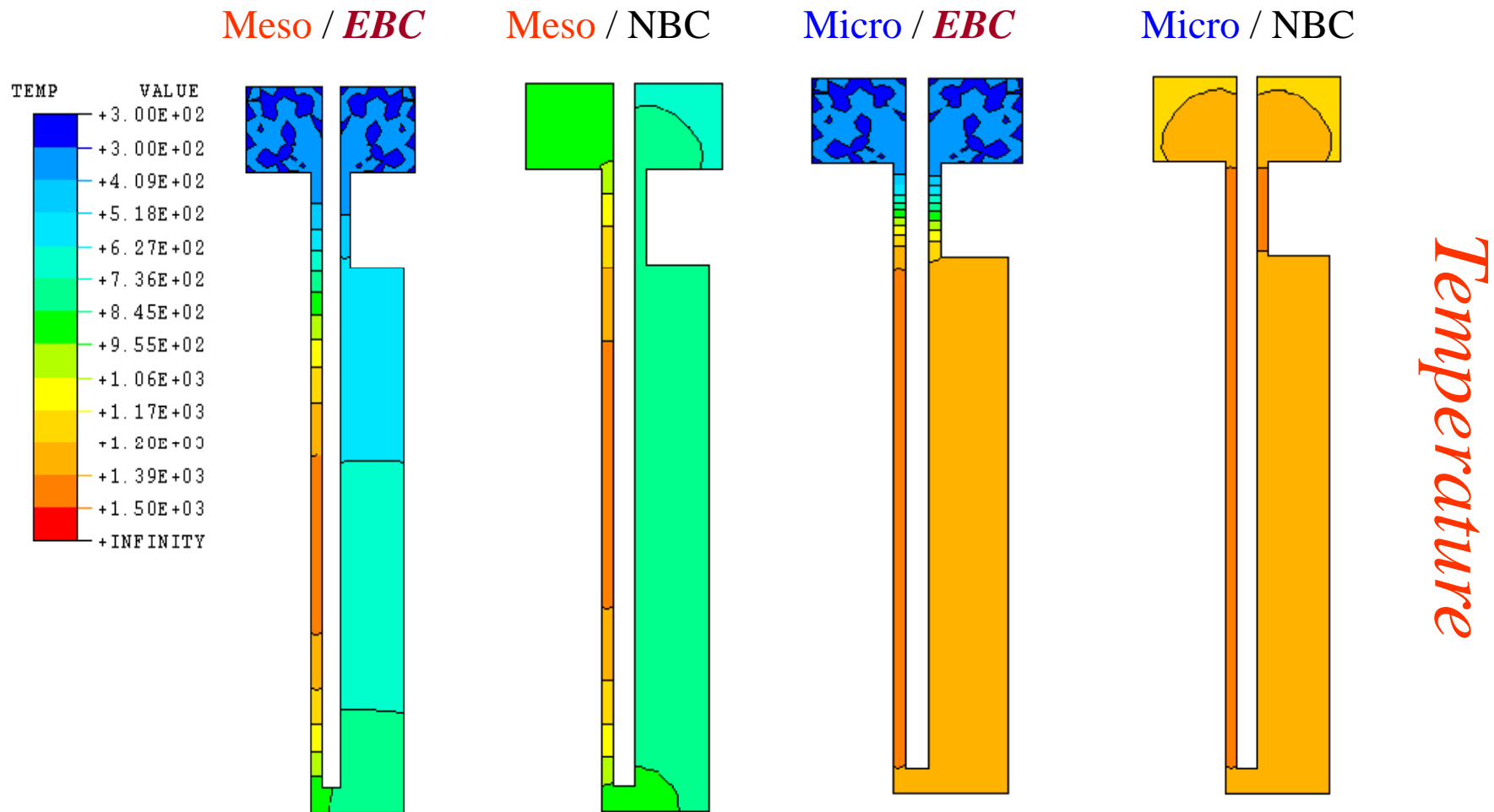
– NBC + Meso

– EBC + Micro

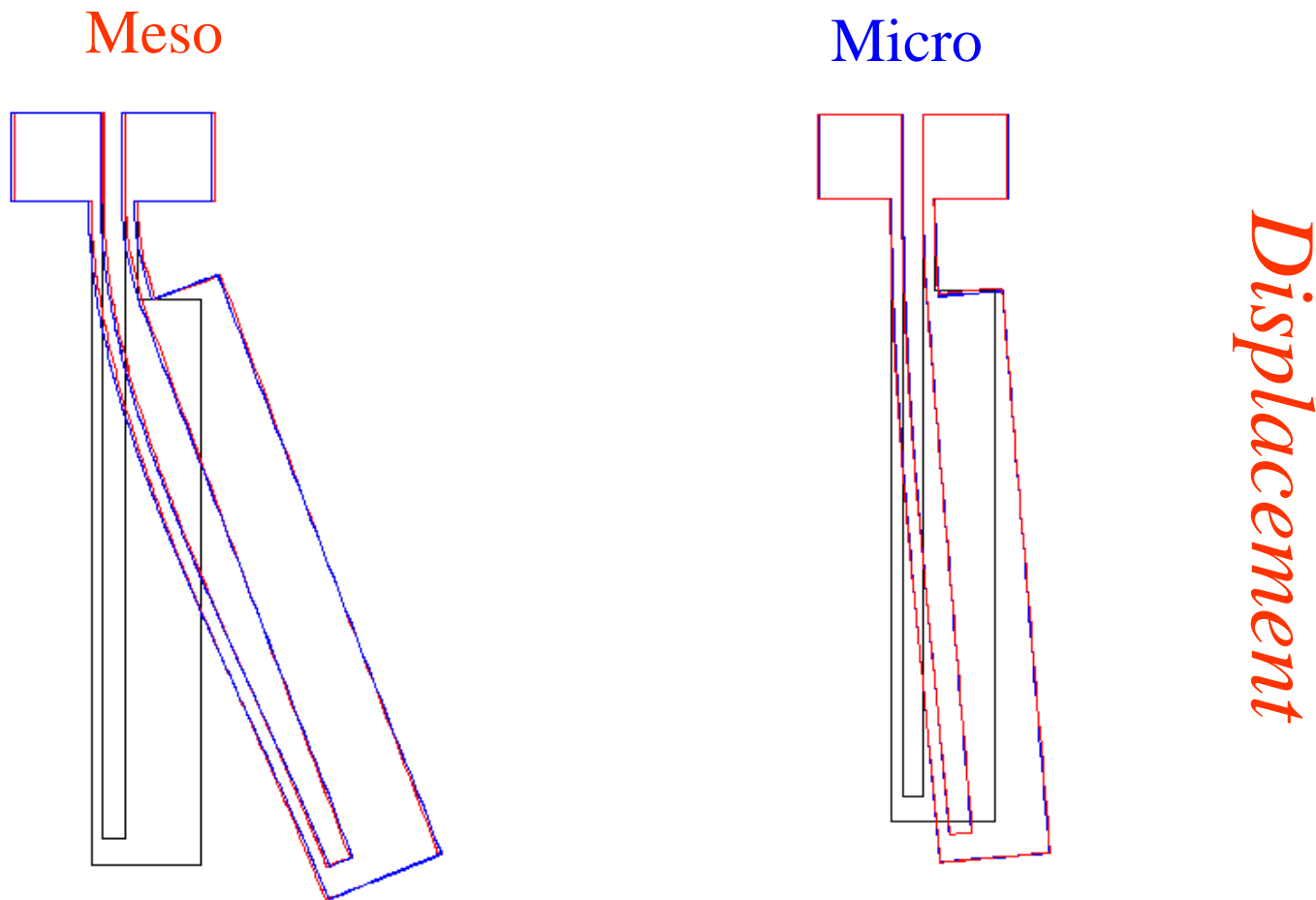
– NBC + Micro

*Experiment*

# Same *Maximum Temperature*

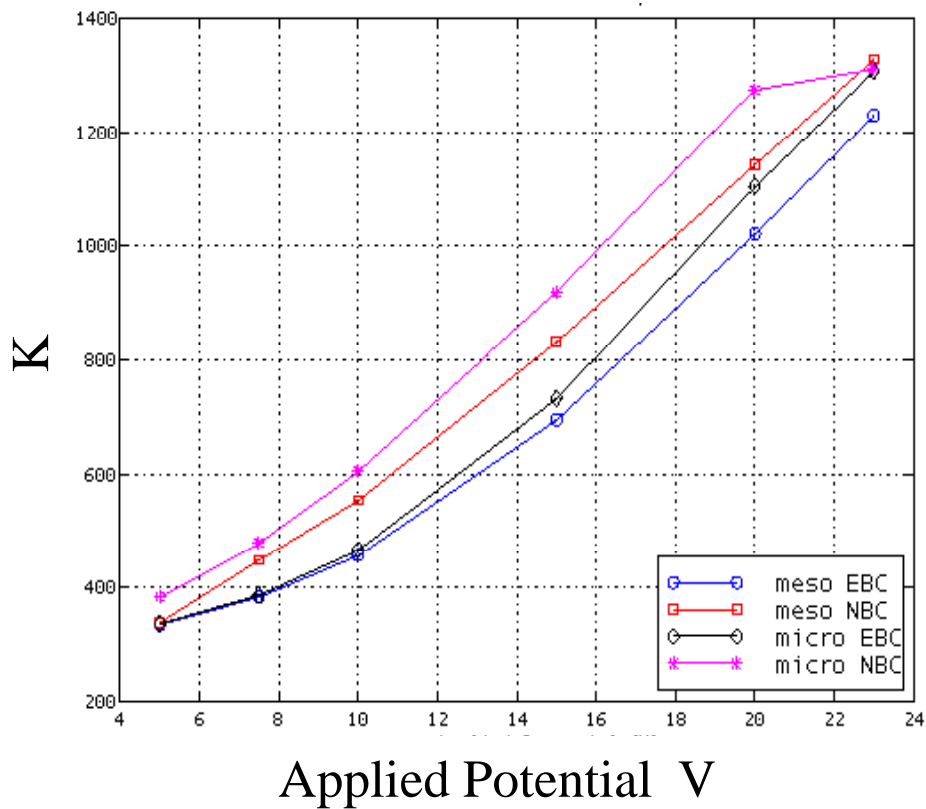


## Same *Maximum Temperature*

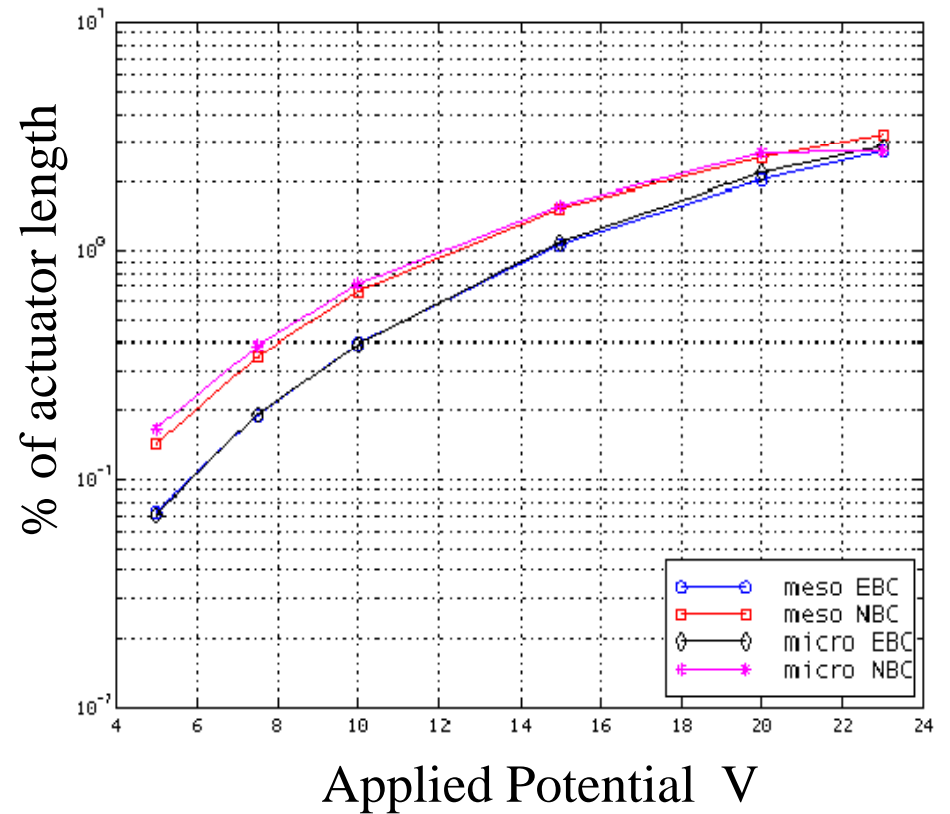


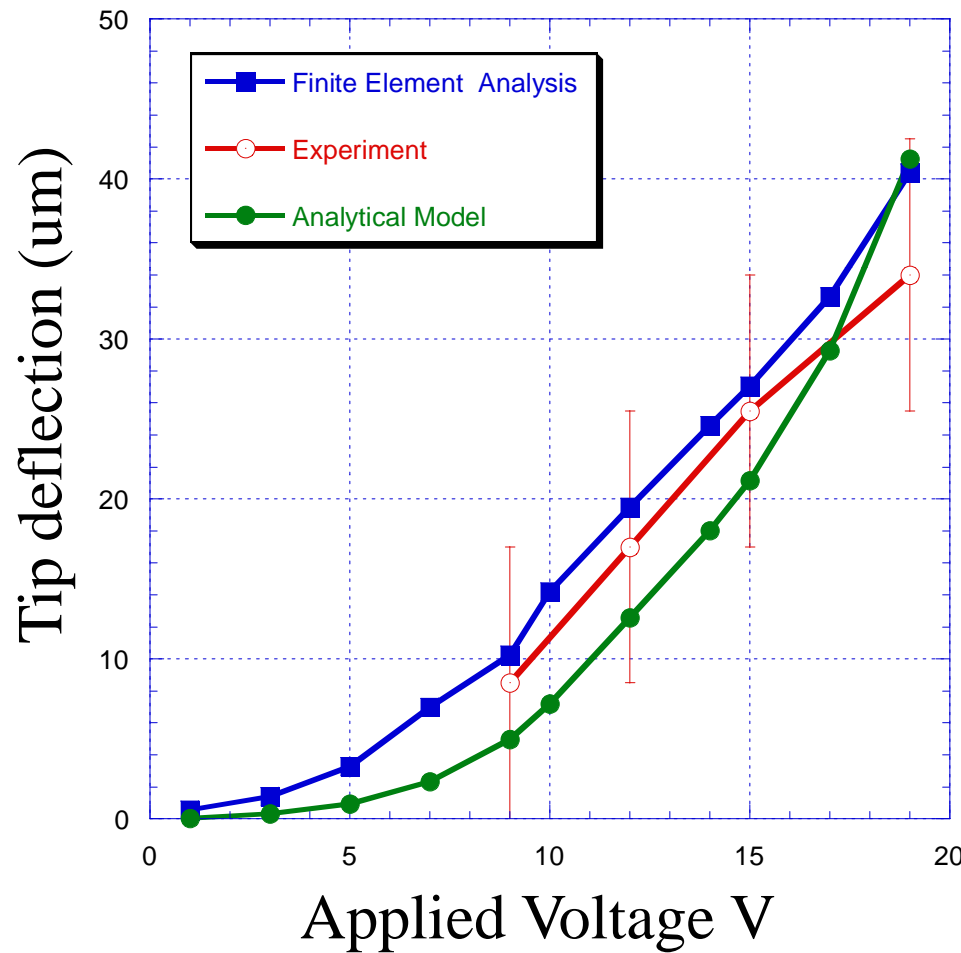
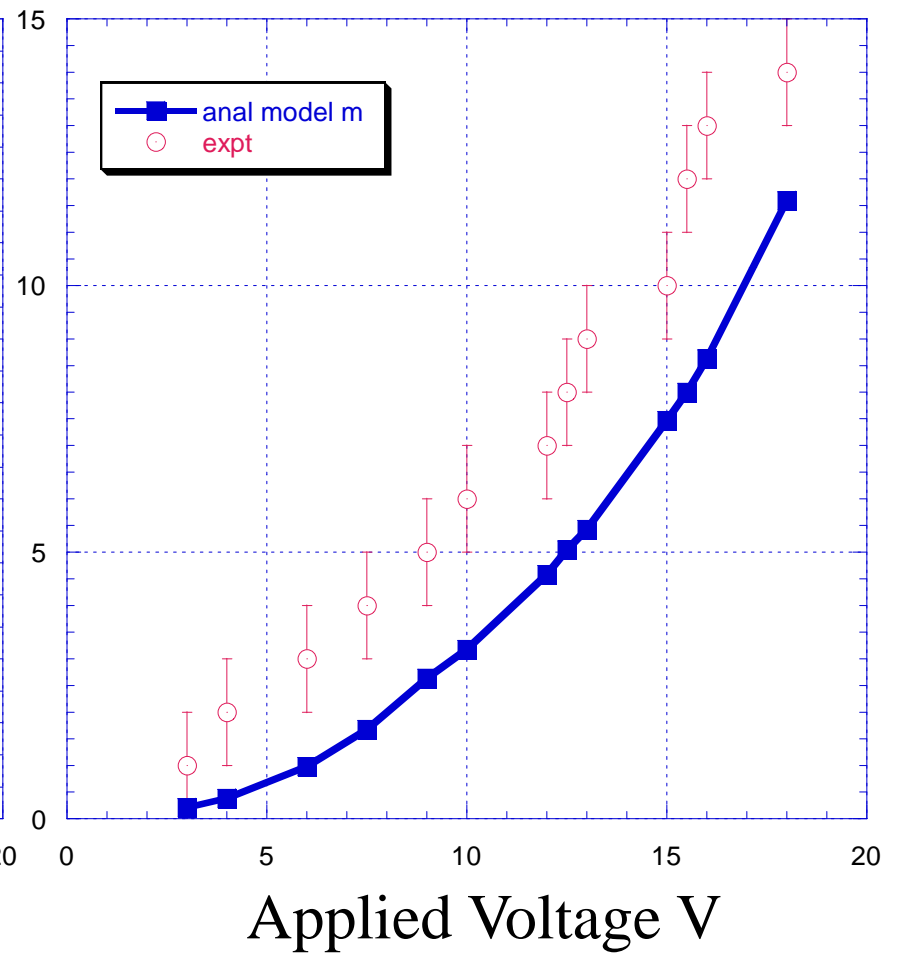
## Same *Applied Voltage*

### Maximum Temperature

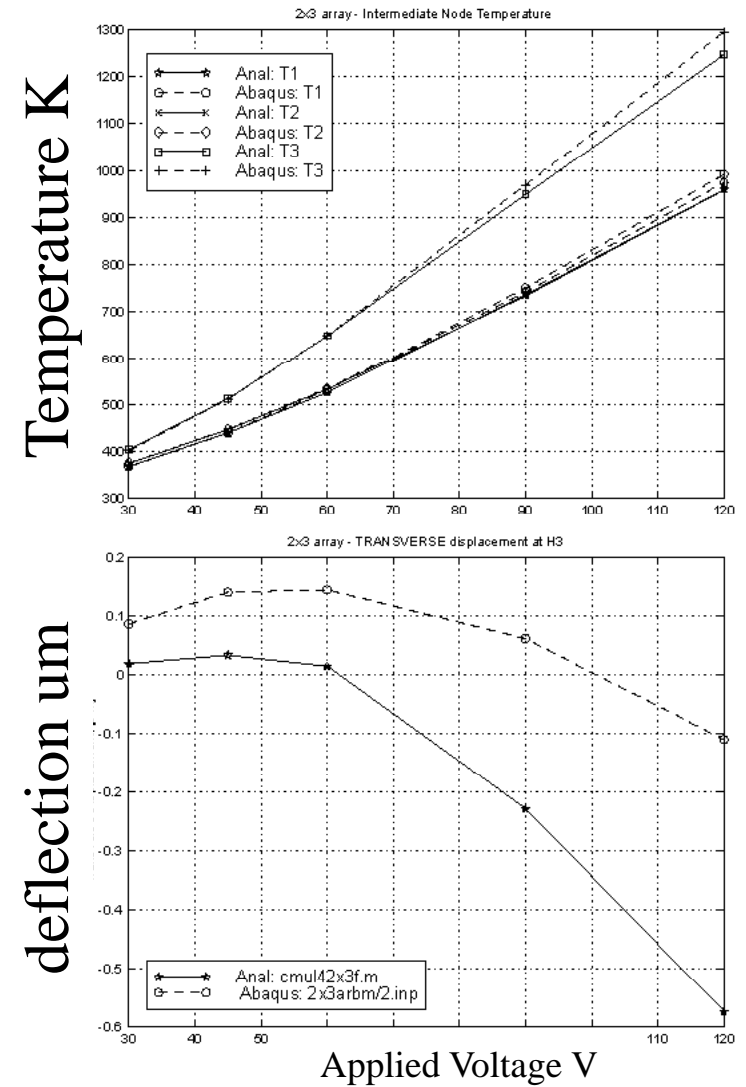
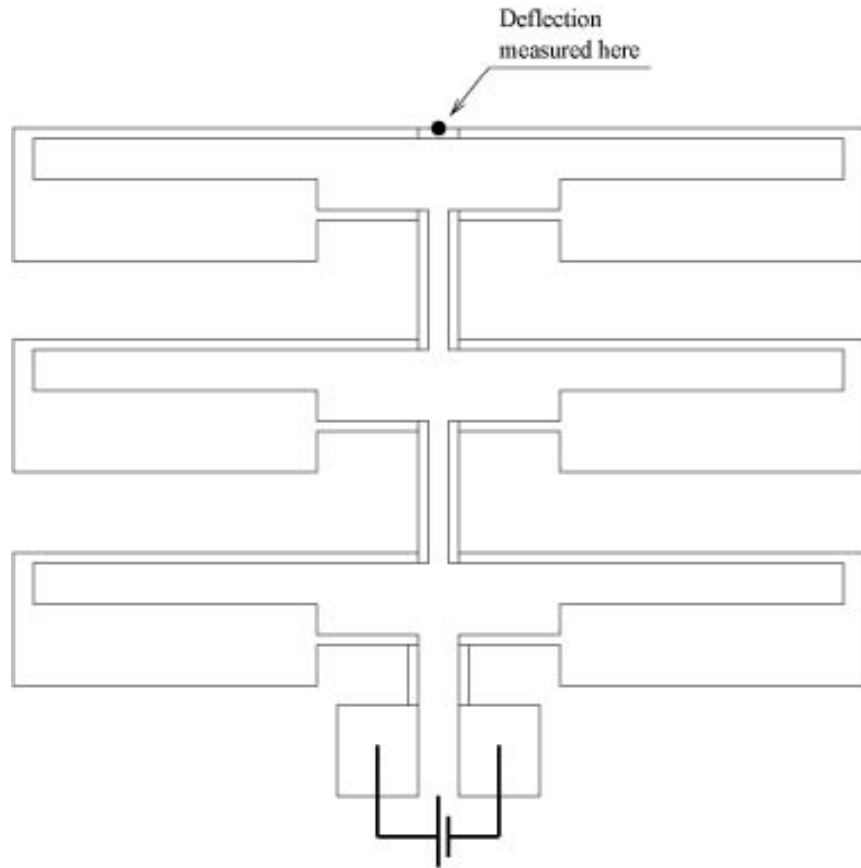


### Normalised Transverse Displ.



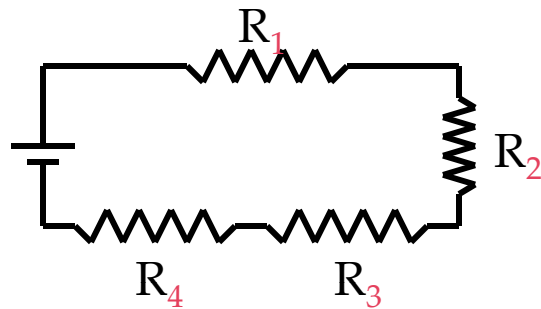
Meso scale *EBC*Meso scale **NBC**

## More complicated geometry

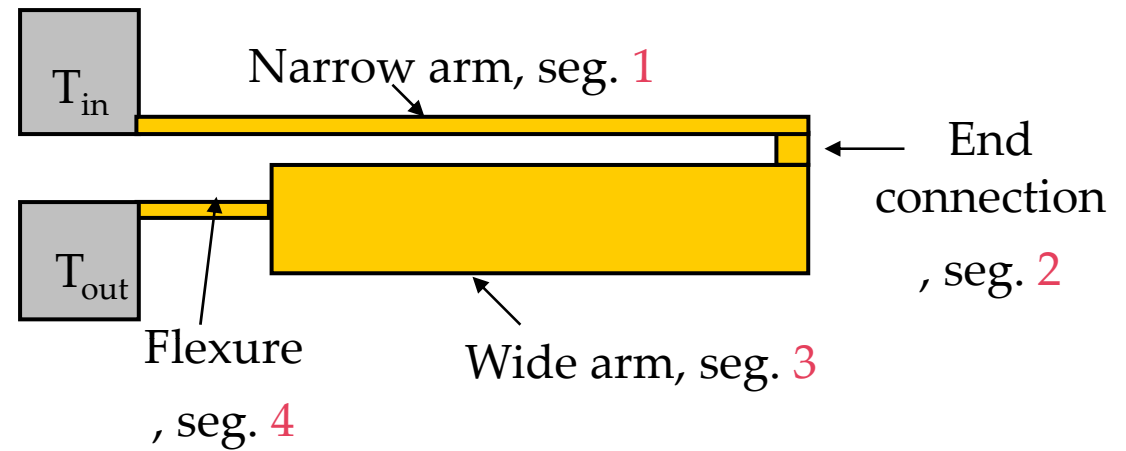




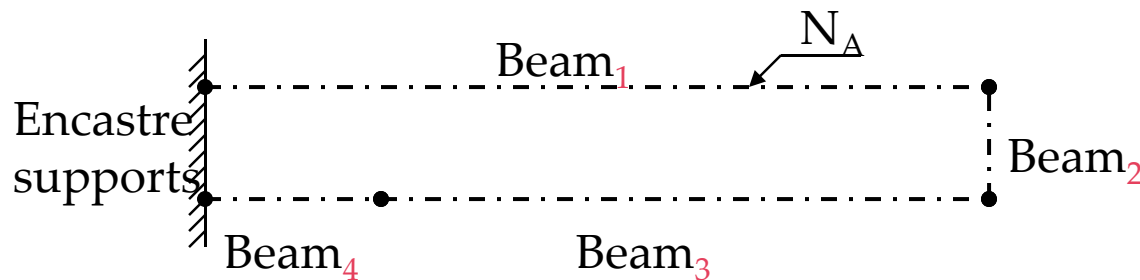
# One dimensional approximation



Electrical Model



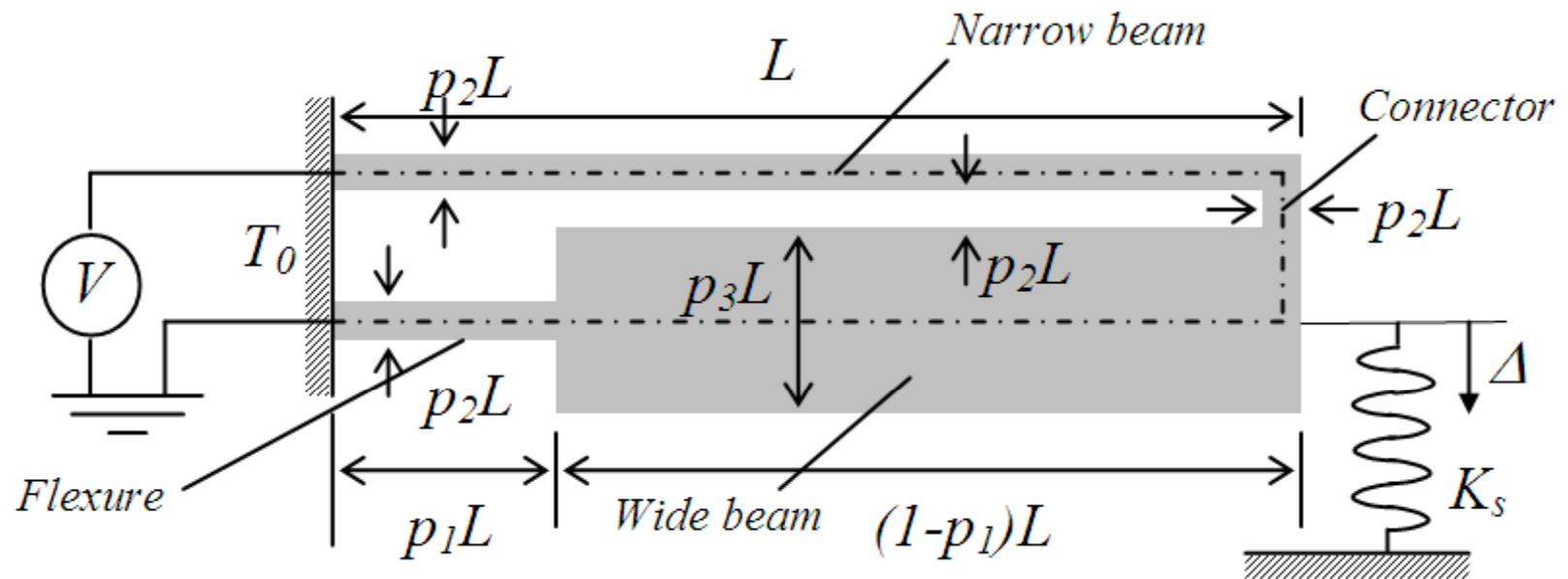
Thermal Model



Elastic Model

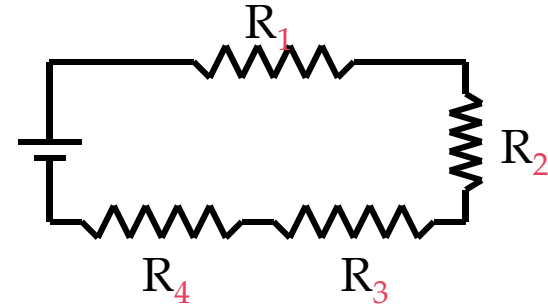
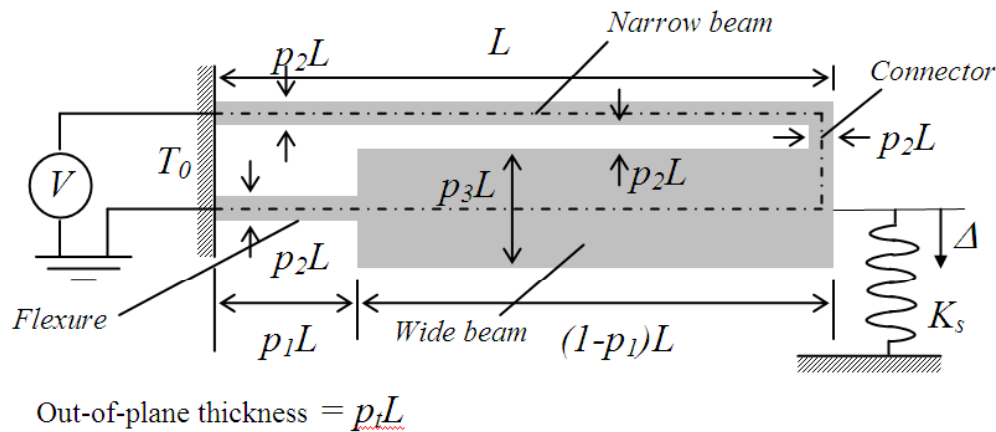
(Maizel's theorem to compute the output deflection)

## Parameters for analytical modeling of electro-thermal-compliant actuator



Out-of-plane thickness =  $p_t L$

# Electrical analysis



$$R_i = \frac{\rho_e L_i}{A_i} \text{ for } i = 1, 2, 3, 4$$

$$R = R_1 + R_2 + R_3 + R_4 = \frac{\rho_e}{L} \left\{ \frac{1}{p_t p_2} + \frac{1}{p_t} + \frac{(1-p_1)}{p_t p_3} + \frac{p_1}{p_t p_2} \right\} = \phi_e \frac{\rho_e}{L}$$

Notice how resistance, current, and dissipated power vary with scaling.

$$J = \frac{V}{R} = \frac{LV}{\phi_e \rho_e}$$

$$P_e = J^2 R = \frac{LV^2}{\phi_e \rho_e}$$

## Coupling between electrical and thermal analysis

---

$$\dot{Q}_{e_i} = \frac{J^2 R_i}{A_i L_i} = \frac{L^2 V^2}{\phi_e^2 A_i^2 \rho_e} \quad i = 1, 2, 3, 4$$

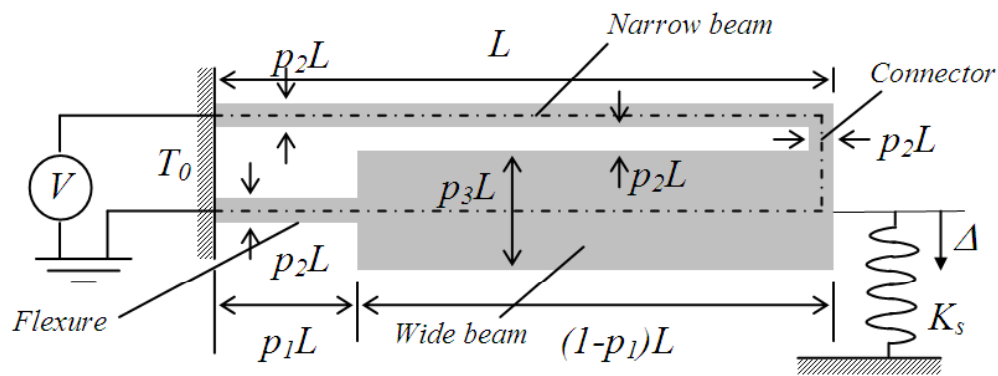
$$\dot{Q}_{e_1} = \frac{V^2}{\phi_e^2 p_t^2 p_2^2 L^2 \rho_e}$$

$$\dot{Q}_{e_2} = \frac{V^2}{\phi_e^2 p_t^2 p_2^2 L^2 \rho_e}$$

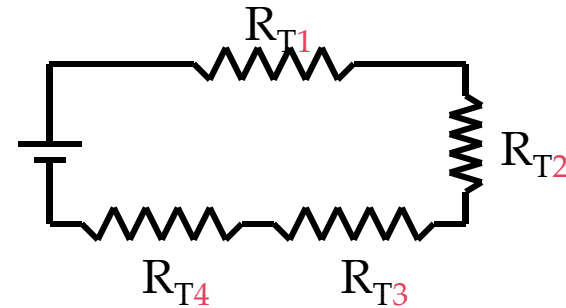
$$\dot{Q}_{e_3} = \frac{V^2}{\phi_e^2 p_t^2 p_3^2 L^2 \rho_e}$$

$$\dot{Q}_{e_4} = \frac{V^2}{\phi_e^2 p_t^2 p_2^2 L^2 \rho_e}$$

# Thermal analysis



Out-of-plane thickness =  $pL$



Temperature profile in the connector is not modeled as it is negligibly short.

$$\frac{d^2 T_i(x)}{dx^2} + \frac{\dot{Q}_{e_i}}{k_t} = 0 \quad i = 1, 3, 4$$

$$T_i(x) = -\frac{\dot{Q}_{e_i}}{2k_t} x^2 + a_i x + b_i \quad i = 1, 3, 4$$

Six constants to be evaluated from the boundary conditions.

## Boundary conditions to solve for constants

$$T_i(x) = -\frac{\dot{Q}_{e_i}}{2k_t} x^2 + a_i x + b_i \quad i = 1, 3, 4$$

1. Temperature raise at the left end is zero.

$$T_1(x = 0) = T_0 \quad \longrightarrow \quad b_1 = T_0$$

2. Temperature raises at the interface of first and third segments are equal.

$$T_1(x = L_1) = T_3(x = 0) \quad \longrightarrow \quad -\frac{\dot{Q}_{e_1}}{2k_t} L^2 + a_1 L + T_0 = b_3$$

3. Thermal equilibrium of the second segment

$$-k_t A_1 \left. \frac{dT_1}{dx} \right|_{x=L_1} - k_t A_3 \left. \frac{dT_3}{dx} \right|_{x=0} + \dot{Q}_{e_2} A_2 L_2 = 0$$

$$-k_t p_t p_2 L^2 \left( -\frac{\dot{Q}_{e_1}}{k_t} L + a_1 \right) - k_t p_t p_3 L^2 a_3 + \dot{Q}_{e_2} p_t p_2^3 L^3 = 0$$

## Boundary conditions (contd.)

$$T_i(x) = -\frac{\dot{Q}_{e_i}}{2k_t} x^2 + a_i x + b_i \quad i = 1, 3, 4$$

4. Temperature raises at the interface of third and fourth segments are equal.

$$T_3(x = L_3) = T_4(x = 0) \quad \Rightarrow \quad -\frac{\dot{Q}_{e_3}}{2k_t} (1-p_1)^2 L^2 + a_3(1-p_1)L + b_3 = b_4$$

5. Heat flux continuity at the interface of third and fourth segments.

$$k_t A_3 \left. \frac{dT_3}{dx} \right|_{x=L_3} - k_t A_4 \left. \frac{dT_4}{dx} \right|_{x=0} = 0 \quad \Rightarrow \quad k_t p_t p_3 L^2 \left( -\frac{\dot{Q}_{e_3}}{k_t} (1-p_1)L + a_3 \right) - k_t p_t p_2 L^2 a_4 = 0$$

6. Temperature raise at the end of the fourth segment is zero.

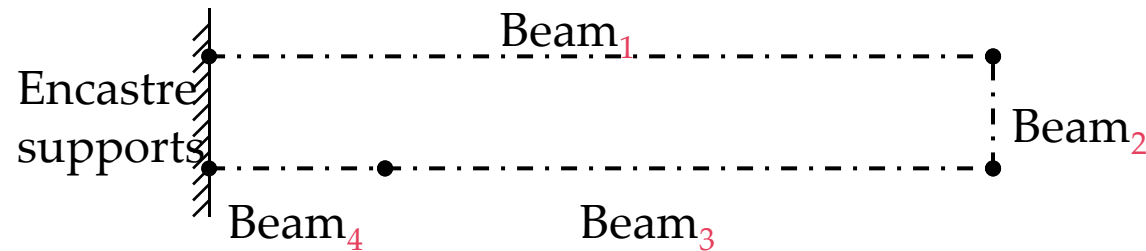
$$T_4(x = L_4) = T_0 \quad \Rightarrow \quad p_1 L a_4 + b_4 = \frac{\phi_{t_5} V^2}{\phi_e^2 \rho_e k_t} + T_0 = c_5$$

$$\text{Total temperature raise} = T_{\max} = \phi_{t_{\max}} \frac{V^2}{\rho_e k_t}$$

Notice the lack of scaling effect!



# Elastic analysis



Maizel's theorem to find the vertical deflection at the tip.

$$\Delta = \sum_{i=1}^4 \left[ \int_0^{L_i} \hat{F}_{axial_i}(x) \alpha \{T(x)_i - T_0\} dx \right]$$

$$\frac{\Delta}{L} = \phi_{\Delta} V^2 \frac{\alpha}{\rho_e k_t}$$

Notice on what quantities the relative deflection depends.

With convection included, it would be different.

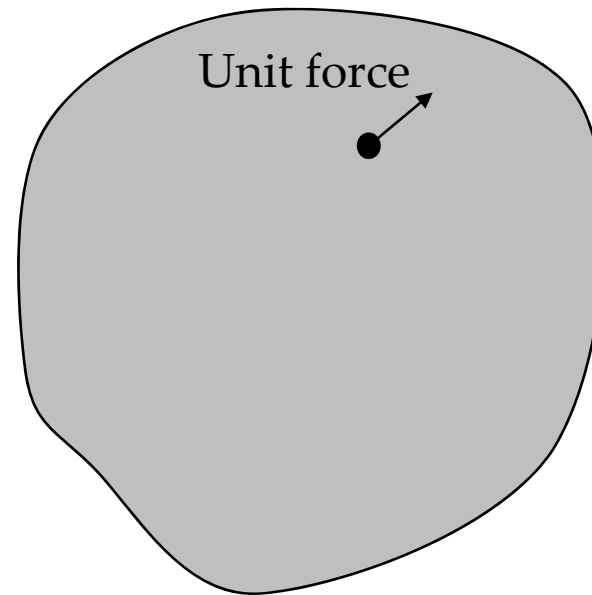
## Maizel's theorem in 3D

$$\Delta = \int_V (\hat{\sigma}_x + \hat{\sigma}_y + \hat{\sigma}_z) (\alpha (T - T_0)) dV$$

$\hat{\sigma}_x$  = normal stress in the  $x$  direction due the application of a unit force at the point of interest in the direction of interest

Similarly, for  $\hat{\sigma}_y, \hat{\sigma}_z$

$T(x, y, z)$  = temperature distribution obtained from thermal analysis



## With convection included...

Without convection...

$$\frac{\Delta}{L} = \phi_{\Delta} V^2 \frac{\alpha}{\rho_e k_t} \propto L^0$$

$$\frac{d^2 T_i(x)}{dx^2} + \frac{\dot{Q}_{e_i}}{k_t} = 0 \quad i = 1..4$$

With convection...

$$\frac{\Delta}{L} \propto \frac{1}{\sqrt{L}}$$

$$\frac{d^2 T_i(x)}{dx^2} - \frac{hp_i(T_1 - T_0)}{k_t A_i} + \frac{\dot{Q}_{e_i}}{k_t} = 0 \quad i = 1..4$$

$$T \propto \frac{\dot{Q}L}{h} \propto \frac{V^2}{\rho_e h L} \propto \frac{1}{L}$$

## Main points

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- Electro-thermal-elastic actuation is easy to implement in practice.
  - Large forces and displacements
  - Slow
- Coupled modelling is sequential, usually...
  - Temperature-dependent properties make it nonlinear.
- Reduced order modelling is convenient and useful.