ME 237	Mechanics of Microsystems	Aug. – Dec., 2010
Final Examination		
Marks: 25	Open notes and open books	Time: 90 minutes

## Question 1 (13 marks)

A bent-beam electro-thermal-compliant actuator is shown in the figure below. Each of its eight inclined beams are l m long, w m wide in the plane, and t m thick out of the plane. Their inclination with the horizontal direction is  $\gamma^{\circ}$ . The mechanical anchors and electrical connections are shown in the figure. The central mass is a rectangular block of length  $l_b$ , in-plane width  $w_b$ , and t thick.

Assume that an ambient temperature of  $T_0$  is maintained at both the anchors. The entire structure except the two anchors has a gap of  $g_0$  m underneath it. Use the following symbols for its material properties.

 $\rho_e$  = electrical resistivity,  $k_t$  = thermal conductivity,  $\alpha$  = thermal expansion coefficient, Y = Young's modulus



- (i) Derive the formula for the lumped electrical resistance between the two anchors. If you make any assumptions regarding the central block, please state them clearly.
- (ii) What current goes through each beam for the applied voltage, V ? Write down the expression.
- (iii) Set up the relevant equations to obtain the temperature profile along each beam. Write also the boundary conditions that will help you solve for the temperature profile. You may neglect convection and radiation.
- (iv) Use Maizel's theorem or any other method to solve for the displacement of the central block for the applied voltage. Show all steps clearly.

## **Question 2** (3×4=12 marks)

(a) A fixed-guided beam of length l m, rectangular cross-section of  $w \times t$  m<sup>2</sup> has a mass of M kg attached to it as shown figure on the right side. Let its deflection under a point transverse load applied on the mass be given by  $(ax^3 + bx^2 + cx)$ . Find the equivalent lumped

## inertia of the beam.

(b1) Compute the fluidic damping force on the plate translating in between two fixed plates. See Figure on the right side. The plate has an area of A. The gap between the top fixed plate the moving plate is  $g_t$ , and that between the bottom fixed plate and the moving plate is  $g_b$ . Use  $\eta$  for denoting the viscosity of the fluid. You may assume Couette flow conditions.

(b2) Show also the velocity profile at the section shown with a dashed line in the figure. Write the expression for the velocity.



(c) A spring of stiffness 0.05 N/m is attached to the moving plate of a parallel-plate capacitor of area  $A = 1500 \ \mu m^2$ . The gap =  $g_0 = 1 \ \mu m$ , and there is a dielectric layer of 0.5  $\mu m$  thickness with relative permittivity  $\varepsilon_r = 3$ . Find: (i) static pull-in voltage, (ii) static pull-up voltage, (iii) dynamic pull-in voltage, and (iv) show the steps for computing the time for pull-in for an applied voltage that is 1.5 times the pull-in voltage.