Final Examination

Marks: 25 Open notes, open books, open Internet **Time: 120 minutes**

```
Use COMSOL MultiPhysics software to solve when it is required.
```

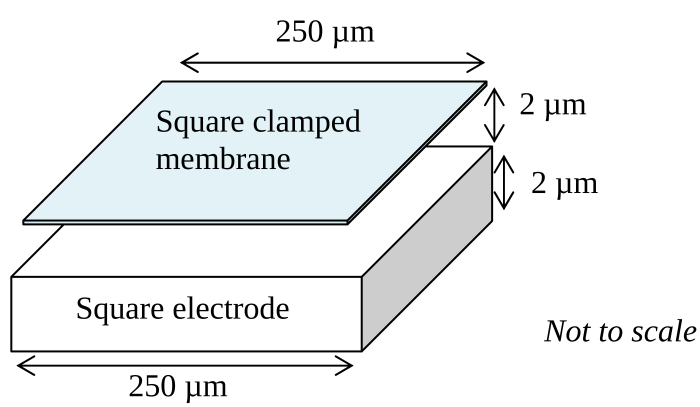
Question 1 *(10 marks) Scaling analysis using parametric sweep*

a. Taking the example of a slender cantilever beam with rectangular cross-section, show analytically using lumped modeling or otherwise that its natural frequency is inversely proportional to the size if the proportions of its dimensions and the material properties are kept constant.
b. Verify the scaling implied in the preceding question using COMSOL MultiPhysics software using parametric sweep.
c. Cantilevers are used for sensing a tiny mass (say, m) attached to them by measuring the shift in the natural frequency, ω. Once again, keeping the dimensional proportions and material properties the same, show analytically how the "change in the frequency with respect to the added mass" (i.e., $\frac{d\omega}{dm}$) varies with size. Note that m should not be changed when you vary the size of the beam. The idea is to decide if miniaturization of cantilevers helps or not in sensing very small m.
d. Verify the scaling trend you derived in part (1c) is in agreement with your simulation in COMSOL, again using parametric sweep.

Question 2 *(15 marks) Coupled analysis*

(i) Consider a pressure sensor consisting of a thin square clamped diaphragm made of silicon (Y = Young's modulus = 155 GPa; ν = Poisson's ratio = 0.3). Let its side be 250 μm and thickness 2 μm. Assume that the process used to make it induced significant residual stress that resulted in membrane tension, τ N/m. We want to model it as a membrane rather than a thin plate.
   a. Using COMSOL, find τ and transverse pressure (in Pa) acting on the membrane that give a maximum transverse deflection of the membrane equal to 1 μm.
   b. What is the maximum stress in the membrane for that value of τ and pressure?

(ii) Let there be an electrode underneath the diaphragm at a distance of 2 μm. The electrode also of square shape with side equal to 250 μm and thickness equal to 2 μm. See Fig. 1. The space between the diaphragm and the electrode and the surrounding area is air.
   a. Use COMSOL to compute the capacitance between the membrane and the electrode when there is no pressure. Check with the parallel-plate approximation and see how close the COMSOL result and the valued calculated with the lumped capacitance formula are.
   b. Couple the equations of the deflection of the membrane and the electrostatic potential distribution between the membrane and the electrode in COMSOL. Now, compute the change in capacitance due to the application of 100 kPa pressure.



Extra credit of 5 marks

Check if the relative change in capacitance ($\Delta C / C_0$) is 1E-5 for 100 Pa pressure on the membrane. If not, what changes would you make dimensions (including gaps) to achieve it?