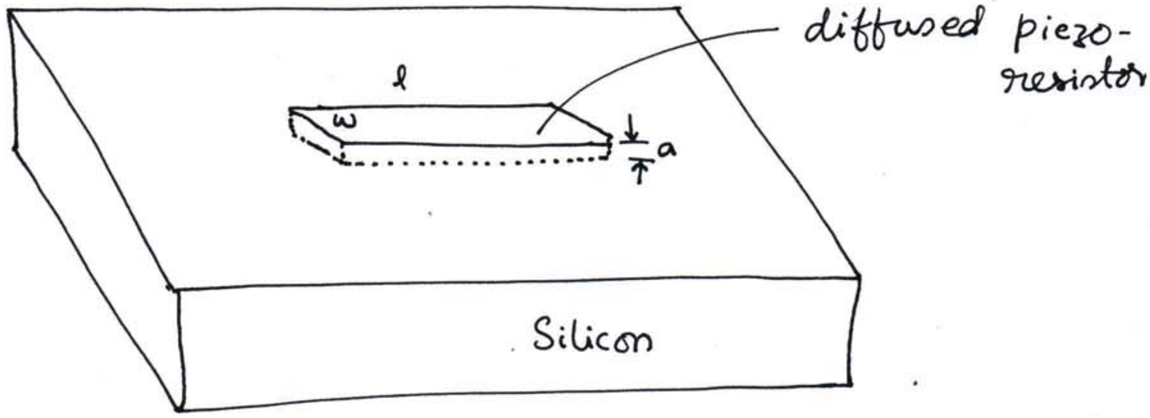
A Self-heated resistor

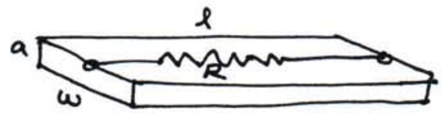
(From "Microsystem Design" by S.D. Senturia) § 11.6.4



As shown in the above figure, a piezo-resistor is formed by ion-implantation and diffusion into a silicon wafer. The dimensions of the resistor are $l \times w \times a$. The top surface of the resistor is flush with the top surface of the silicon substrate.

✱ Electrical resistance of the piezo-resistor

$$= R_0 = \frac{l}{k_e A} = \frac{l}{k_e a w}$$



With $k_e = 1500$ Siemens/m

$l = 300\ \mu m$, $w = 4\ \mu m$, and $a = 2\ \mu m$,

$R_0 = 2.5 \times 10^4 = 25\ k\Omega$

✱ A piezo-resistor responds to mechanical strain by in the structure on which it is mounted by a change ΔR in its resistance. By measuring ΔR, we can measure strain and thereby a signal that caused the strain. Change in resistance is usually measured with Wheatstone's bridge. For this, we need to apply some electric potential across the piezo-resistor. This leads to a current passing through it. Electric current causes the resistor to heat up due to Joule heating. Ensuing temperature rise changes R_0 to $R = R_0(1 + \alpha_{TR} T)$ where α_{TR} = temperature-coefficient of resistance and T the rise in temperature.

A Wheatstone bridge circuit measures total ΔR but cannot distinguish between ΔR caused by strain and ΔR caused by the temperature rise due to passage of current through the resistor (for the purpose of measuring).

$$\underbrace{\Delta R}_{\text{Measured}} = \underbrace{(\Delta R)_{\text{strain}}}_{\substack{\text{what we}\\ \text{want to}\\ \text{measure}}} + \underbrace{(\Delta R)_{\text{temperature rise}}}_{\substack{\text{Disturbance}\\ \text{in measurement}}} .$$

So, we need to make $(\Delta R)_{\text{temp. rise}}$ as small as possible. That is,

$(R_0 \alpha_{TR} T)$ should be as small as possible.

In a typical piezo-resistor used in MEMS, R_0 may change by 1%. That is, $\frac{(\Delta R)_{strain}}{R_0} = 0.01$. Let us say that the relative disturbance $\frac{(R_0 \alpha_{TR} T)}{R_0}$ be 1% of $\frac{(\Delta R)_{strain}}{R_0}$. That is,

$$\alpha_{TR} T = (0.01)(0.01) = 10^{-4}.$$

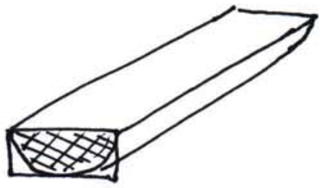
So, we can tolerate only $\frac{10^{-4}}{\alpha_{TR}}$ of temperature rise in the piezo-resistor. With $\alpha_{TR} = 2500 \times 10^{-6}/K$, we have

$$T_{R,max} = \frac{10^{-4}}{2500 \times 10^{-6}} = 0.04 \text{ K}$$

We will now do a calculation to determine what voltage can be applied across the piezo-resistor so that its rise in temperature does not exceed 0.04 K.

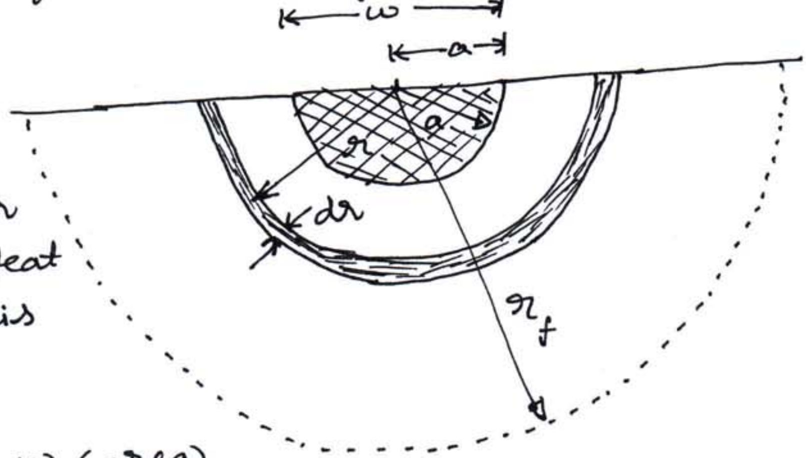
* We now need to do thermal conduction analysis on the piezo-resistor. Convection and radiation are deemed to be negligibly small.

* Computing the electrical resistance was easy because electric current flows only along the piezo-resistor. But Joule heat generated in the resistor gets conducted throughout the substrate underneath. It is semi-infinite medium. So, we make an assumption.



We assume that the temperature in the entire semi-cylinder (or is it hemi-cylinder?) is constant. Let it be T_a.

Let the radius of the semi-cylinder be a.



Let us consider a semi-circular ring at radius r with width dr. Heat flowing through this is given by

$$I_h = (\text{heat flux})(\text{area})$$

$$= J_h (\pi r l) = \text{"heat current"}$$

$$J_h = -k_{th} \frac{dT}{dr}$$ (Fourier's law; constitutive equation in heat conduction)

Now, $$I_h = -k_{th} \frac{dT}{dr} \pi r l$$

$$\Rightarrow \quad dT = -\frac{I_h}{k_{th}\pi l} \cdot \frac{dr}{r}$$

Integration on both sides gives: $$\int_{T_a}^{T_f} dT = -\frac{I_h}{k_{th}\pi l} \int_a^{r_f} \frac{dr}{r}$$

$$\Rightarrow \quad T_f - T_a = -\frac{I_h}{k_{th}\pi l} \ln\left(\frac{r_f}{a}\right)$$

$$\Rightarrow \quad T_a = T_f + \frac{I_h}{k_{th}\pi l} \ln\left(\frac{r_f}{a}\right)$$

Lumped thermal resistance $$= \frac{T_a - T_f}{I_h} = \frac{\ln(r_f/a)}{k_{th}\pi l} = R_{th}$$

With $k_{th} = 148\ W/(K\text{-}m)$

$r_f = 10a$ (assumed to be far enough to model the semi-infinite medium)

$$R_{th} = \frac{\ln(10)}{k_{th}\pi l} = 16.51\ K/W$$

Notice this strange unit!
$(1/R_{th})$ = thermal conductance
= power per unit temperature.

* We still do not know T_a. This is determined by heat capacity of the piezo-resistor.

$$\underbrace{\text{Heat capacity}}_{\text{Thermal capacitance}} = C_{th} = \rho_m \underbrace{awl}_{\text{volume of the piezo-resistor}} c_m \approx 4\times10^{-9}\ J/K$$

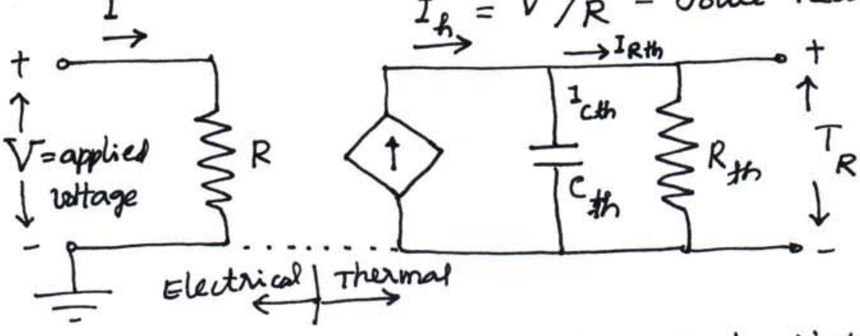
mass density $= 2{,}330\ kg/m^3$

specific heat $= 712\ J/(kg\text{-}K)$ = Energy required to heat unit mass by 1 K rise in temperature

Heat used to raise the temperature of the piezo-resistor at steady state = Joule heat generated − heat conducted to the substrate

We can use lumped thermal resistance (R_{th}) and lumped thermal capacitance (C_{th}) in a circuit along with Joule heating as a "current source". Let us also couple the resulting circuit with the electrical circuit powering the piezo-resistor.

$I_h = V^2/R$ = Joule heat current



Since $R = R_0(1 + \alpha_{TR} T)$, and since $I_h = V^2/R_0(1+\alpha_{TR}T)$, the electrical and thermal circuits are coupled.

From the electro-thermal circuit,

$$I_h = I_{Cth} + I_{Rth}$$

$$\Rightarrow \frac{V^2}{R_0(1+T_R\alpha_{TR})} = C_{th}\frac{dT_R}{dt} + \frac{T_R}{R_{th}} \quad \left\{ \begin{array}{l} \text{By applying circuit analogy} \\ T_R \rightarrow \text{"voltage" in the} \\ \text{thermal circuit} \end{array} \right.$$

$$\Rightarrow \frac{dT_R}{dt} = \left(\frac{V^2}{R_0(1+\alpha_{TR}T_R)} - \frac{T_R}{R_{th}} \right) \frac{1}{C_{th}}$$

We make an approximation by assuming that $(\alpha_{TR}T)$ is very small.

$$\frac{1}{1+\alpha_{TR}T_R} \approx 1 - \alpha_{TR}T_R.$$

Now, we have: $\frac{dT_R}{dt} = \left\{ \frac{V^2}{R_0}(1-\alpha_{TR}T_R) - \frac{T_R}{R_{th}} \right\} \frac{1}{C_{th}}$

$$\Rightarrow \frac{dT_R}{dt} = \frac{V^2}{R_0C_{th}} - \frac{V^2\alpha_{TR}R_{th}}{R_{th}C_{th}R_0}T_R - \frac{1}{R_{th}C_{th}}T_R$$

$$\Rightarrow \frac{dT_R}{dt} = \frac{V^2}{R_0C_{th}} - \frac{1}{R_{th}C_{th}}\left(\frac{\alpha_{TR}R_{th}V^2}{R_0} + 1 \right) T_R$$

Solution of this differential equation is given as

$$T_R = \frac{R_{th}C_{th}}{\left(\frac{\alpha_{TR}R_{th}V^2}{R_0} + 1 \right)} \left\{ \frac{V^2}{R_0C_{th}} - e^{-\left(\frac{\alpha_{TR}R_{th}V^2}{R_0} + 1 \right) \frac{1}{R_{th}C_{th}} t} \right\}$$

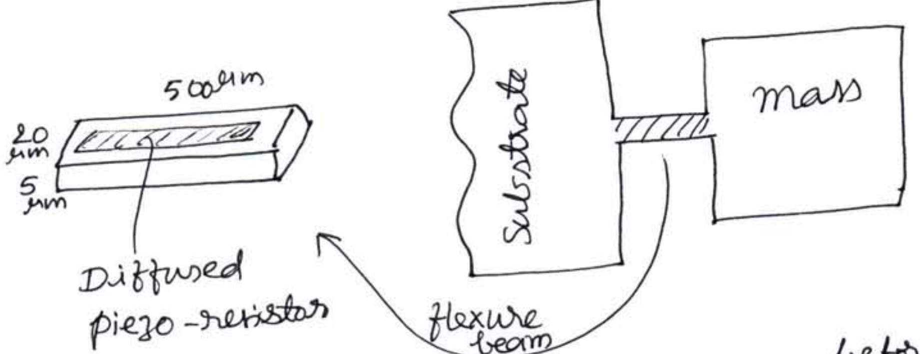
$$\text{Time constant} = \frac{R_{th}C_{th}}{\frac{\alpha_{TR}R_{th}V^2}{R_0} + 1} \simeq \tau$$

$$T_{R_{t\to\infty}} = \frac{V^2R_{th}/R_0}{\left(\frac{\alpha_{TR}R_{th}V^2}{R_0} + 1 \right)} = T_{R_{max}} = 0.04\ K$$

$$\Rightarrow V = 7.78\ V \quad \text{and} \quad \tau = 65.72\ ns$$

Thus, unless we apply 7.78 V, the temperature rise
will not be high enough to cause more than 1%
error in measuring strain. Notice also that the
time constant is very small. In any case, since
7.78 V is much larger than voltage across a
single resistor, we are safe here.

But what if a piezo-resistor is on a
50 μm × 20 μm × 5 μm flexure beam with the
substrate on one side and a suspended mass
on the other? See the figure below.



Electrical resistance will be the same as before.
Fortunately, thermal resistance can be calculated
rather easily now because the rectangular
parallelopiped has big thermal masses on both the
side. So, $R_{th} = \frac{l}{K_{th} A} = \frac{500 \times 10^{-6}}{148 \times (20 \times 10^{-6} \times 5 \times 10^{-6})}$

$$= 337.8 \times 10^{3} \, K/W.$$

Now, V for 0.04 K rise in temperature
is given by 0.17 V — a very small voltage
indeed. So, electronic circuit needs to be
designed carefully so as not to apply more than
0.17 V across this piezo-resistor.
Now, the time constant is ~135 μs — a longer wait
to reach the steady state as compared to 65 ns
earlier.