Incorporating Residual Stresses in 2D Finite Element Code

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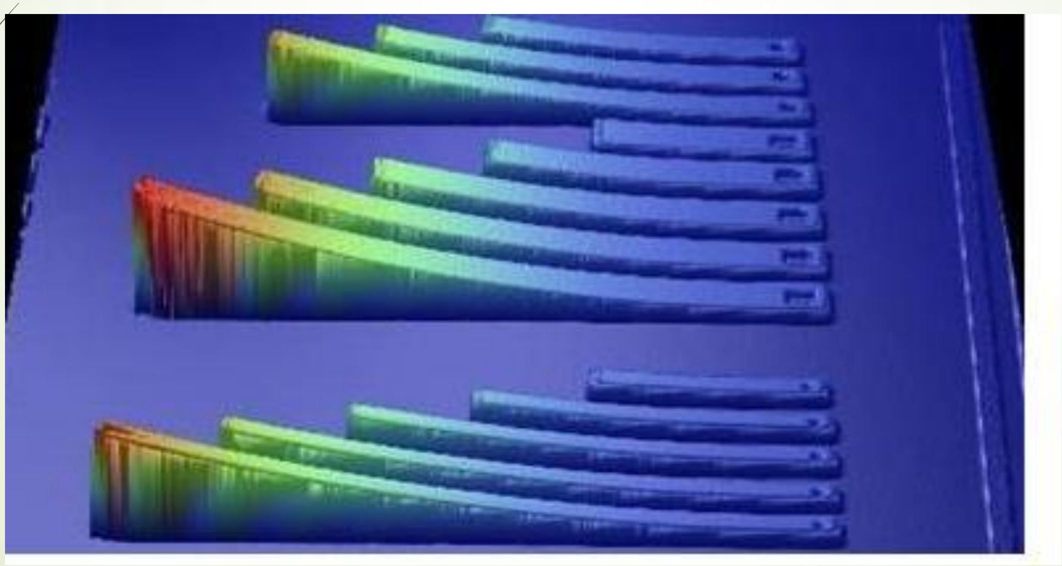
What is Residual Stress

Residual stresses are the stresses that remain in a solid material after the original cause of the stresses has been removed.



Taken from Wikipedia

Reasons of Residual Stress

- Due to different coefficients of thermal expansions
- Oxidation
- Substitutional doping
- Ion Implantation
- Mismatch of lattice spacing in epitaxial growth
- Rapid addition of materials

Governing equation and weak formulation

The governing equation with residual stress is given by

$$\mathrm{YI}\frac{d^4w}{dx^4} + \sigma_r bt\,\frac{d^2w}{dx^2} = \mathrm{q_r}$$

The weak formulation is given by

$$\int_0^L YI\,\frac{d^2w}{dx^2}\frac{d^2v}{dx^2}\,\mathrm{dx} - \int_0^L \sigma_r bt\,\frac{dw}{dx}\frac{dv}{dx}\,dx = \int_0^L q_r\,dx$$

Finite element formulation

- The governing equation is

$$\nabla \cdot \sigma + b = 0$$

$$\sigma = C\varepsilon + \sigma_{\mathrm{r}}$$

Over the entire domain

Where b = body force

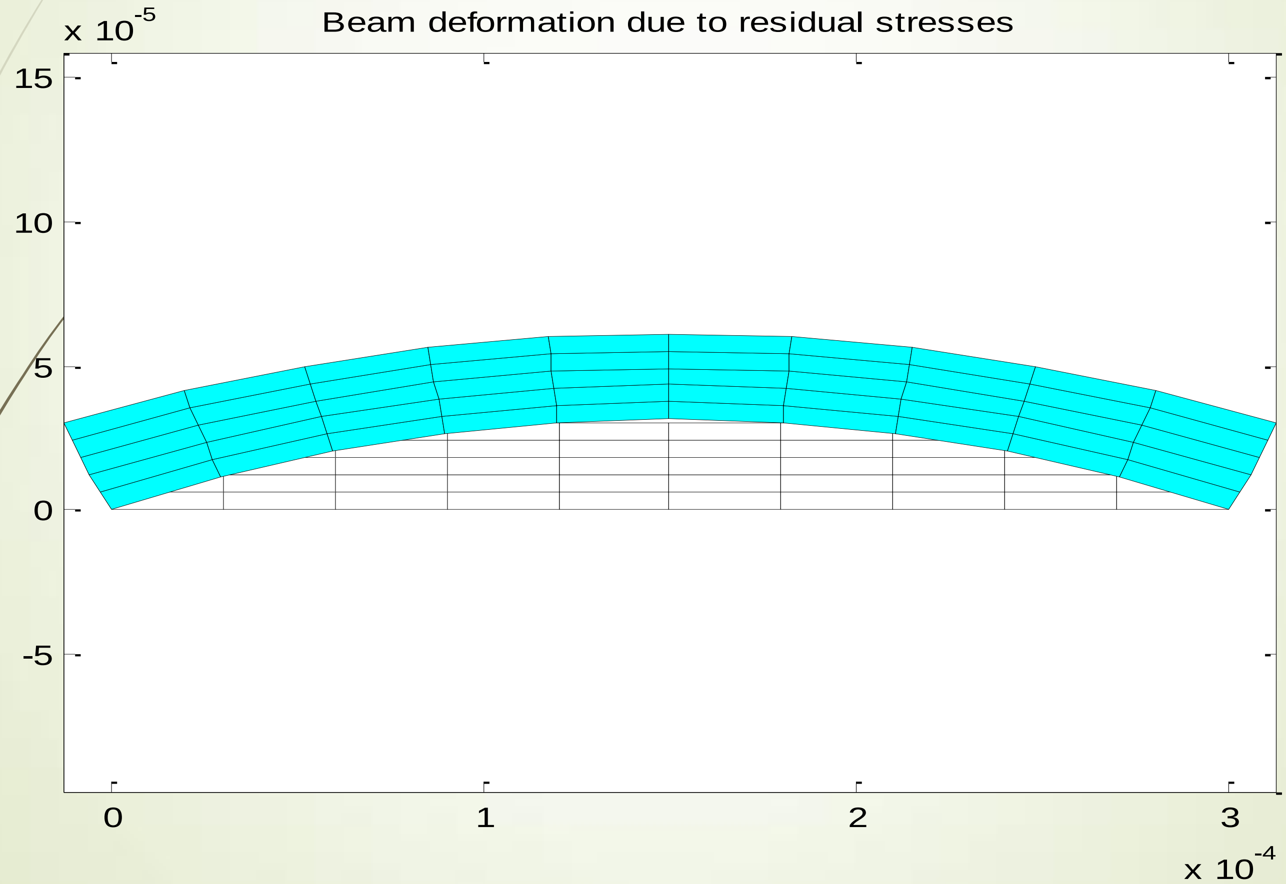
The finite element formulation for the continuum plane stress elements is given as

$$\int B^{\mathrm{T}} C B \, d\Omega = \int N^{\mathrm{T}} b \, d\Omega - \int B^{\mathrm{T}} \sigma_{\mathrm{r}} \, d\Omega$$

over the entire domain.

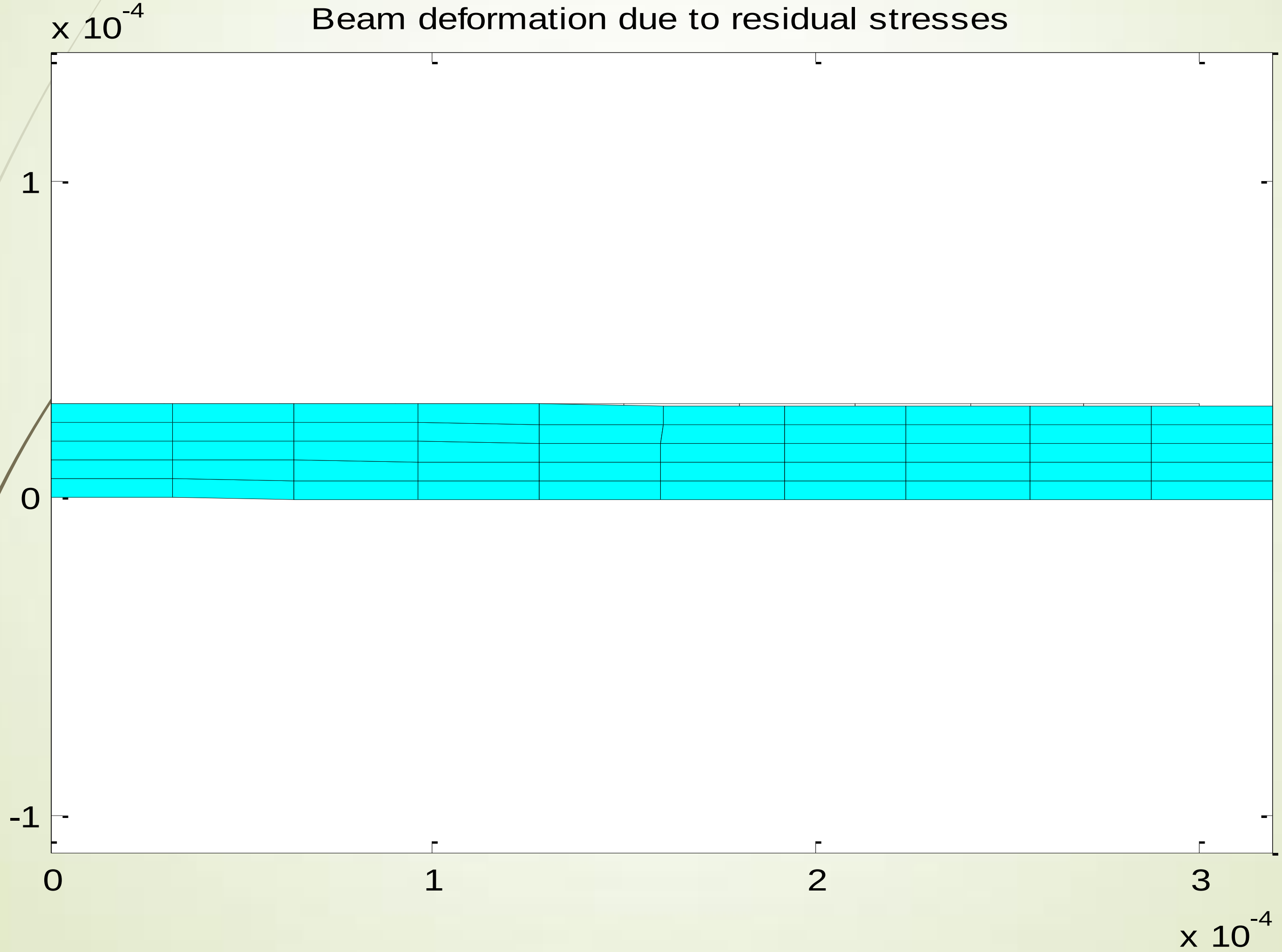
By this we would be able to convert residual stress term into load vector

FEA Results



Beam deformation due to residual stresses
x 10^-5
15
10
5
0
-5
0
1
2
3
x 10^-4

FEA Results (Cont.)

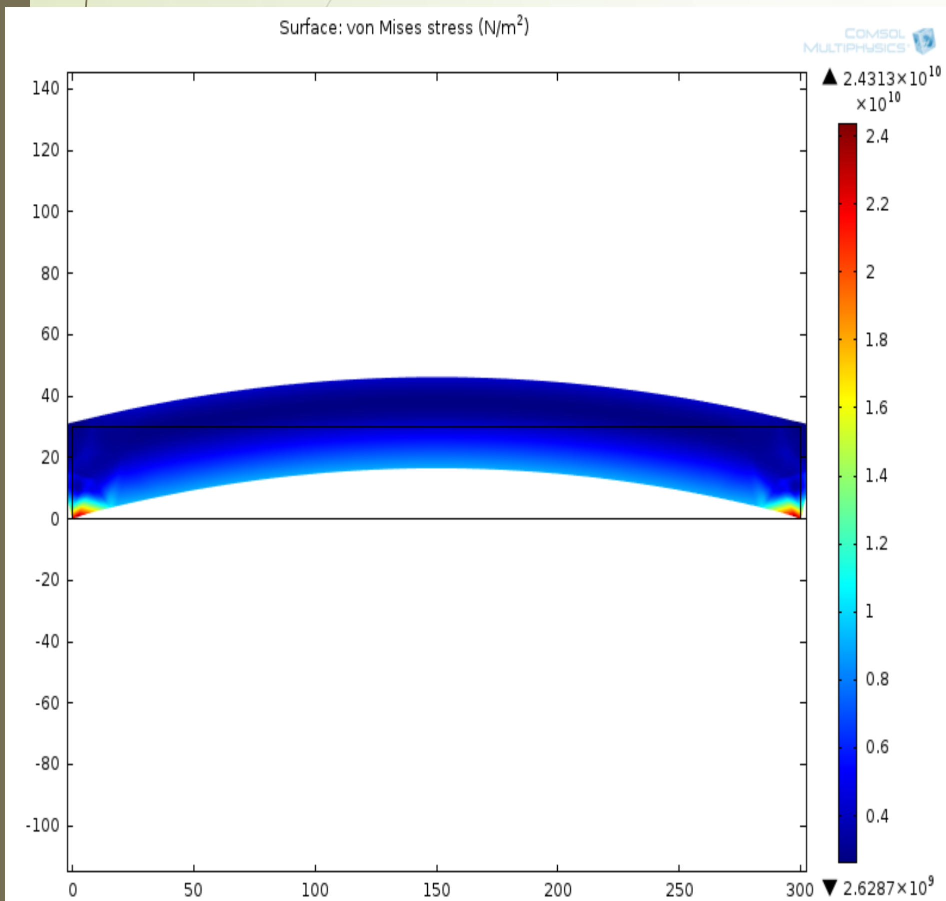


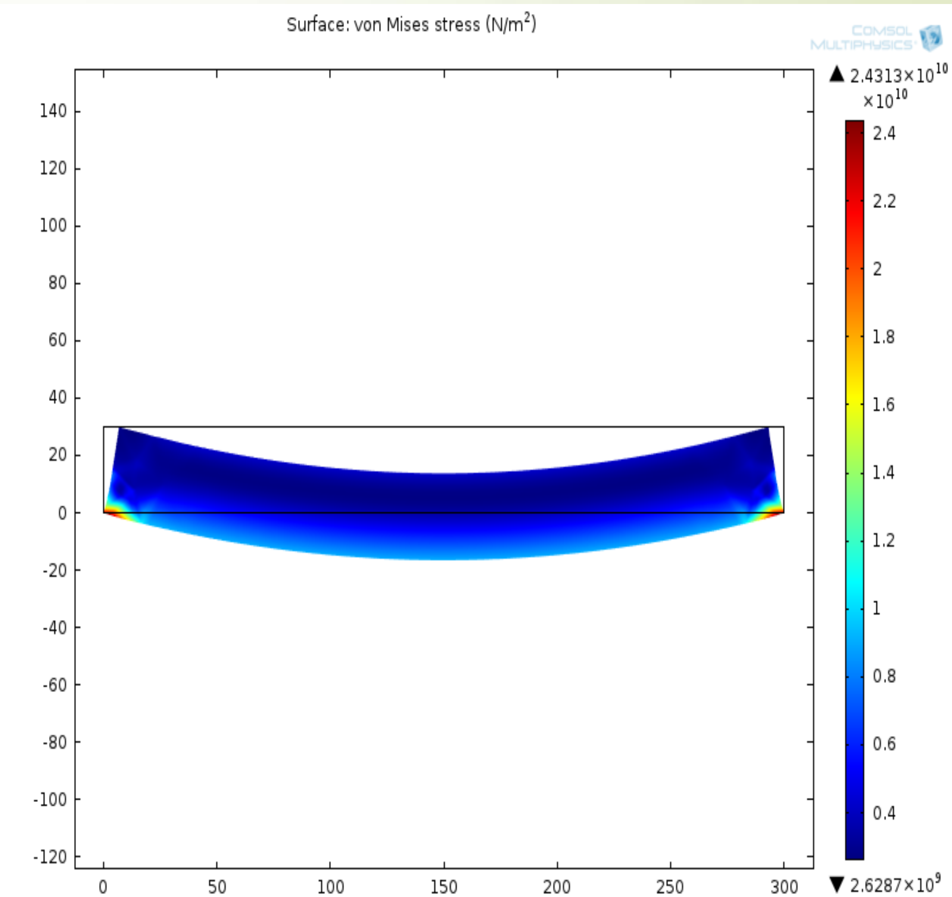
Results and conclusion

As we are applying positive i.e. tensile residual stress, the cantilever will try to contract and vice versa

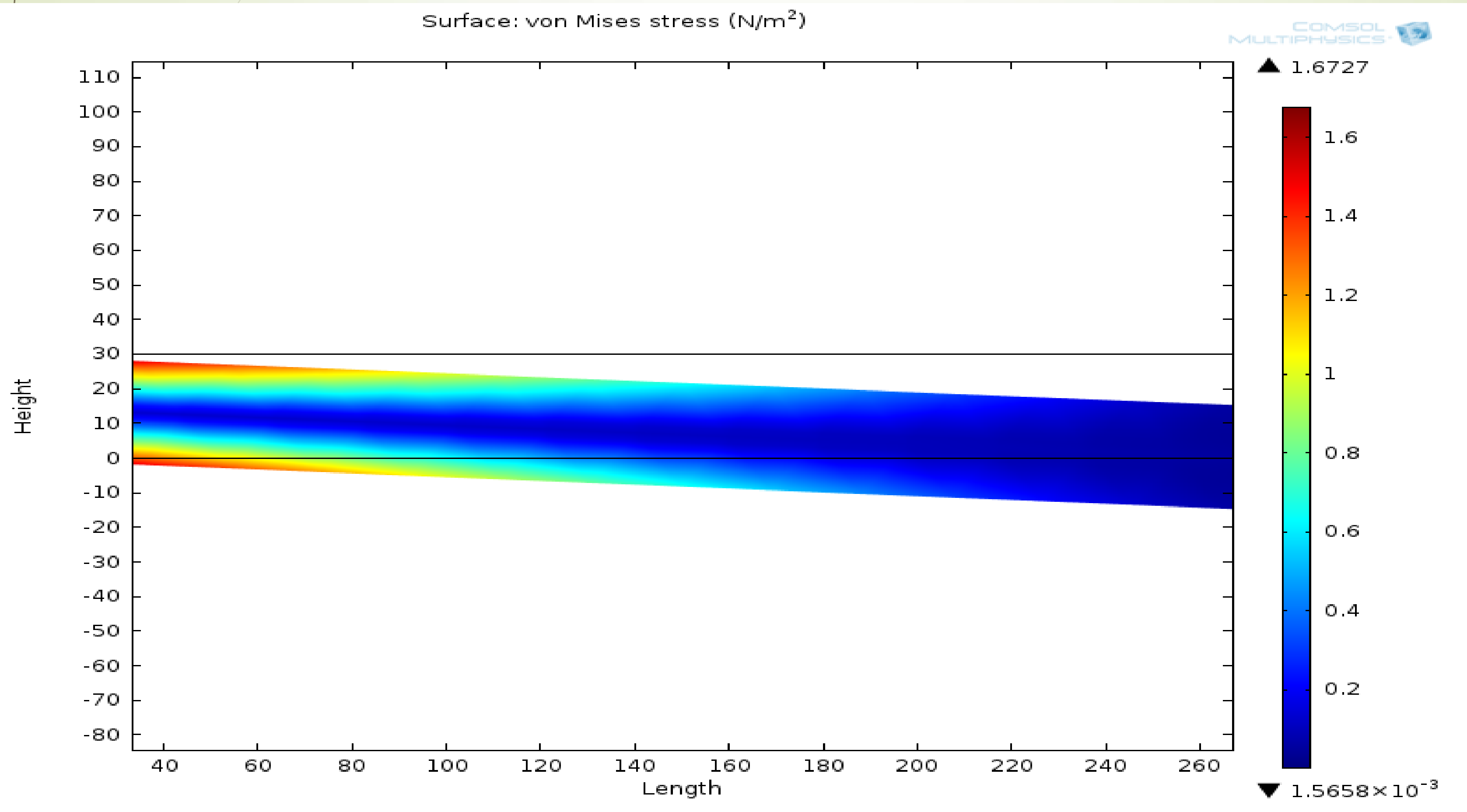
For the simply supported beam, the beam will deflect upwards as it will try to contract again.

COMSOL Simulation





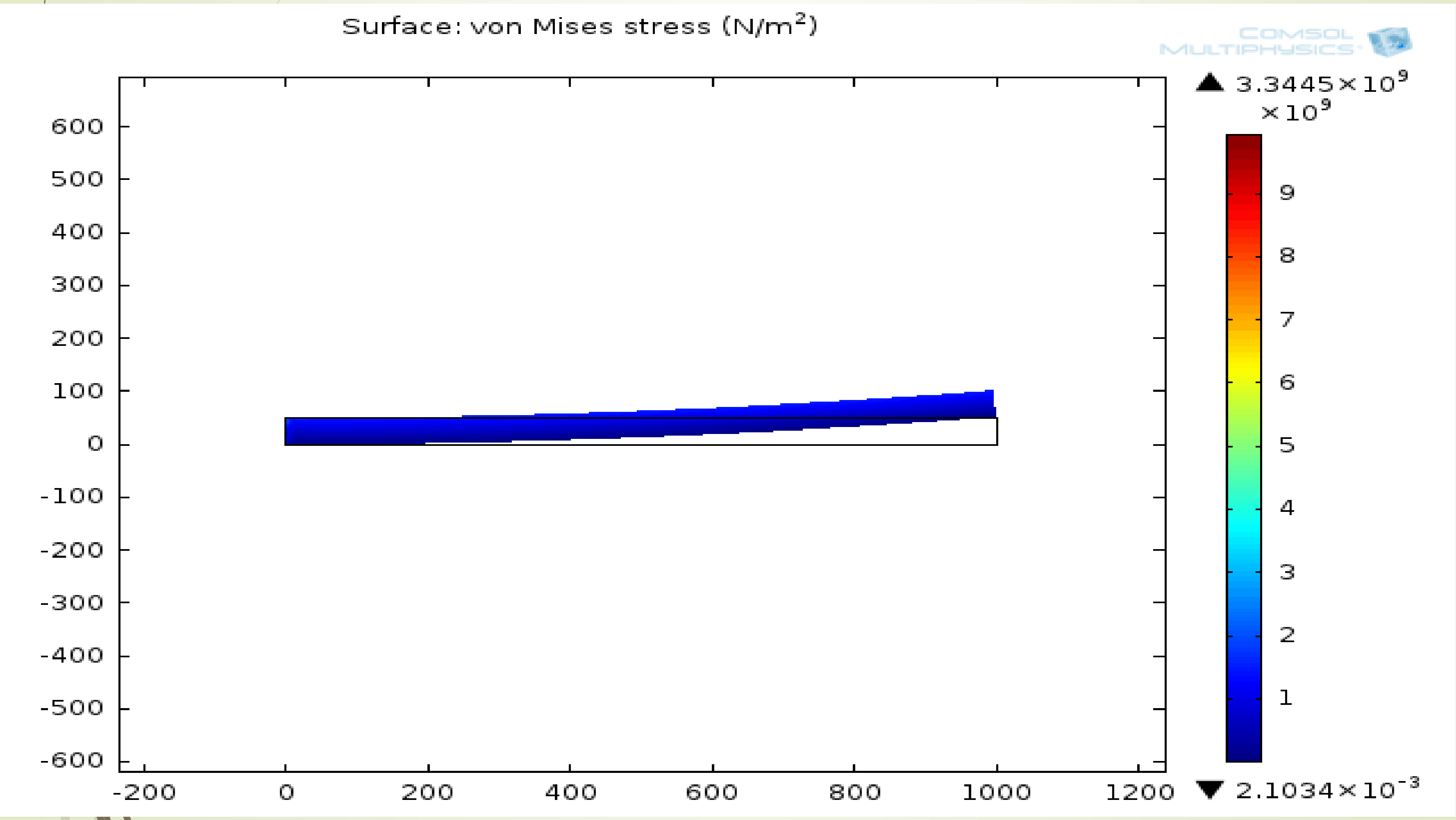
COMSOL Simulation (Cont.)



Stress Gradient

When there is a residual stress gradient in some direction in the beam, then it causes a bending moment at each cross section. The deflection profile concludes that the curvature is constant and thus beam bends into a circular arc.

Curling of cantilever



Something interesting

We can incorporate geometric nonlinearities to consider large displacements of the beams.

By applying stress gradients and obtaining a polar array of curled up cantilever beams and also considering anticlastic effect we can form a cell cage for capturing biological cells.

References

Micro and Smart Systems by G.K. Ananthasuresh, K.J. Vinoy, S. Gopalakrishnan, K.N. Bhat, V.K. Aatre

- THANK YOU