Design of a compliant micromechanical suspension to prescribed stiffness and frequency

NE-211 – Micro and Nano Mechanics

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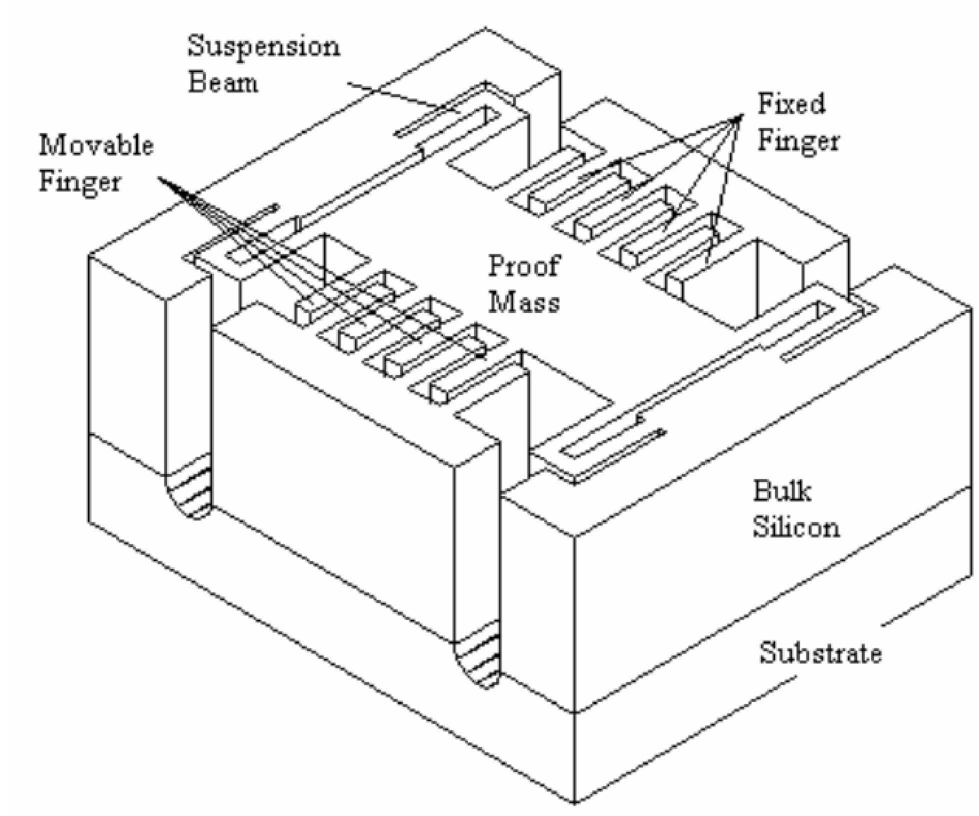
Contents

Automotive crash accelerometer

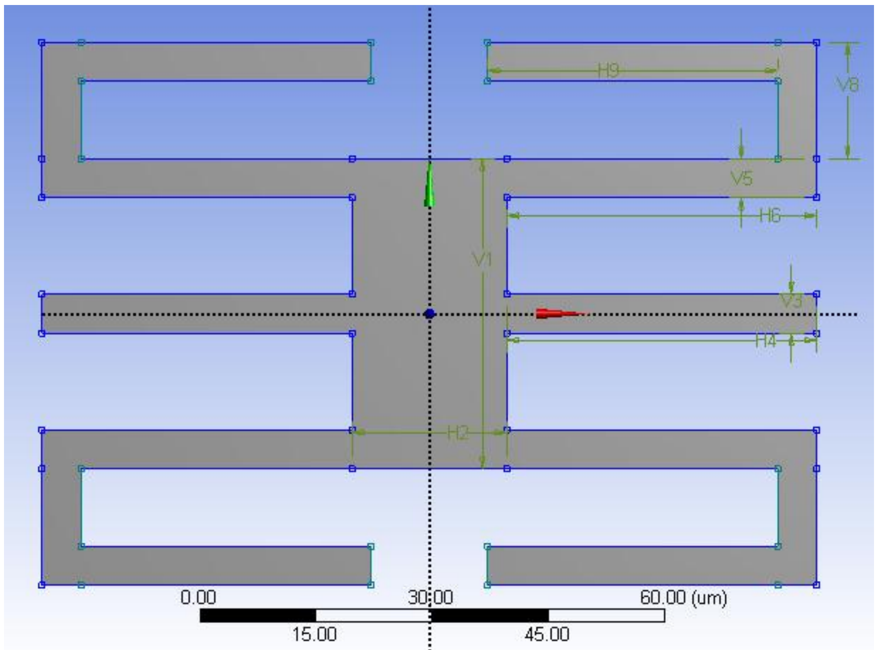
- Typical force: 1000g – Stiffness co-relation
- Frequency: 5-10 KHz
- Resonance effect
- Required fundamental frequency: ~10^4 -10^5 Hz

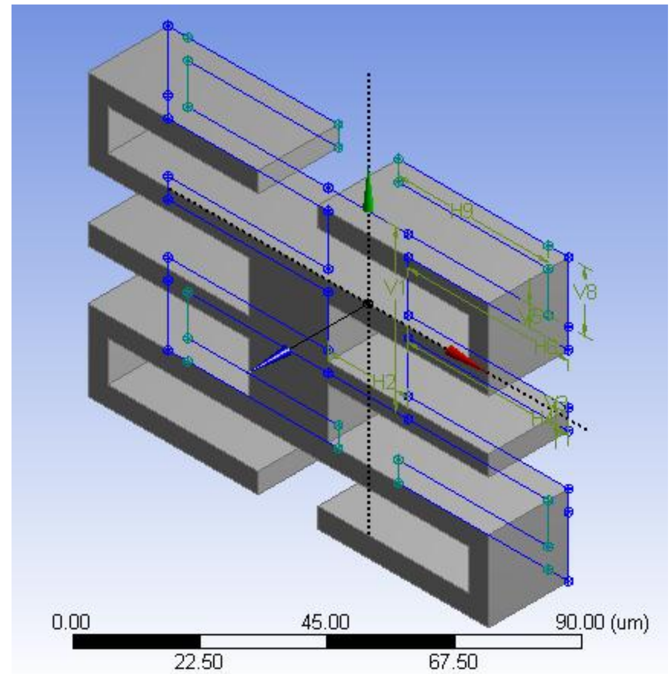
The Folded Beam Structure



Wai-Chi et.al, Arch. Mech, 62

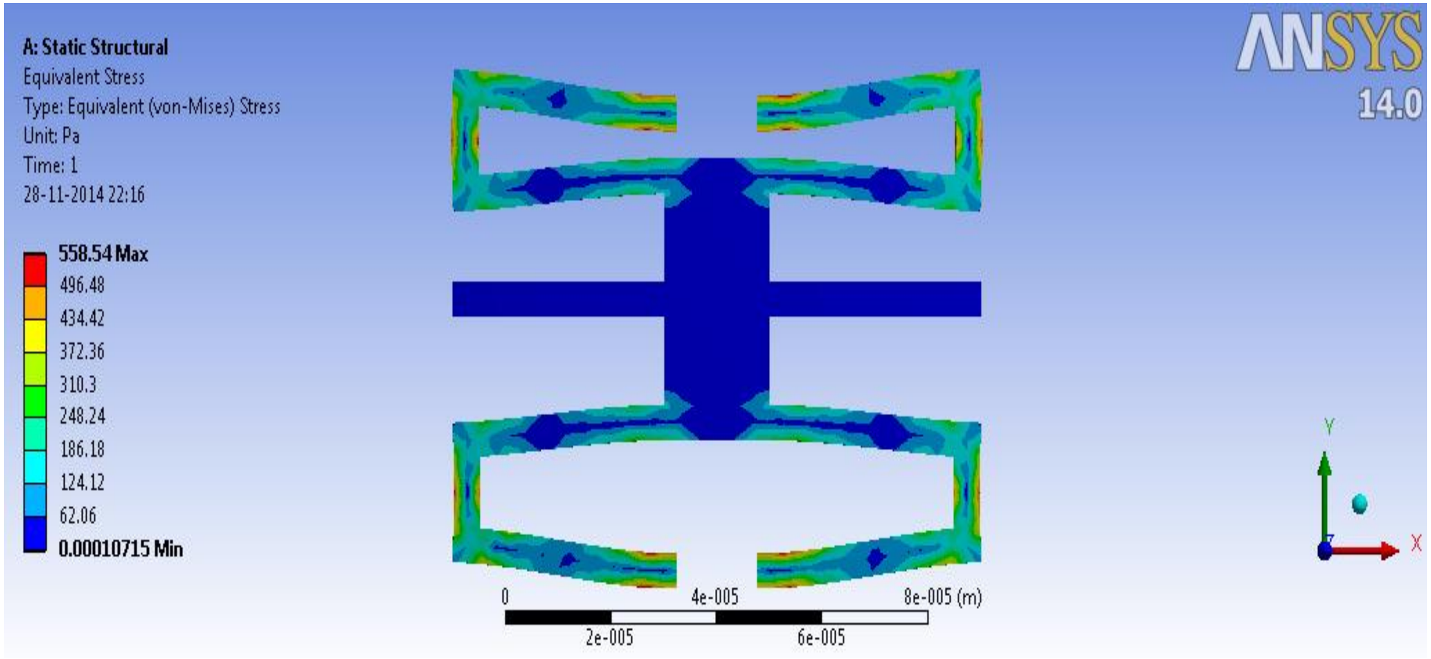
The Folded Beam Structure



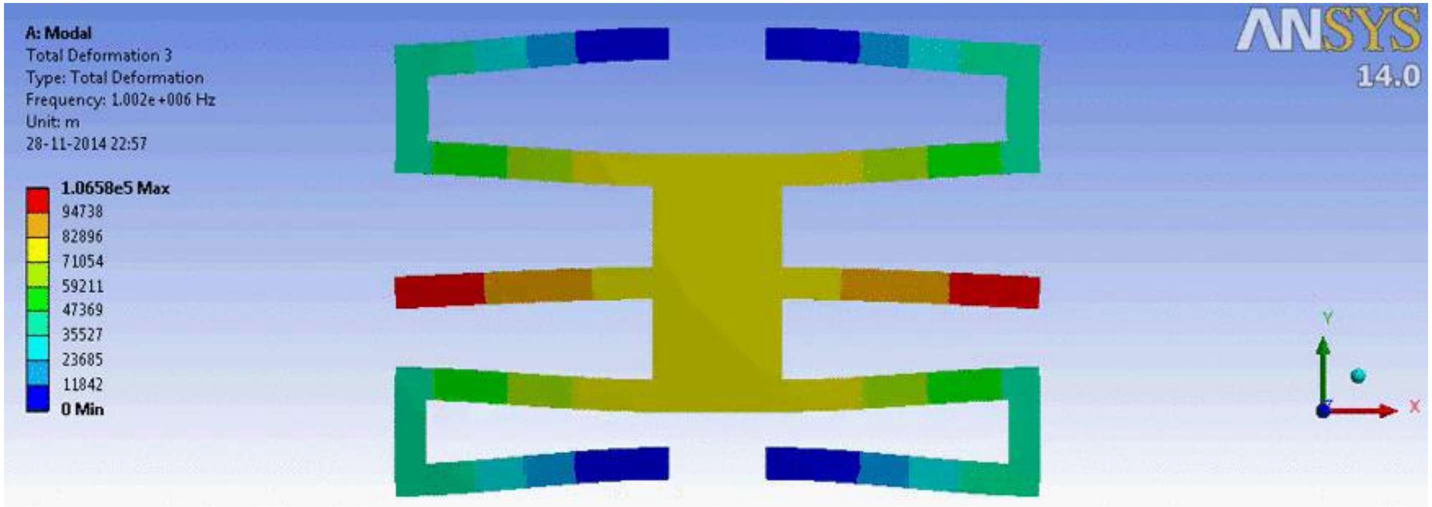


Deflection:
For a force corresponding to 1000g

Stress:
For a force corresponding to 1000 g



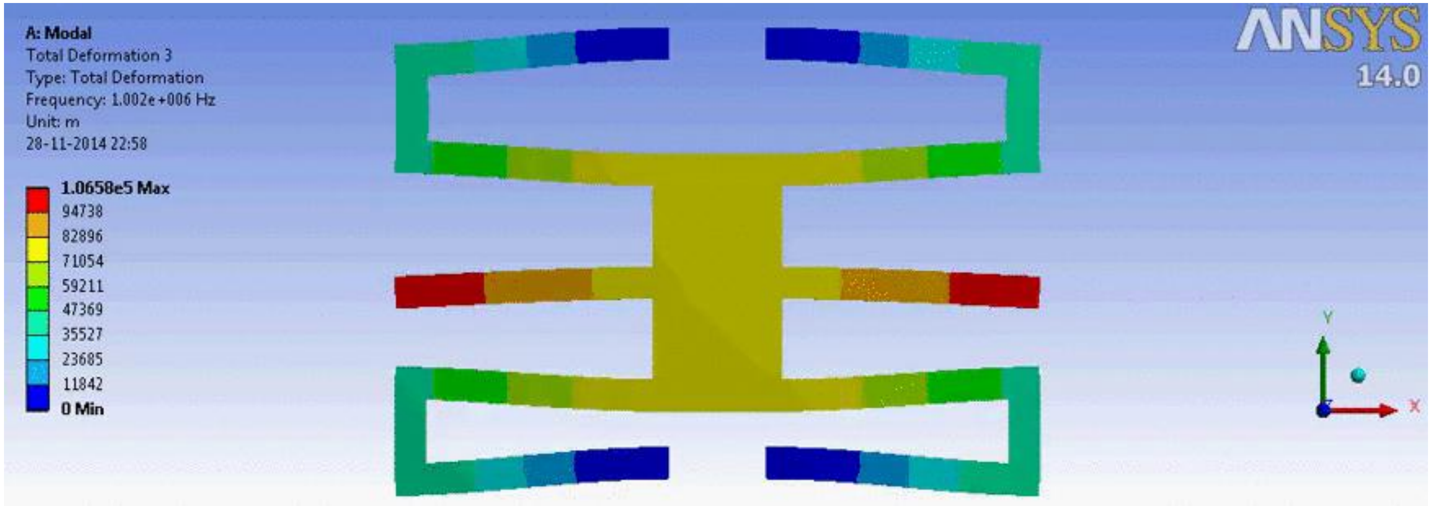
Various mode shapes



Fundamental frequency : ~10^6 Hz

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eff}}}$$

Various mode shapes



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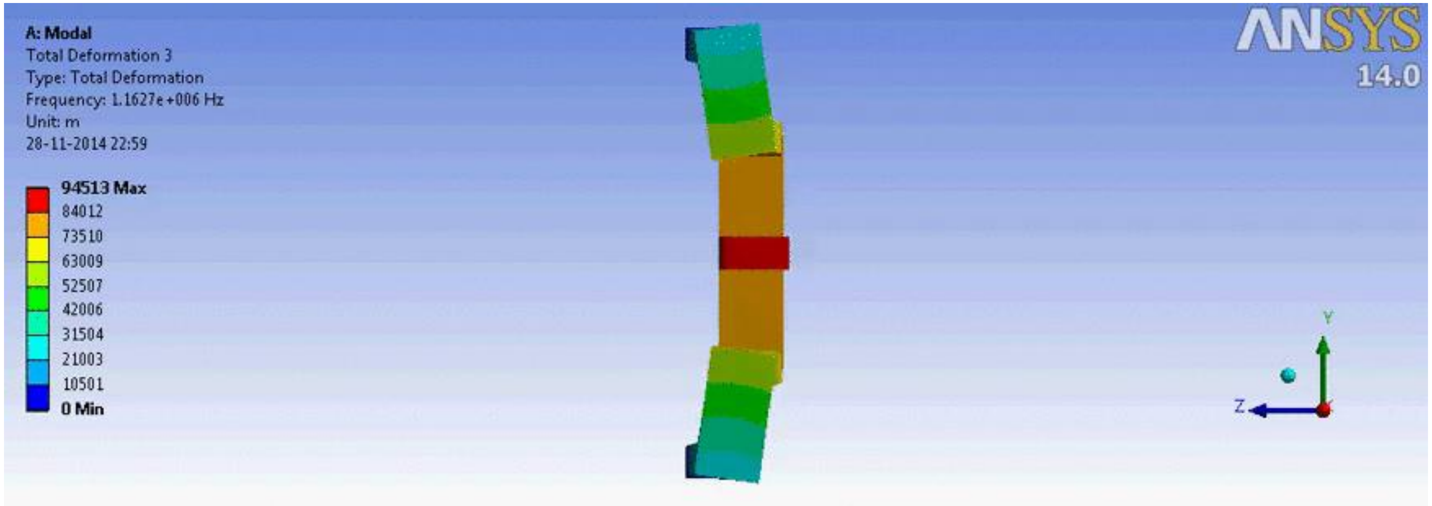

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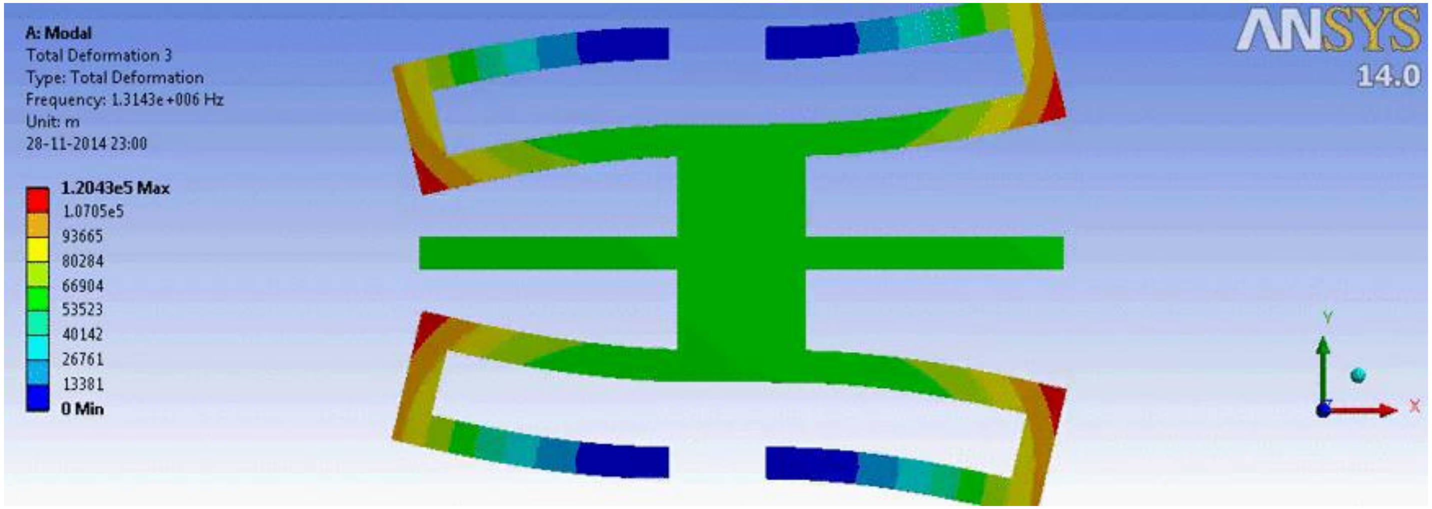

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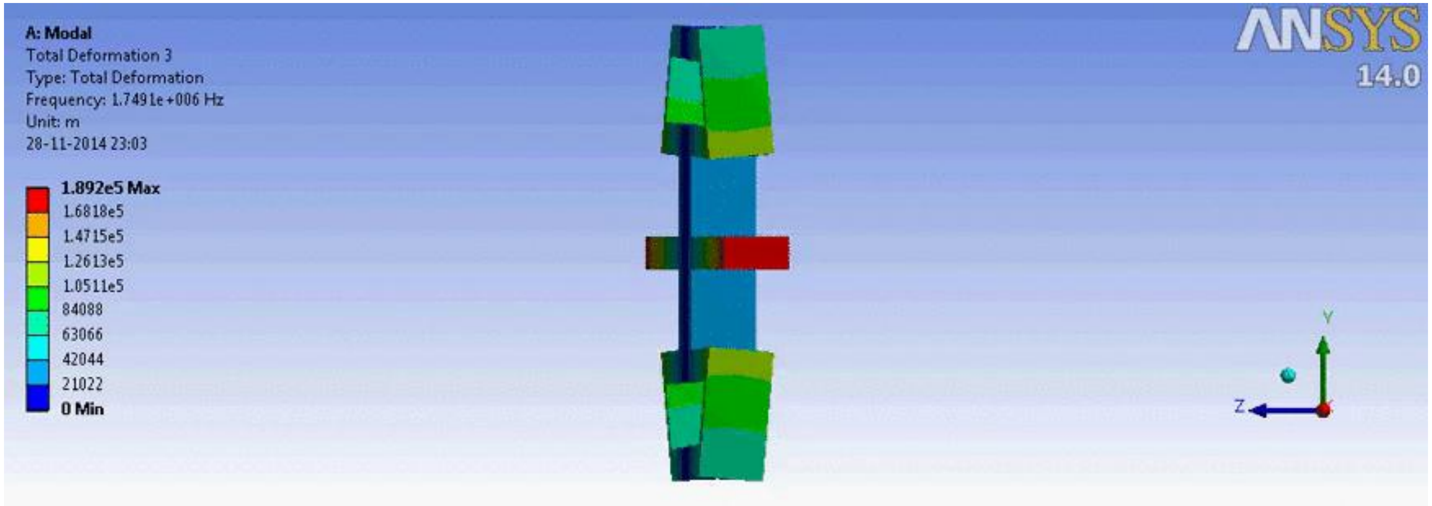

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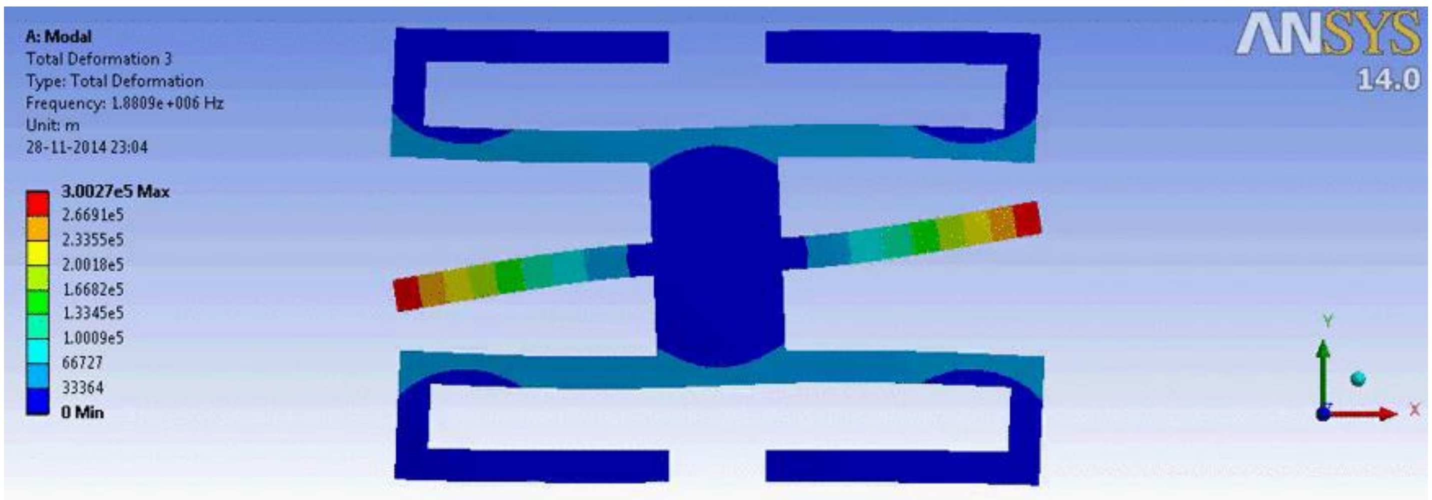


Fundamental frequency : ~10^6 Hz

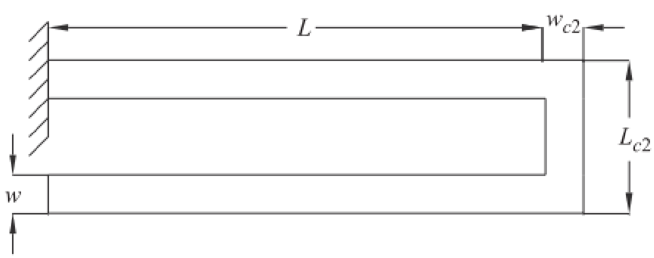
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eff}}}$$

Stiffness – Castigliano's theorem

$$U = \int_V \frac{1}{2}\frac{\sigma^2}{E} dV \qquad \sigma = My / I \qquad I = \int_A y^2 dA$$

$$U = \int_0^L \int_A \frac{M^2}{2EI^2} y^2 dA dx = \int_0^L \frac{M^2}{2EI} dx \qquad \delta_c = \frac{\partial U}{\partial R} = \frac{\partial}{\partial R}\int_0^L \frac{M^2}{2EI} dx$$

$$\frac{1}{k_{eff}} = \frac{1}{Et} \times \left[\frac{L^3}{2w^3} + \frac{3(1+\gamma)L}{5w} + \frac{l}{4w_l} - \frac{3Ll^2}{4w_l^3}\right]$$



Wai-Chi et.al, Arch. Mech, 62

Effective mass – Rayleigh principle

- For a beam of cross sectional area A, length L, and displacement at any point Δ(x), velocity at any point $\frac{d(\Delta(x))}{dt}$, and distribution function $N(x) = \frac{\Delta(x)}{\Delta_{max}}$, effective mass is given by
- $m = \rho \int_0^L N^2(x).A(x)\,dx$

$$m_{eff} = 8\left(\frac{13}{35}\rho A L\right) + 4\left(\frac{1}{3}\rho A_l l\right) + m_{proof\ mass} + m_{comb\ fingers}$$

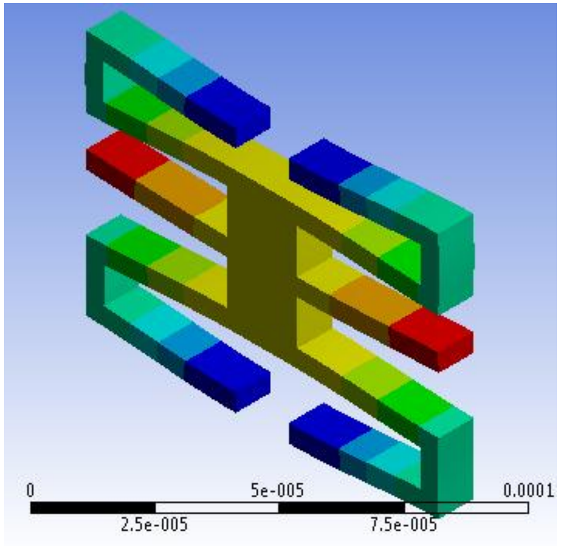
Wai-Chi et.al, Arch. Mech, 62

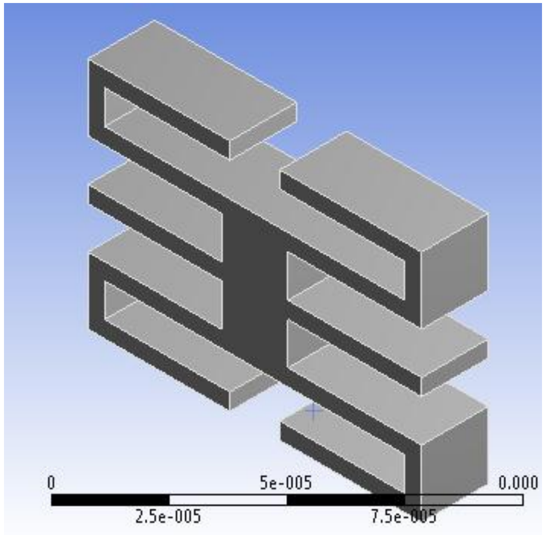
Parameters that 'might' affect resonant frequencies

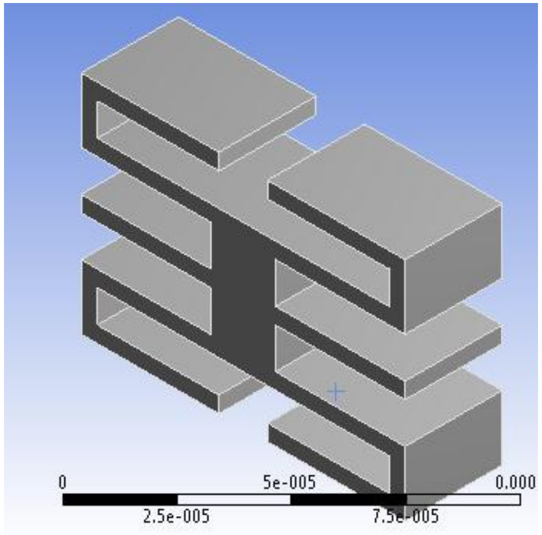
- Thickness
- Relative size of proof mass
- Asymmetry
- Leg dimensions
- Scaling

Frequency – dimensional parameters - thickness

Thickness





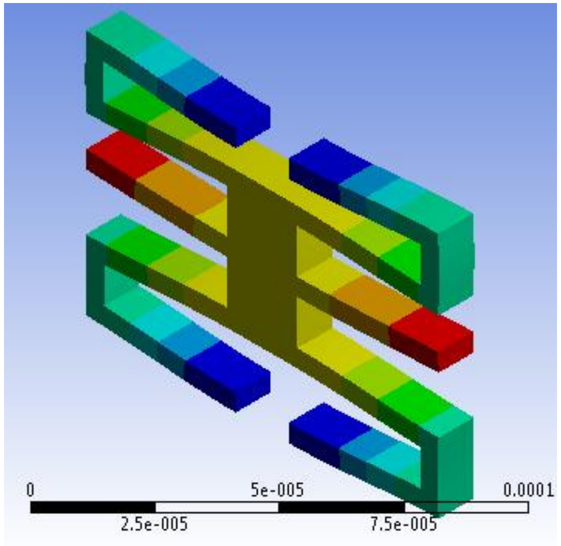


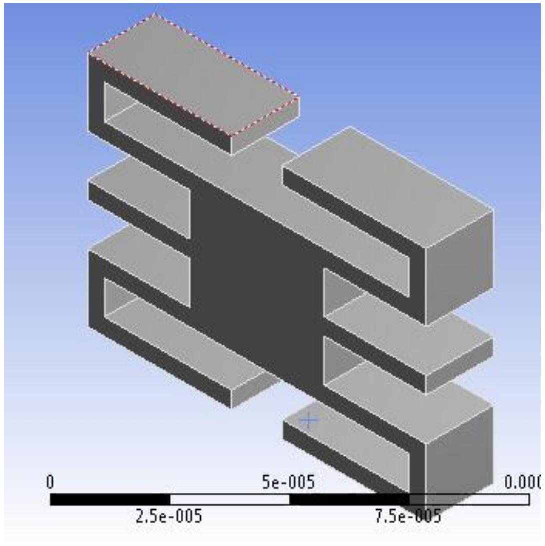
1.002×10^6 Hz

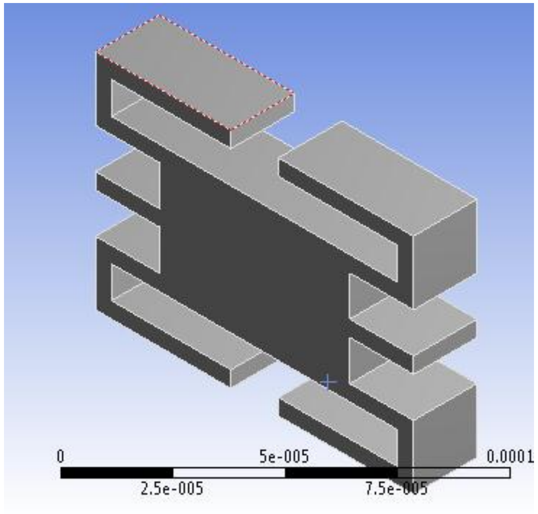
1.016×10^6 Hz

1.028×10^6 Hz

Proof mass





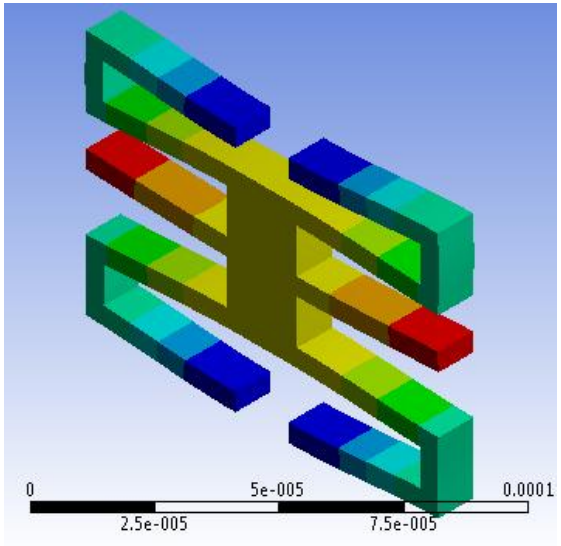


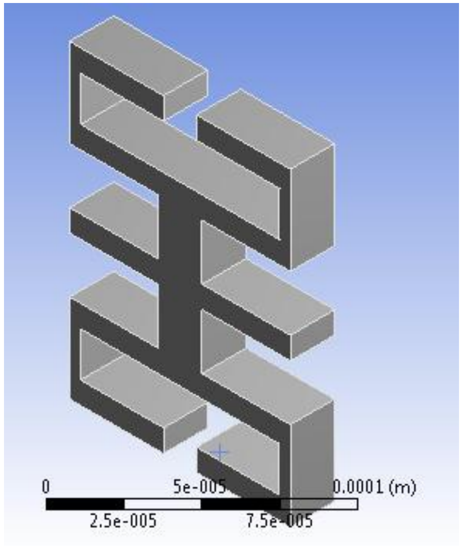
1.002×10^6 Hz

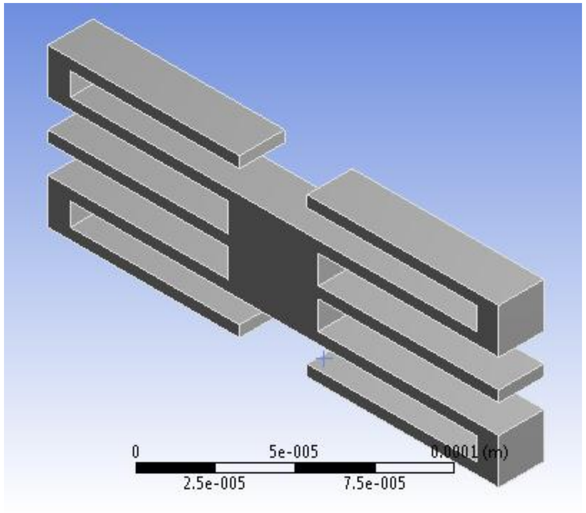
1.125×10^6 Hz

1.011×10^6 Hz

Asymmetry





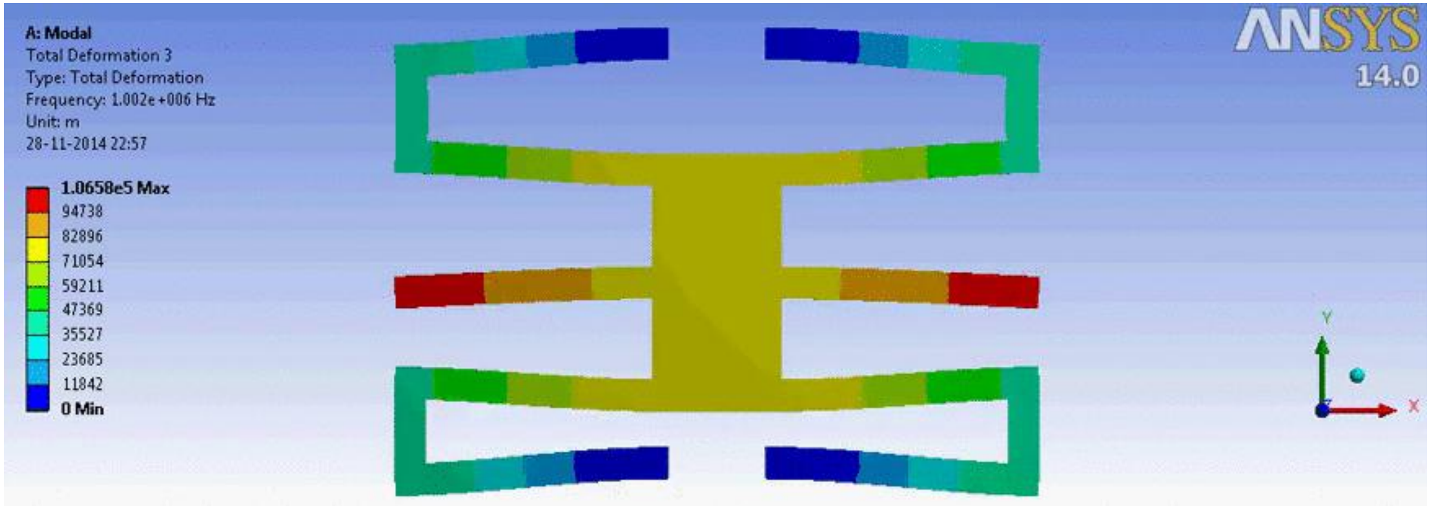


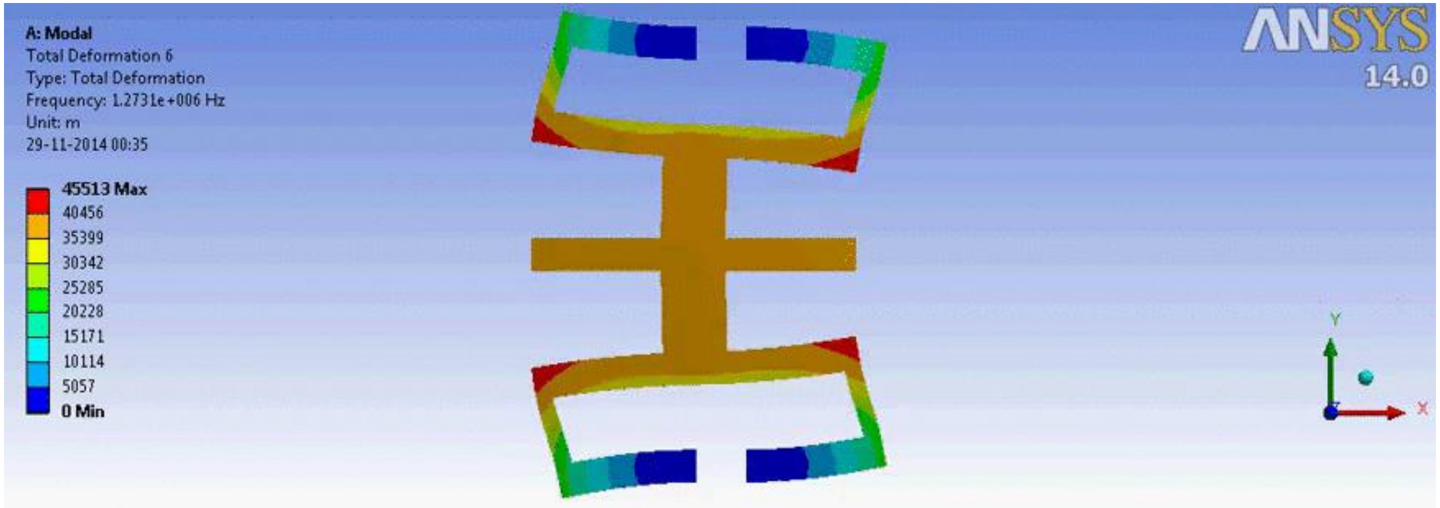
1.002×10^6 Hz

1.273×10^6 Hz
First Mode Shape Changes!

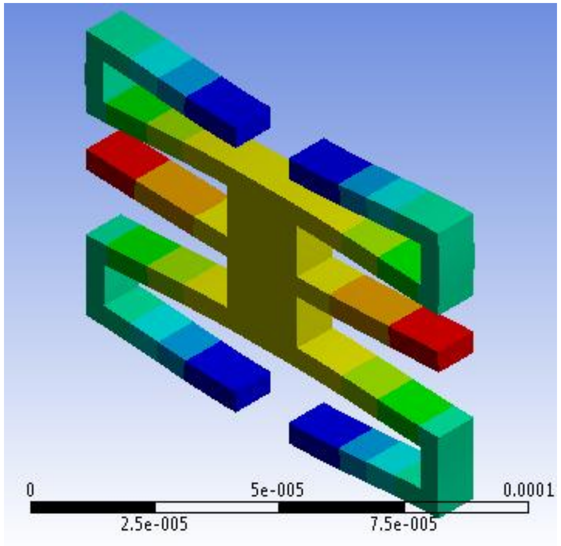
3.013×10^5 Hz

Asymmetry

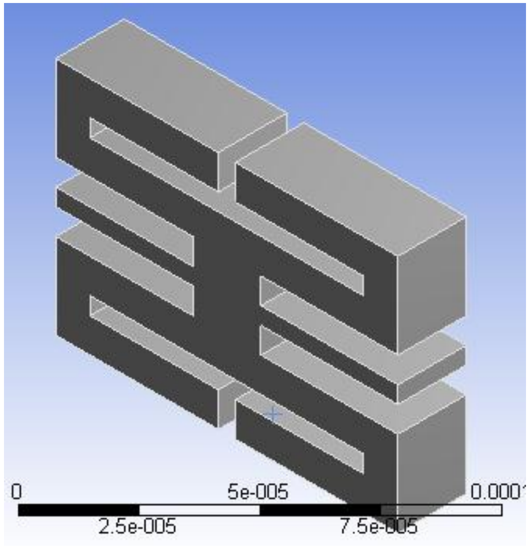

A: Modal
Total Deformation 3
Type: Total Deformation
Frequency: 1.002e+006 Hz
Unit: m
28-11-2014 22:57
1.0658e5 Max
94738
82896
71054
59211
47369
35527
23685
11842
0 Min
ANSYS
14.0
Y
X


A: Modal
Total Deformation 6
Type: Total Deformation
Frequency: 1.2731e+006 Hz
Unit: m
29-11-2014 00:35
45513 Max
40456
35399
30342
25285
20228
15171
10114
5057
0 Min
ANSYS
14.0
Y
X

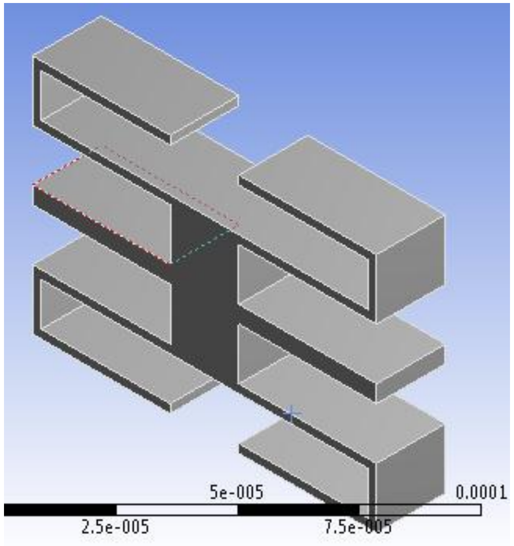
Leg dimensions



1.002×10^6 Hz

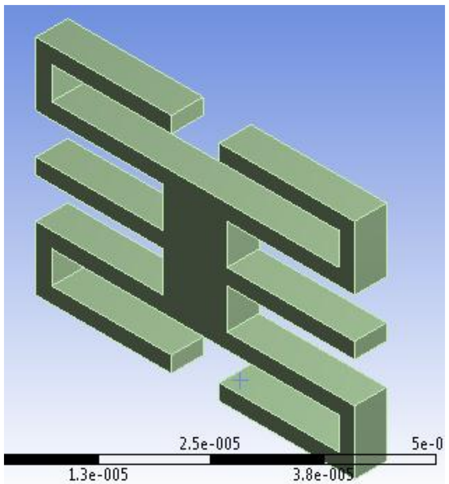


2.067×10^6 Hz

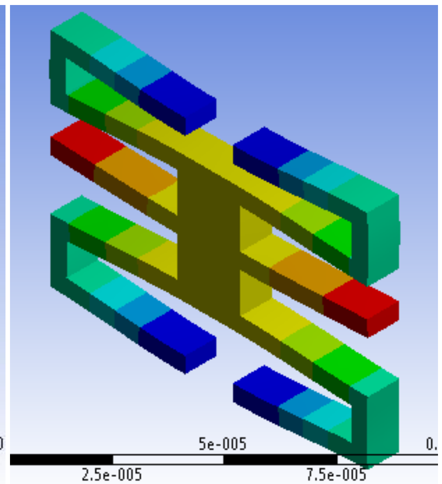


4.05×10^5 Hz

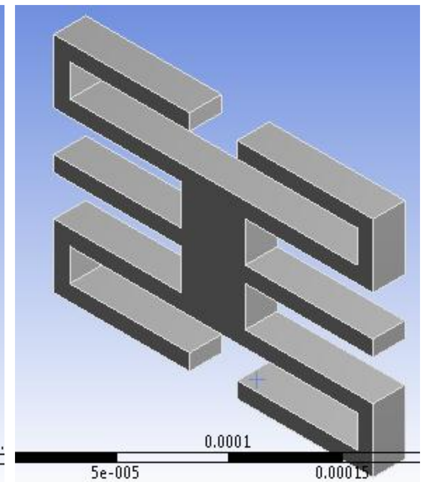
Scaling



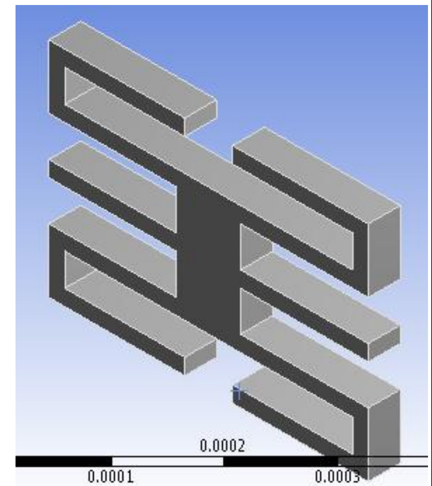
2.004×10^6 Hz



1.002×10^6 Hz



5.009×10^5 Hz

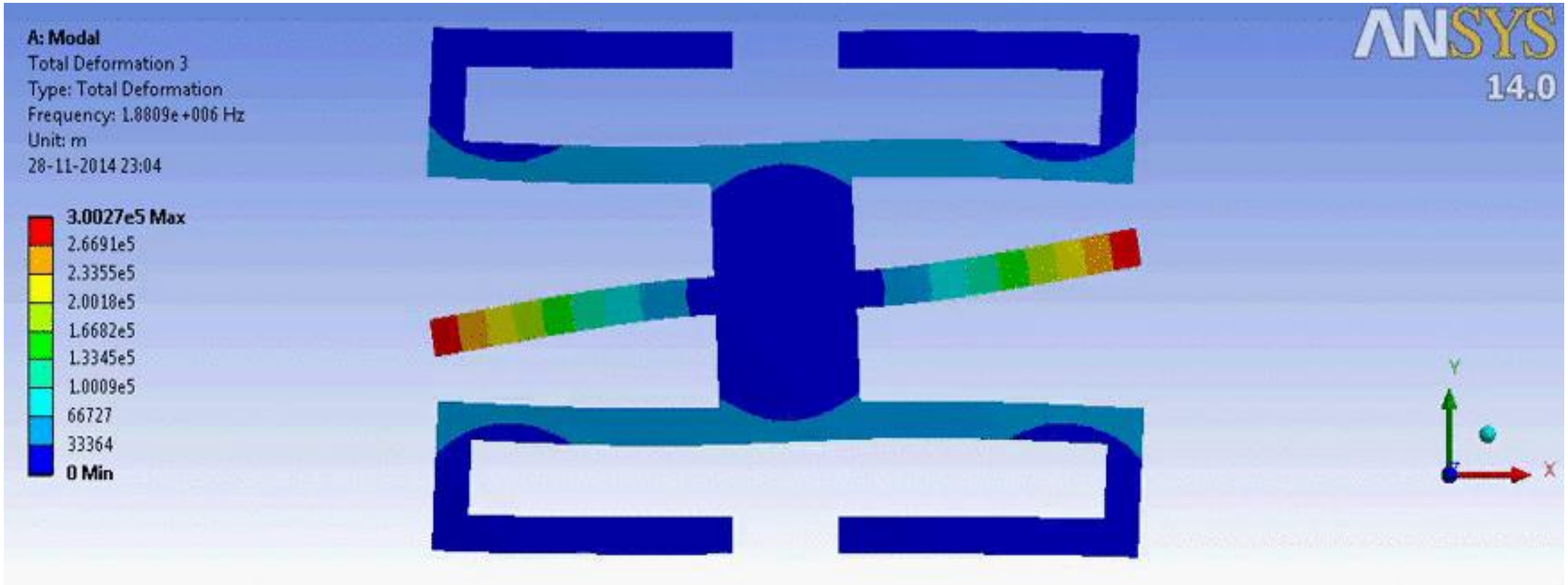


2.505×10^5 Hz

For an area of 0.5mm x 0.5 mm, folded beam structure can be designed for frequency values ranging over two orders of magnitude!

Summary

- A viable design for automotive crash test accelerometers has been proposed.
- Frequency and stiffness values are found to be within the prescribed limits.
- Resonant frequency is fairly independent of thickness and relative size of proof mass.
- Resonant frequency is a strong function of symmetry of the structure, leg dimensions, and scaling.



A: Modal
Total Deformation 3
Type: Total Deformation
Frequency: 1.8809e+006 Hz
Unit: m
28-11-2014 23:04
3.0027e5 Max
2.6691e5
2.3355e5
2.0018e5
1.6682e5
1.3345e5
1.0009e5
66727
33364
0 Min
ANSYS
14.0
Y
X

Thank You!