Mechanics of Microsystems : Micro/nano Mechanics
(NE 211)
Course Project

Coding isothermal Reynolds equation in Matlab and comparing with analytical solution for the ‘OOOO’ boundary condition

More S. K.
Cense – M.Tech

Outline

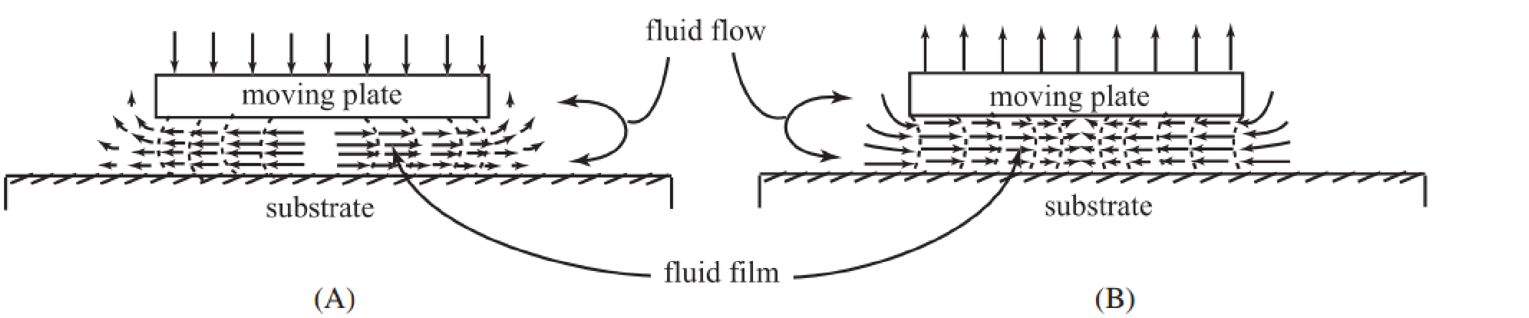
- What is Squeeze Film Damping
- What is the Governing Equation
- FE formulation
- Formulation of Matlab Code
- Results

Squeeze Film Damping

- Cavities, filled with air or some other gas, are common in MEMS.
- Squeeze film effect is natural in such devices.

Consider a air gap between movable plate and fixed underlying structure.

Figure 4: A schematic diagram of squeeze film flow (A) downward normal motion; (B) upward normal motion.[20]



Squeeze Film Damping

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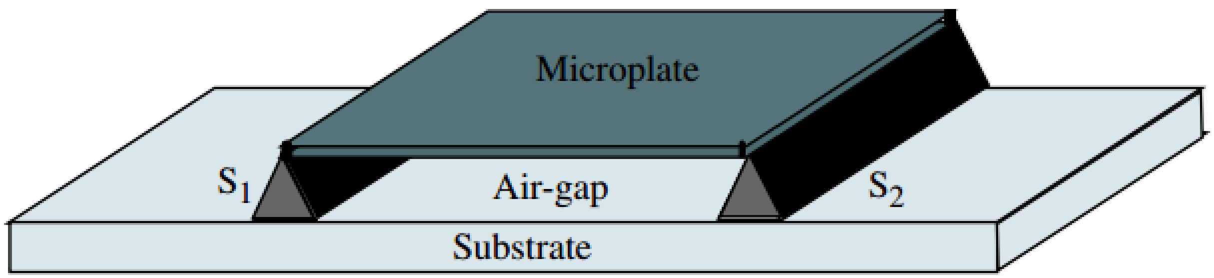
Figure 2: Schematic of a flexible structure under squeeze film damping.



S_1, S_2 - Anchors

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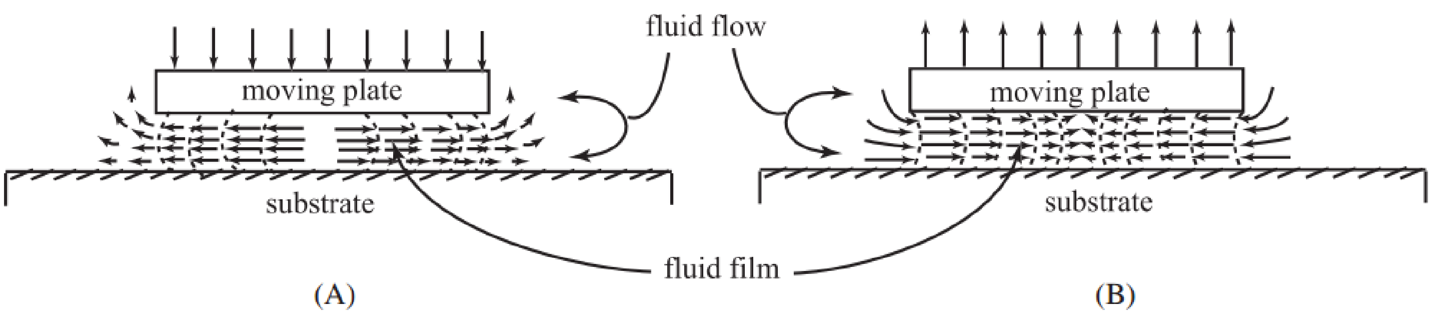

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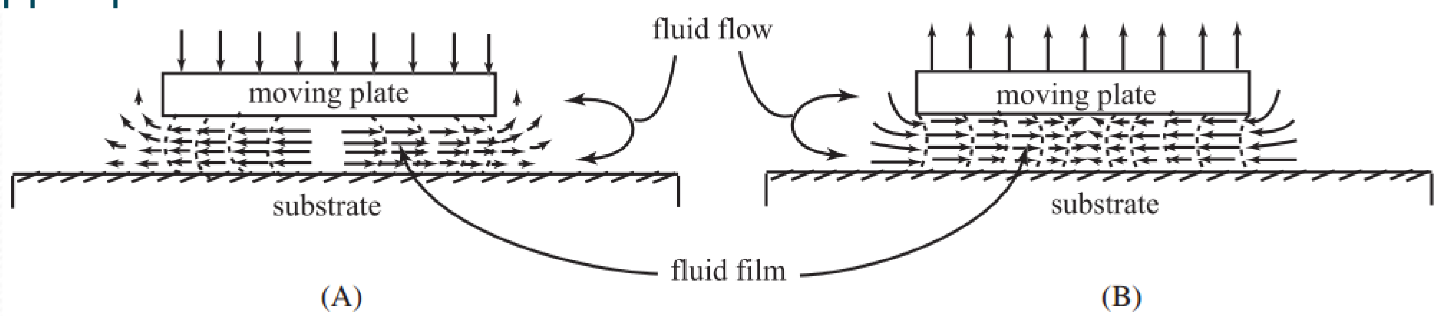
The upper plate moves down.



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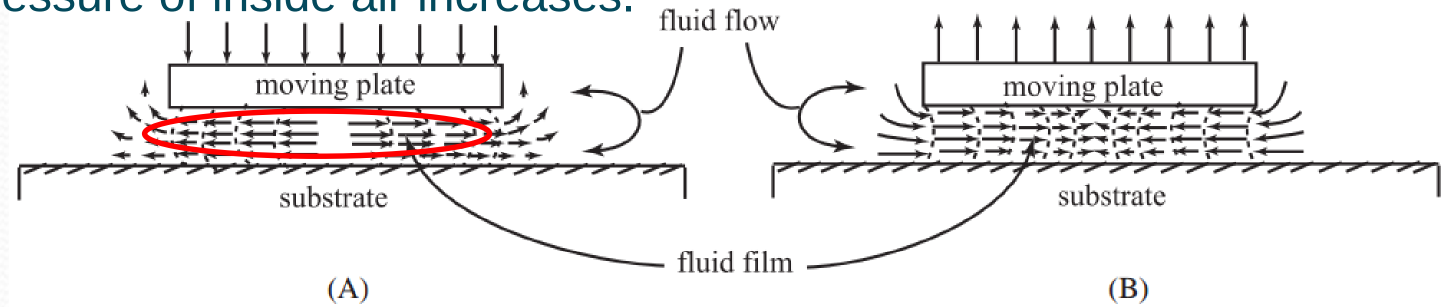
Pressure of inside air increases.



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Squeeze Film Damping

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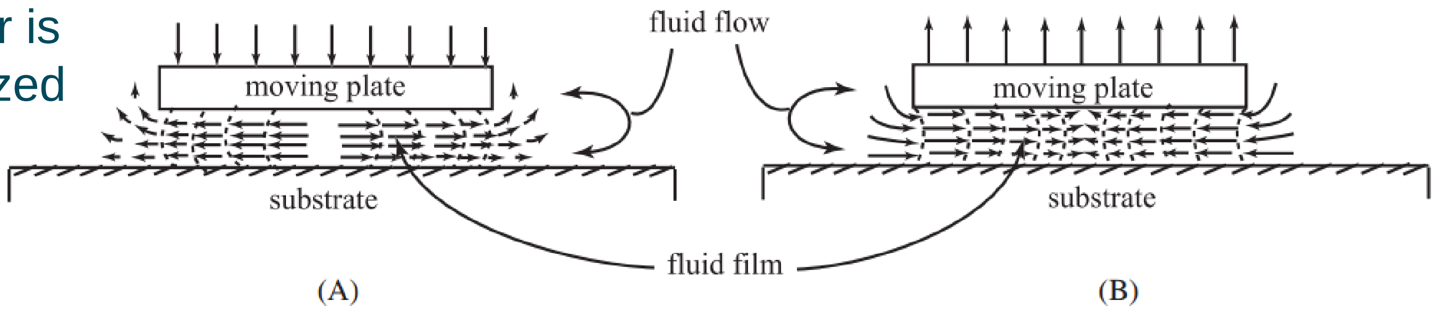
The air is squeezed out



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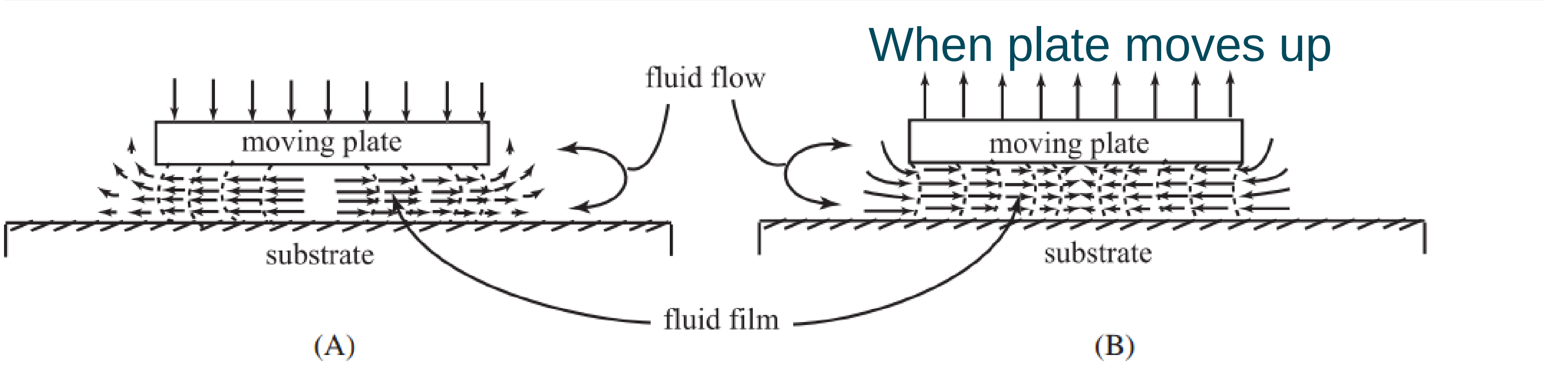

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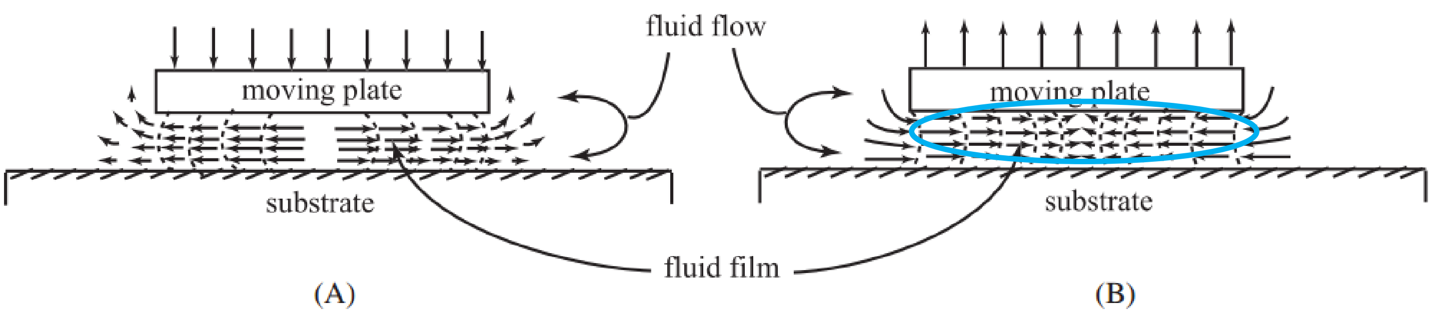

Pressure drops

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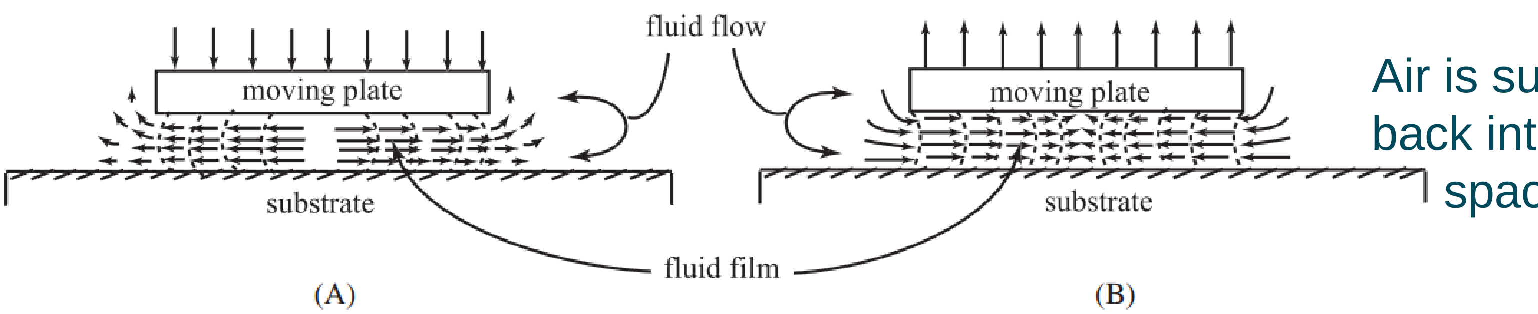

Air is sucked back into the space

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Squeeze Film Damping

The viscous drag during the flow creates a dissipative mechanical force on the plate opposing the motion.

This dissipative force is called

Squeeze Film Damping

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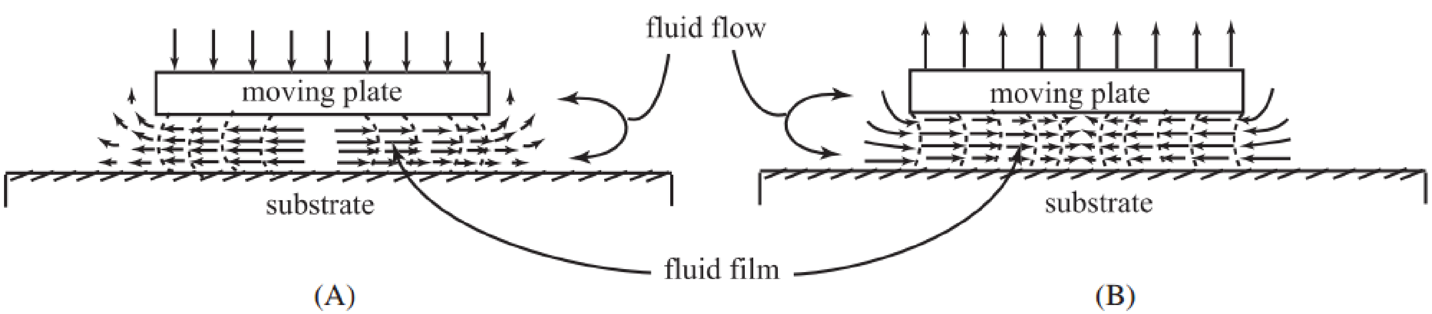

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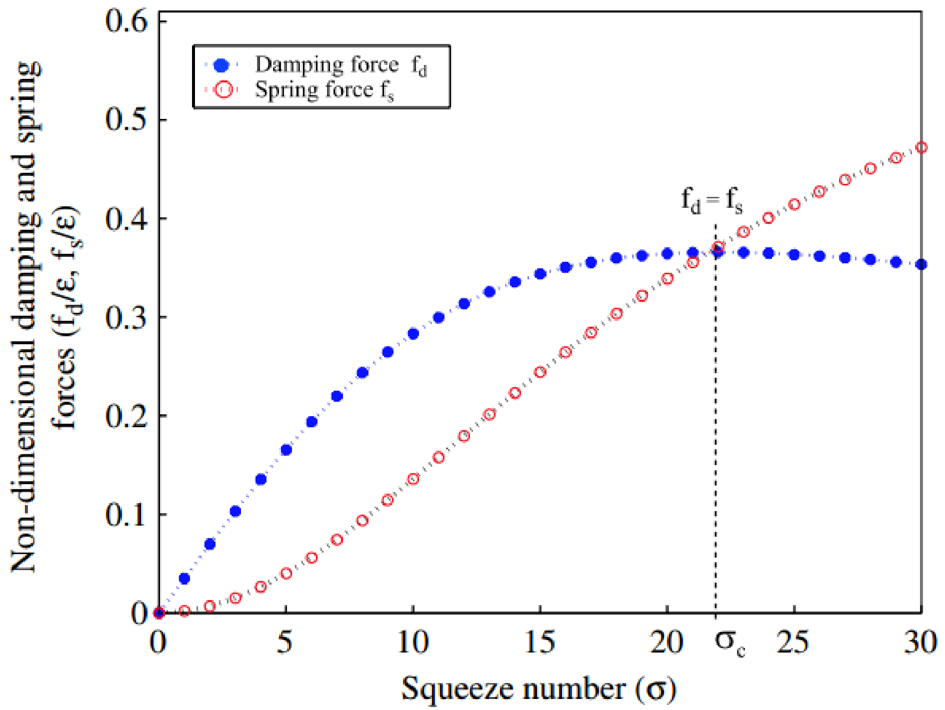
The behavior of trapped gas film at different frequencies of vibration of the plate is of particular interest

- For slow motions of moving plate the effect of the squeezed film is pure damping force, independent of pressure.
- The fluid motion is in phase with the moving plate.
- At higher frequencies of oscillations it often happens that fluid motion lags the plate motion by complete 180deg
- In such a case fluid can't escape from the cavity even if the boundary is open.
- The squeeze film behaves like a spring.

Squeeze Film Effect

Figure 6: Variation of non-dimensional damping force (f_d/ϵ) and spring force (f_s/ϵ) with squeeze number σ; the point of intersection of the two curves (i.e., f_d=f_s) corresponds to the cut-off squeeze number (σ_c).



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Governing Equation

Linearised Reynolds Equation

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} = \alpha^2 \left[\frac{\partial H}{\partial t} + \frac{\partial P}{\partial t} \right]$$

$$\alpha^2 = \frac{12\mu}{h_0^2 P_a}$$

μ = effective viscosity

P_a = ambient air pressure

Non Dimensionalizing by guessing solutions

$$P = \tilde{P} e^{i\omega t} \qquad H = \delta \phi e^{i\omega t}$$

$$X = L x$$

$$Y = L y$$

$$\sigma = \alpha^2 L^2 \omega$$

$$\frac{\partial^2 \tilde{P}}{\partial x^2} + \frac{\partial^2 \tilde{P}}{\partial y^2} = \sigma \left[i\delta\phi + i\tilde{P} \right]$$

$\tilde{P}$ = Pressure amplitude

ϕ = shape of plate

δ = displacement amplitude

A Roychowdhury, S patra Variational formulation for the Reynolds equation.

Weak Form or Weighted Integral

$$\int \frac{\partial \tilde{P}}{\partial x_i}\frac{\partial V}{\partial x_i}\, d\Omega + i\sigma \int (\tilde{p} + \phi\delta) V \, d\Omega = 0$$

V = weight function

After substituting shape functions for V and Nondimentionalized P

$$\sum_{j=1}^{n}\left[\int_{\Omega_e}\left\{\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\cdot\frac{\partial N_j}{\partial y} + i\sigma N_i N_j\right\} dx\,dy\right]\tilde{P} = -i\sigma\delta \sum_{j=1}^{n}\left[\int_{\Omega_e} N_i N_j\, dx\,dy\right]\phi_j$$

A Roychowdhury, S patra Variational formulation for the Reynolds equation.

Which can be represented in compact for as

$$[K_1 + K_2]\tilde{P} = -\delta K_2 \Phi$$

$$[K_1] = [S^{11}] + [S^{22}]$$

$$[K_2] = i\sigma [S^{33}]$$

$$[S^{11}] = \int_{\Omega_e} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x}\, dx\, dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} \frac{\partial N_i}{\partial \xi} \cdot \frac{\partial N_j}{\partial \xi}\, d\xi\, d\eta$$

$$[S^{33}] = \int_{\Omega_e} N_i N_j\, dx\, dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} N_i N_j\, d\xi\, d\eta$$

$$[S^{22}] = \int_{\Omega_e} \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y}\, dx\, dy$$

$$[S^{22}] = \int_{-1}^{1}\int_{-1}^{1} \frac{\partial N_i}{\partial \eta} \cdot \frac{\partial N_j}{\partial \eta}\, d\xi\, d\eta$$

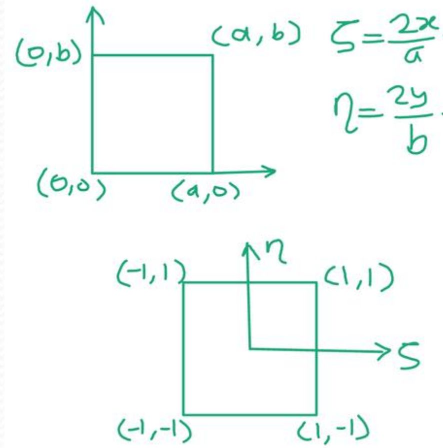
Shape Functions for Bilateral Square Element in Natural Coordinate System

$$N_1 = \frac{1}{4}(1-\zeta)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\zeta)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\zeta)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\zeta)(1+\eta)$$



$$\zeta = \frac{2x}{a} - 1$$

$$\eta = \frac{2y}{b} - 1$$

And the change of Coordinates

$$\frac{\partial N}{\partial x} = \frac{\partial N}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial N}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$\frac{\partial N}{\partial y} = \frac{\partial N}{\partial \zeta}\frac{\partial \zeta}{\partial y} + \frac{\partial N}{\partial \eta}\frac{\partial \eta}{\partial y}$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

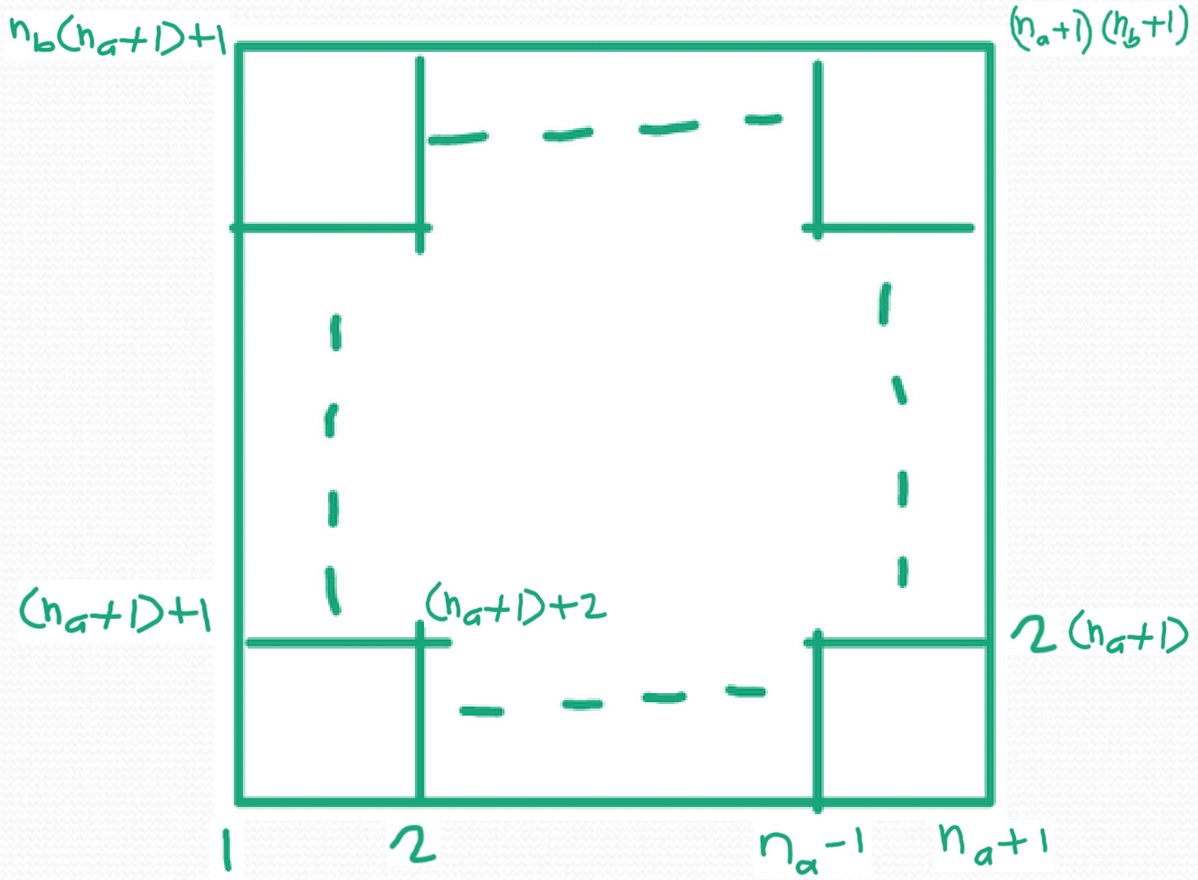
$$dx\,dy = \det[J]\,d\zeta\,d\eta$$

A Roychowdhury, S patra Variational formulation for the Reynolds equation.

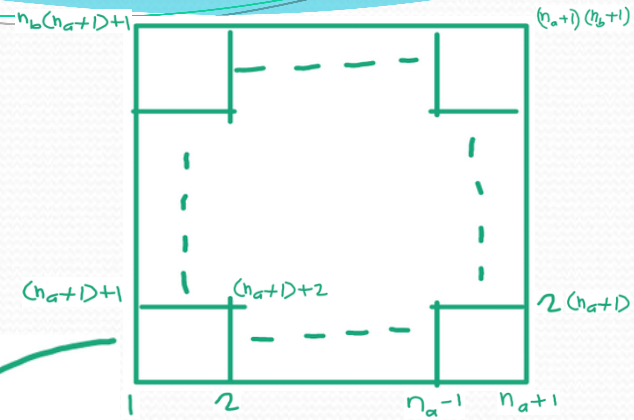
Flow Chart for FEA solution of Reynolds equation

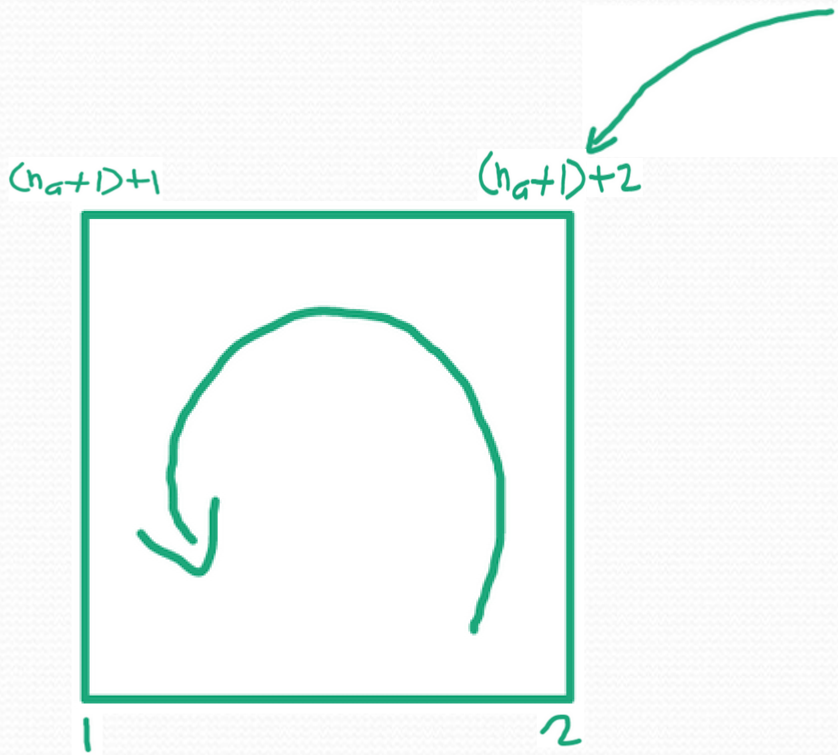
- Preprocessing
  - Node and Element Matrix
    - Contain Global and Local coordinates of nodes
    - Nodal connectivity of Elements
  - Apply Boundary Conditions
    - i.e. Save all nodes which are constrained.
    - Save all nodes other than constrained in different matrix
  - Prepare Mode Shape Matrix
    - Node Displacements according to first mode shape

- Node and Element Matrix



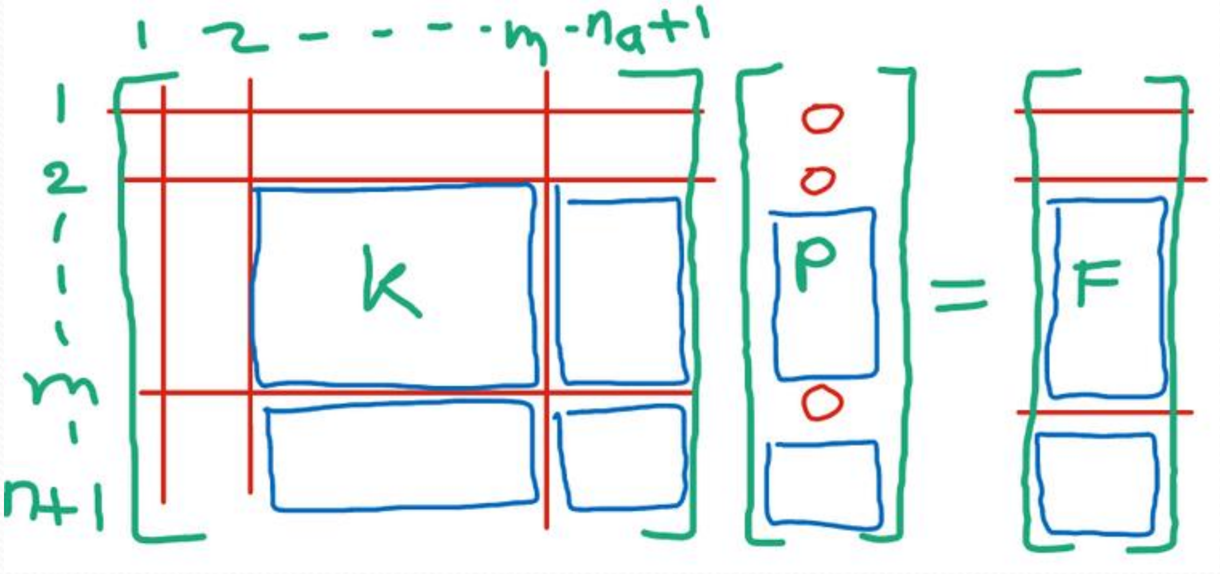
- Node and Element Matrix
    - Local Node Numbering for Elements
    - Local Node Connectivity



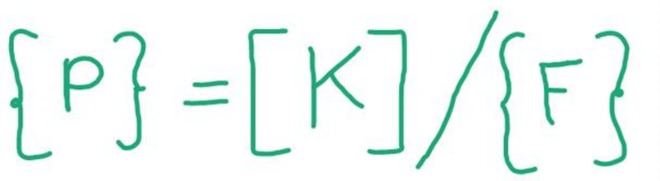


- Building Element Matrix
  - Element Matrix for each element
  - Assembly of all element matrices in Global Matrices K1_g and K2_g
  - Building Global Force matrix

- Solving
  - Applying Boundary Conditions.

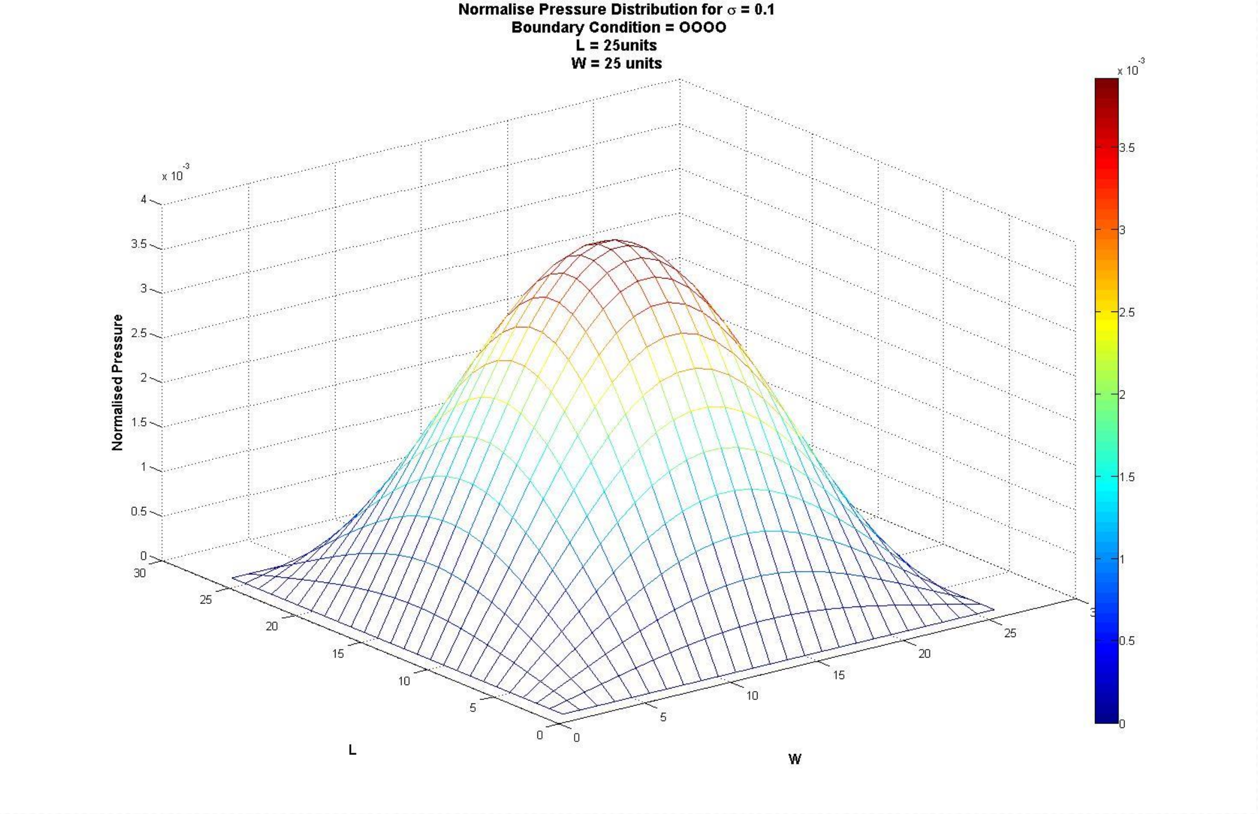


- Solving for Pressure distribution.



Results

- Boundary Condition = “OOOO”



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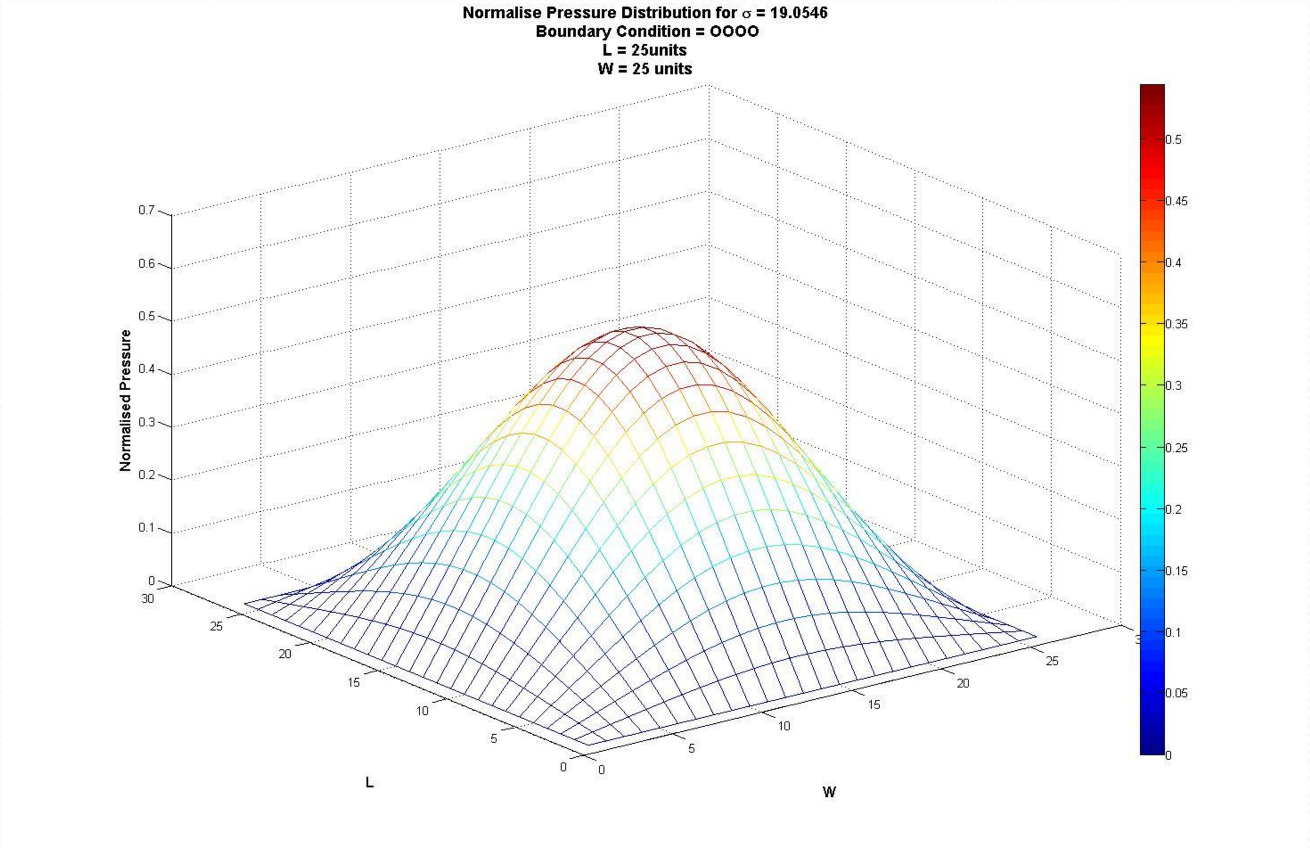

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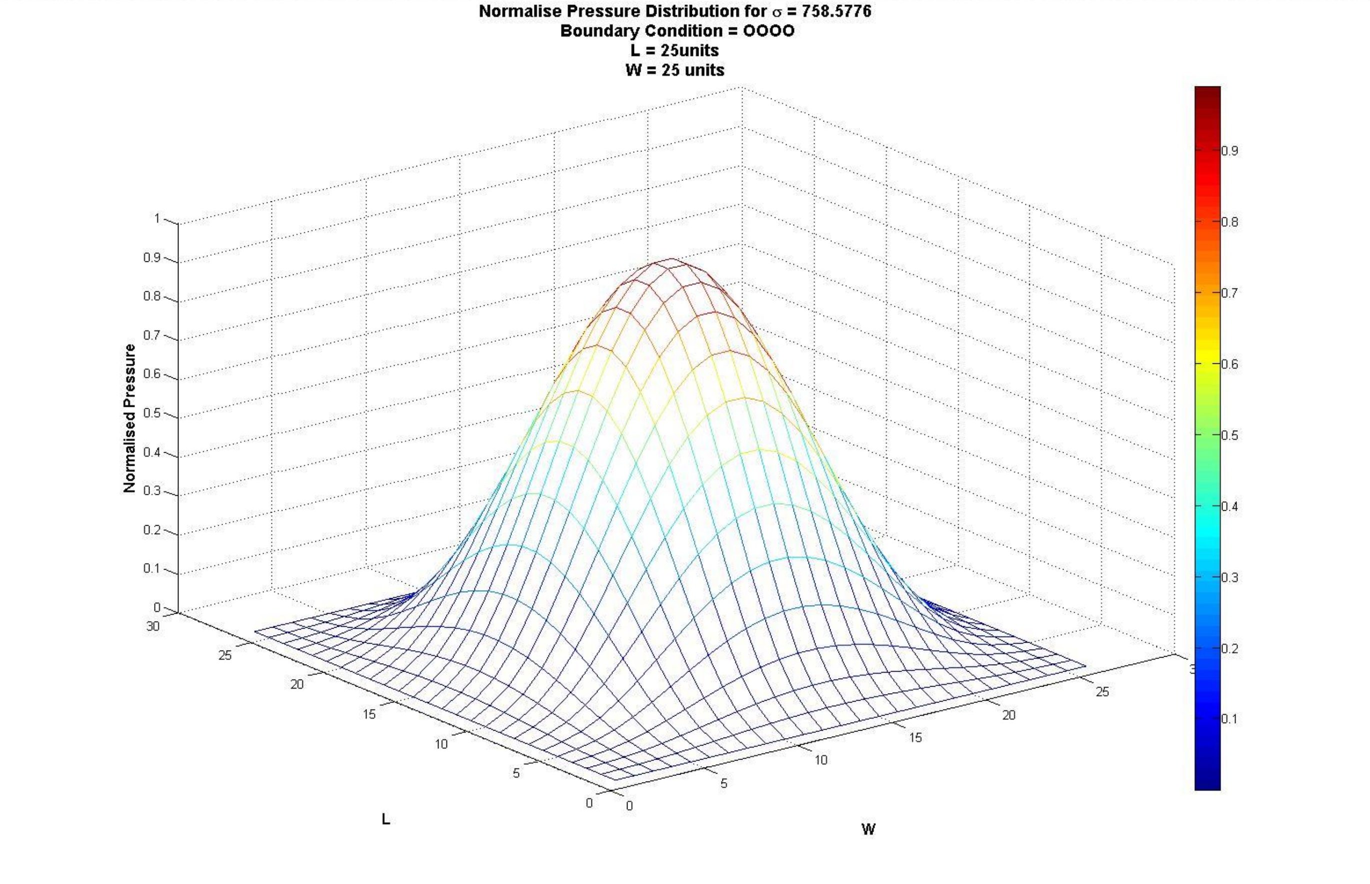

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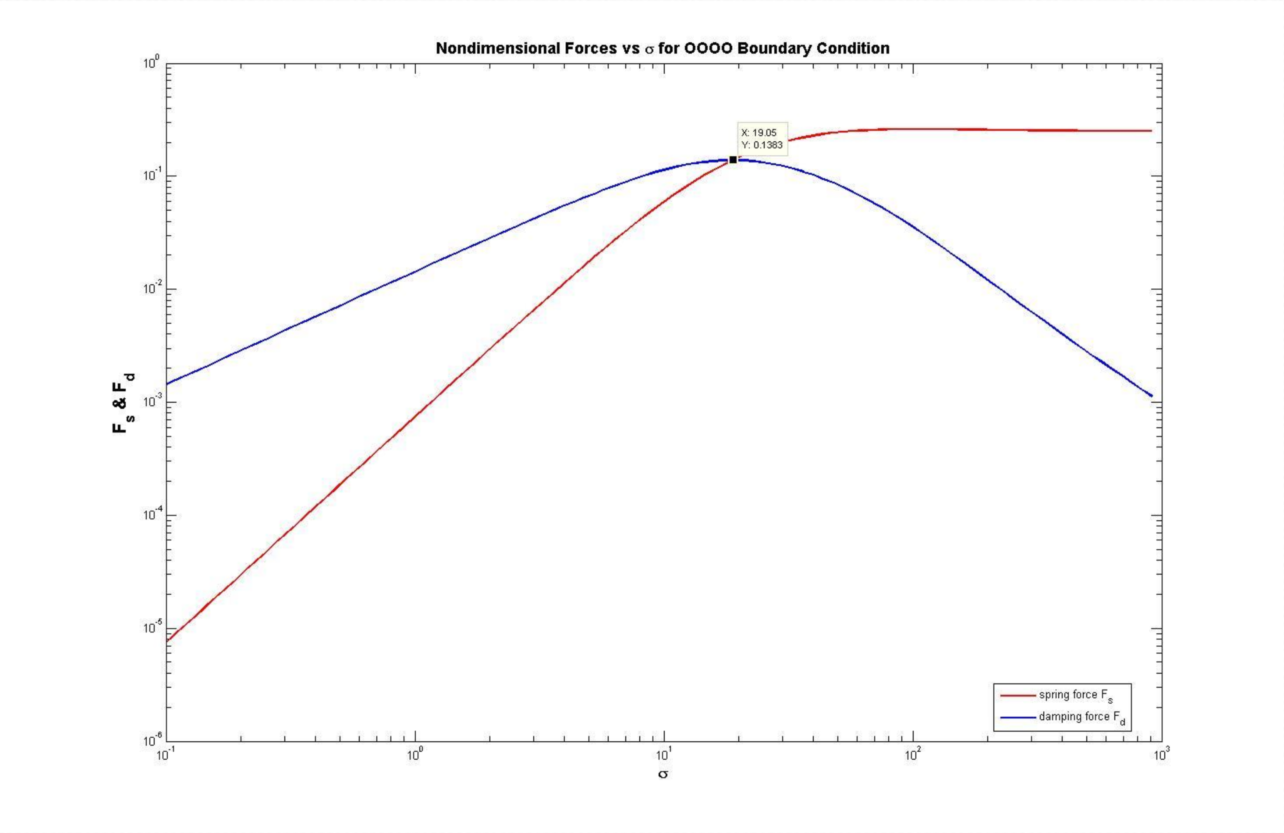

References

- Pratap R, Mohite S, Pandey A K 2007 Squeeze Film Effects in MEMS devices *Journal of the Indian Institute of Science* VOL 87, NO 1 (2007), PP 75-94.
- A Roychowdhury, S patra Variational formulation for the Reynolds equation.
- Reddy J. N. An Introduction to Finite Element Method.
- S. D. Senturia, Micro system design

Acknowledgement

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