

The background of the slide is a grayscale microsystem layout, likely a MEMS device. It features various rectangular and circular structures, interconnected by thin lines. A scale bar on the left side indicates a length of 14 micrometers. The overall appearance is that of a detailed technical drawing or photomicrograph of a microfabricated component.

Scaling effects in microsystems

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Effects of scaling in microsystems

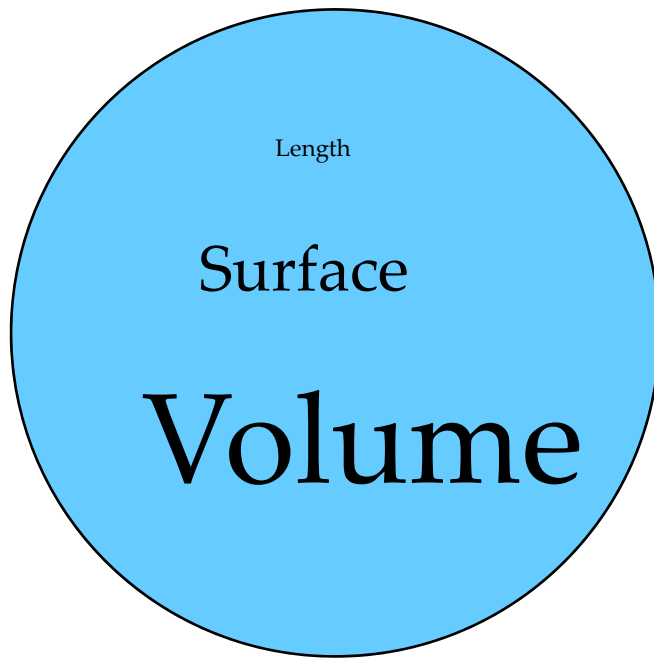
- Mechanical
- Thermal
- Electrostatic
- Magnetic
- Fluidic
- Optical
- Bio-chemical systems
- Acoustic
- Power
- **Matter of units**



Basic scaling law

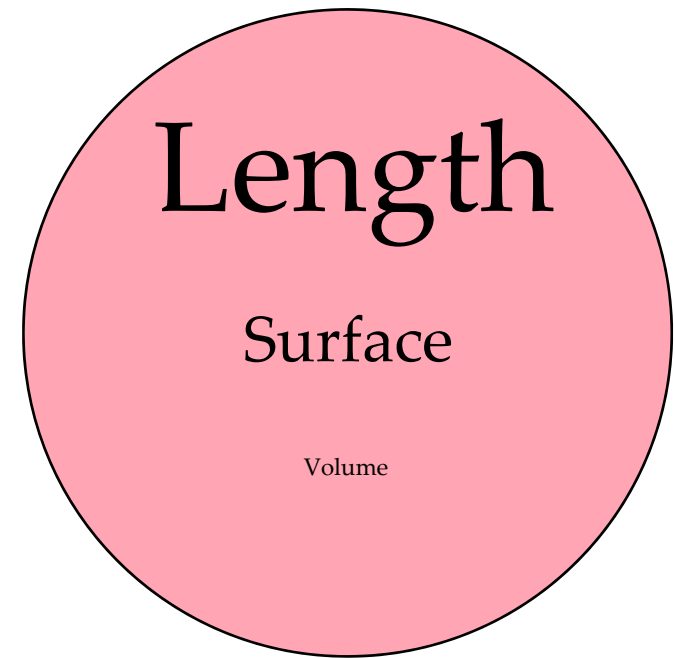
- **Volume-related** phenomena decrease much more rapidly than **surface-related** phenomena, which in turn decrease much more rapidly than **length-related** phenomena.

Macro world



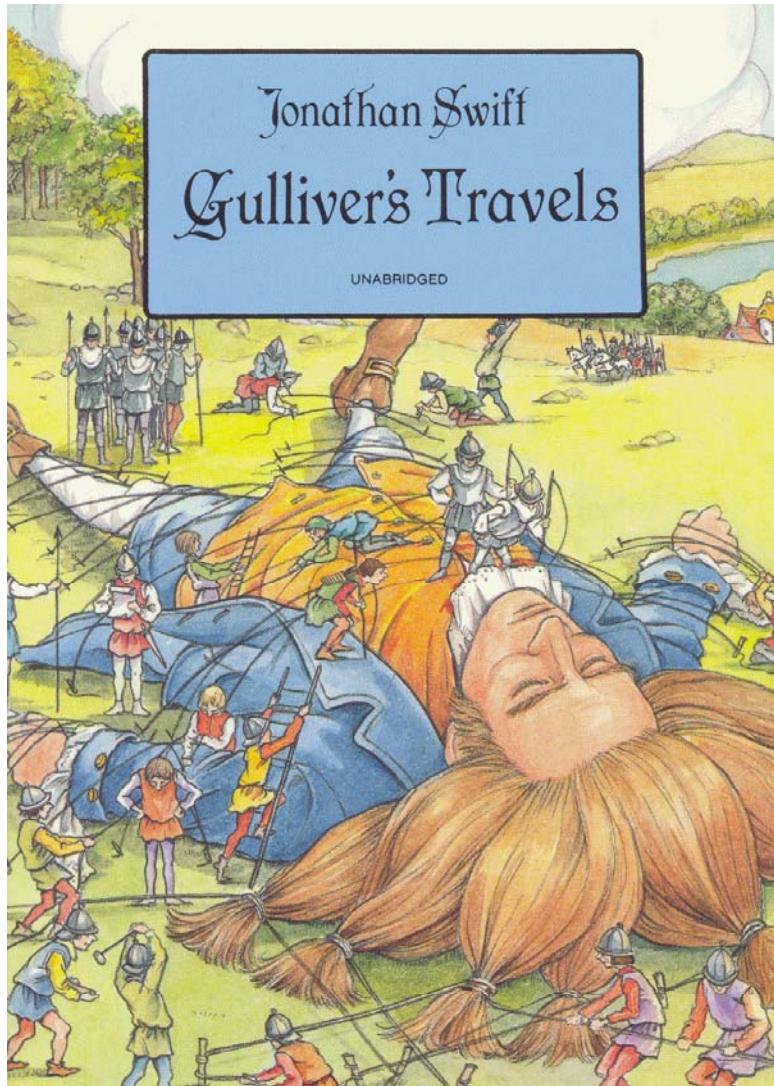
10	L	0.1
100	L^2	0.01
1000	L^3	0.001

Micro world



4

How much more food does Gulliver need than Lilliputians do?



“THESE People are most excellent Mathematicians and arrived to great Perfection in Mechanics by the Countenance and Encouragement of the Emperor, who is a Renowned Patron of Learning.”

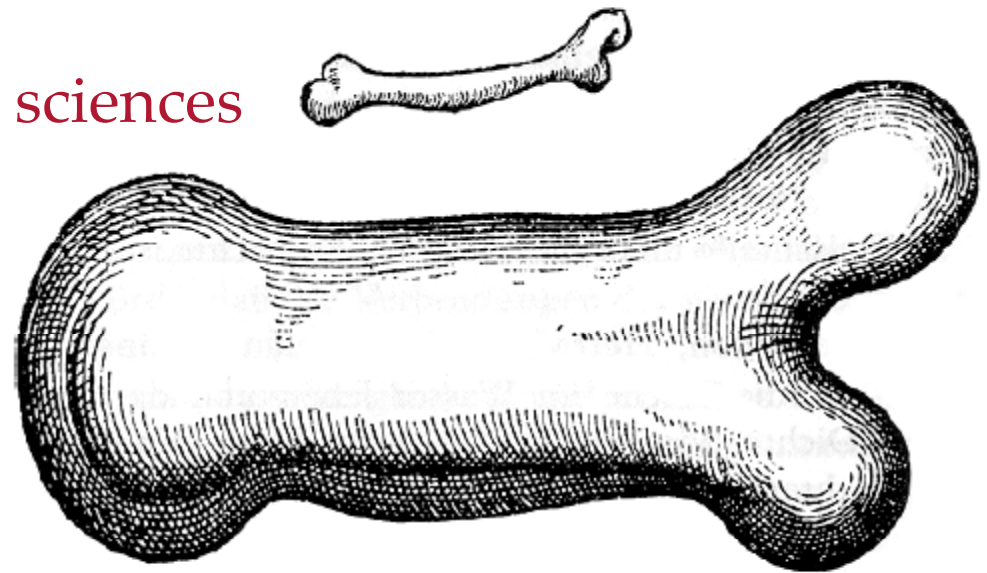
– Jonathan Swift

Galileo's bones



To illustrate briefly, I have sketched a bone whose natural length has been increased three times and whose thickness has been multiplied until, for a correspondingly large animal, it would perform the same function which the small bone performs for its small animal. From the figure shown here you can see how out of proportion the enlarged bone appears. ... Whereas, if the size of a body be diminished, the strength of that body is not diminished in the same proportion; indeed the smaller the body the greater its relative strength.

Dialogues concerning two new sciences
by Galileo Galilei





Allometry: differential growth in body parts depending on size

$$y = \alpha x^\beta$$

In isometric
scaling,
 $\alpha = \beta = 1$

Shape depends on
function.

	Exponent length	Exponent mass
Surface	1.95	0.65
Skeleton weight (terrestrial)	3.25	1.08
Skeleton weight (whales)	3.07	1.02
Muscle mass	3.00	1.00
Metabolic rate	2.25	0.75
Lung volume	3.09	1.02
Respiration frequency	-0.78	-0.26
Heart weight	2.94	0.98
Heart rate	-0.75	-0.25
Liver mass	2.55	0.85
Kidney mass	2.61	0.87
Brain mass (excluding primates)	2.25	0.75
Eyes mass	1.80	0.60
Life span (age)	0.60	0.20
Oxygen consumption	2.25	0.75
Blood volume	2.97	0.99

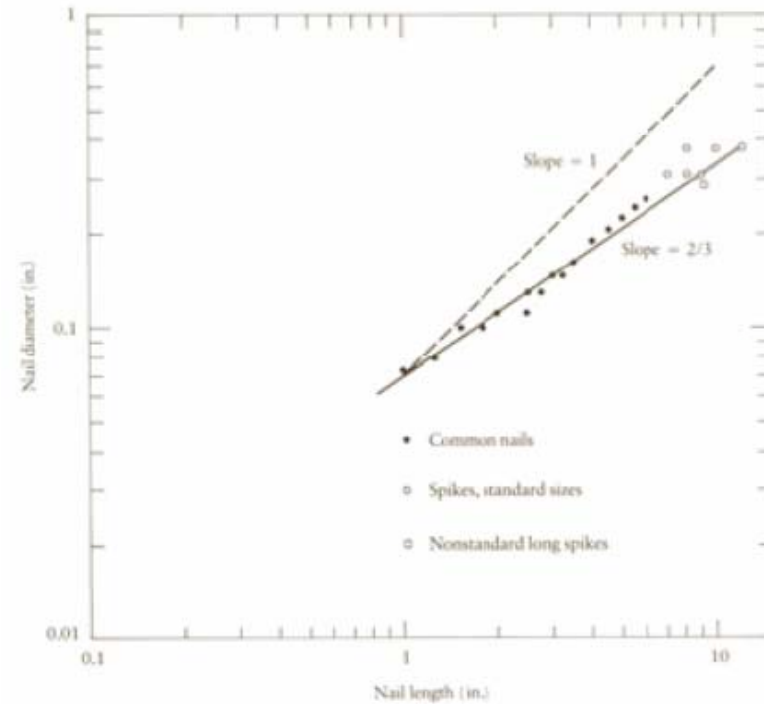
McGowan, From diatoms to dinosaurs: The size and Scale of living things.

Nailing down the scaling issue



Common nails arranged by size from 60 penny (6 inches) to 2 penny (1 inch).

On Size and Life
by McMohan and Bonner



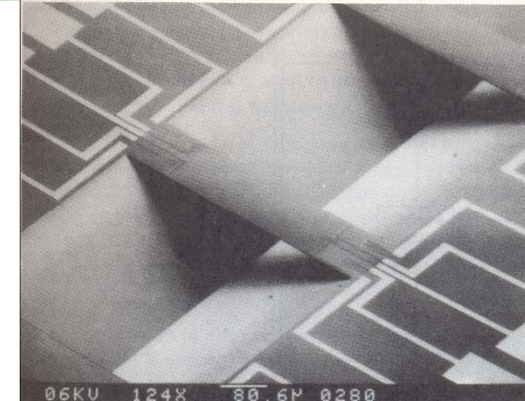
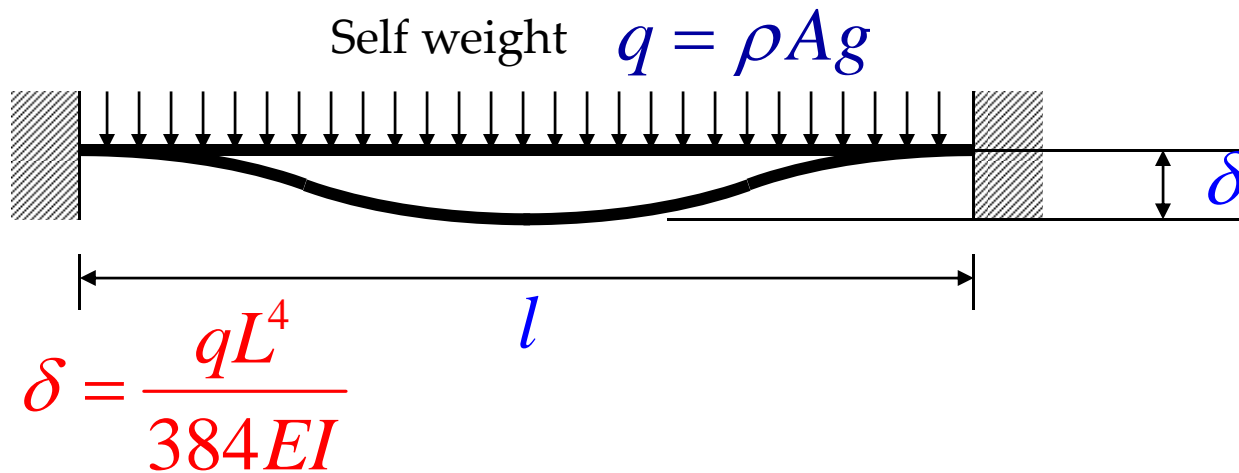
Nail diameter vs. nail length on a log-log plot, showing the allometric formula $d = 0.07l^{2/3}$. A broken line of slope 1.0, representing strict isometry, is also shown.



Some scaling questions at the micro regime

- Is self-weight important?
- Are inertial forces always negligible?
- Why is electrostatic force attractive?
- Isn't magnetic force attractive?
- How does (or doesn't) scaling favor thermal actuation?
- What changes in micro-fluidics?
- Can all optical phenomena be scaled down favorably?
- What about scaling in micro power generation?
- Why is scaling down useful for chemical micro reactors?
- Scaling effects on bioMEMS
- Scaling in acoustics
- Effect of scaling on manufacturing

9 Is self-weight important in micromechanical devices?



Boustraw et al. 1990; flow-rate sensor

$$I = \frac{bh^3}{12} = \frac{(\alpha_1 L)(\alpha_2 L)^3}{12} = \frac{\alpha_1 \alpha_2^3 L^4}{12}$$

$$\delta = \frac{12qL^4}{384E\alpha_1\alpha_2^3L^4} = \frac{q}{32\alpha_1\alpha_2^3E} = \frac{\rho Ag}{32\alpha_1\alpha_2^3E} = \frac{\rho g\alpha_1\alpha_2L^2}{32\alpha_1\alpha_2^3E} = \frac{\rho gL^2}{32\alpha_2^2E}$$

$$\Rightarrow \frac{\delta}{L} = \frac{\rho gL}{32\alpha_2^2E} \propto L$$

Relative deflection decreases with decreasing size.

10 Strength against self-weight: Galileo's bones revisited



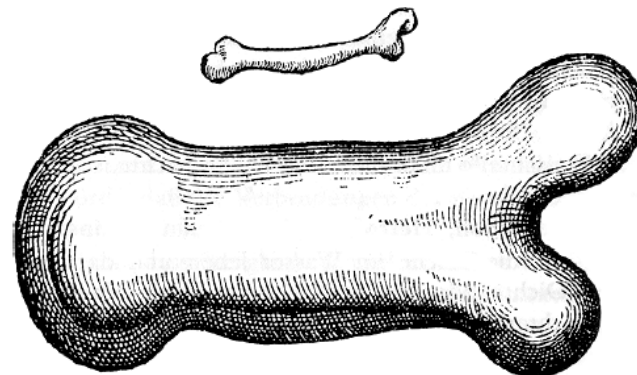
Maximum stress in a beam due to self-weight

$$\sigma = \frac{My}{I} = \frac{(qL^2)(p_t L)}{p_t L^4} = \frac{(\rho g L^2 L^2)(p_t L)}{p_t L^4} = \frac{\rho g p_t L}{p_t} \propto L$$

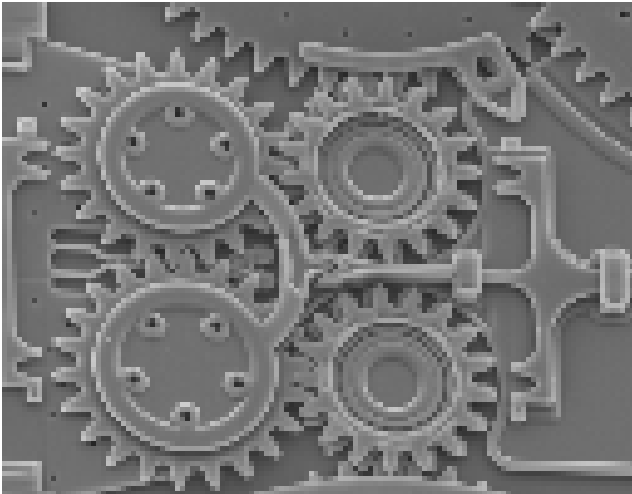
So, stress increases with size if the density is the same.

If we assume that the material strength does not change with size, we have a problem.

Therefore, the bones of bigger animals need to be fatter while those of smaller ones are slender.



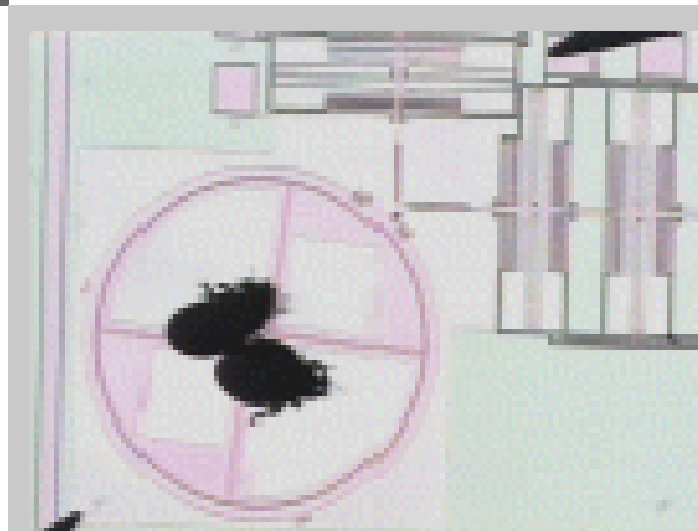
What about inertial forces in general?



The inertia may be insignificant but velocities are relatively huge (million rpm is not uncommon).

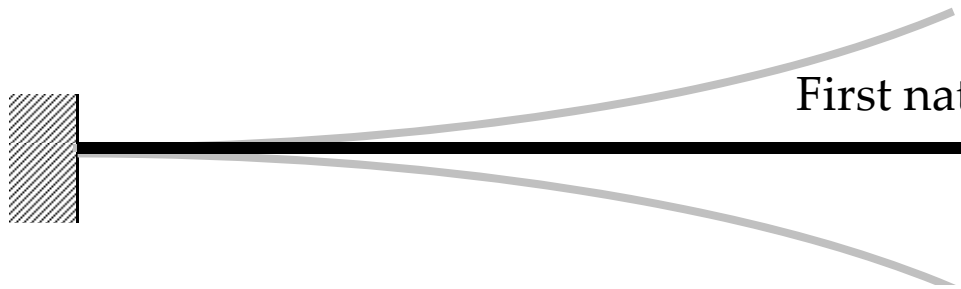
Therefore, inertial forces could be substantial.

Figures: courtesy of Sandia National Laboratory





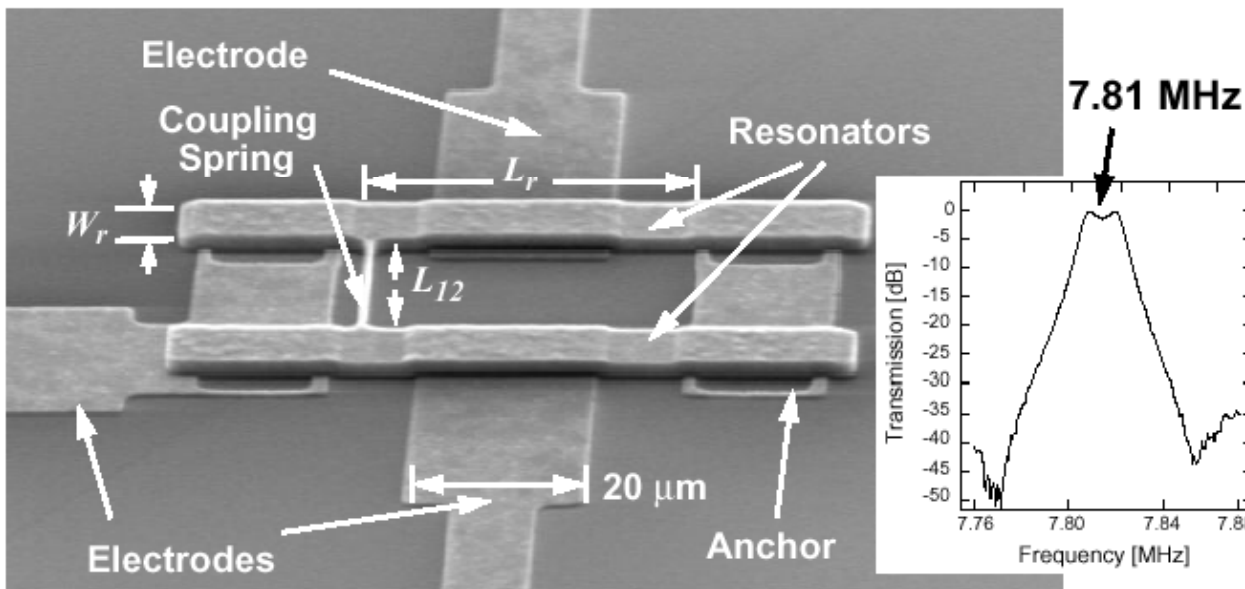
Smaller things can move faster



First natural frequency of a cantilever beam:

$$\omega = \frac{22.4}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} \propto \frac{1}{L}$$

HF Spring-Coupled Micromechanical Filter



Natural frequency increases with decreasing size.

2-Resonator HF
(4th Order)
[Bannon, Clark,
Nguyen 1996]



Performance
 $f_o=7.81\text{MHz}$, $BW=15\text{kHz}$
Rej.=35dB, I.L.<2dB

Residual stresses and stress gradients

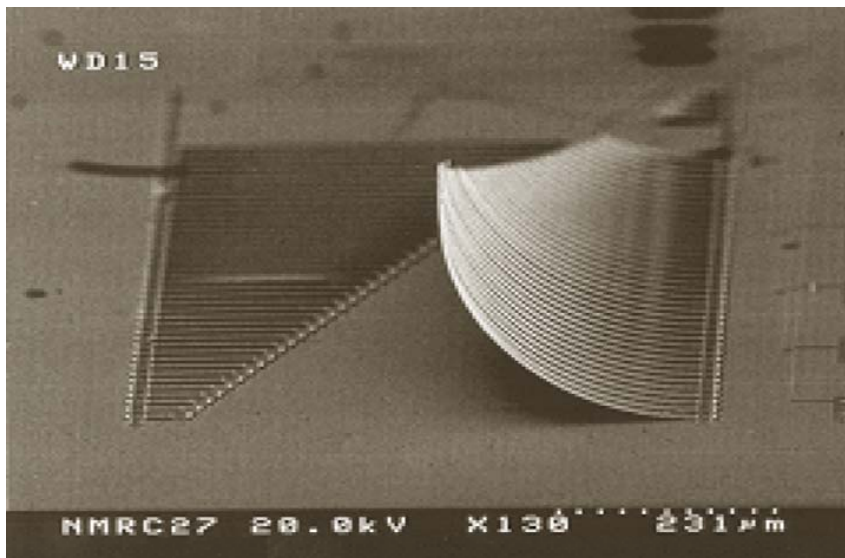


➤ How do they affect micromechanical structures?

Residual stresses modify the effective stiffness of the structures.

They may also cause delamination at the interfaces.

They cause bowing of multi-layer structures and thus making them non-planar.



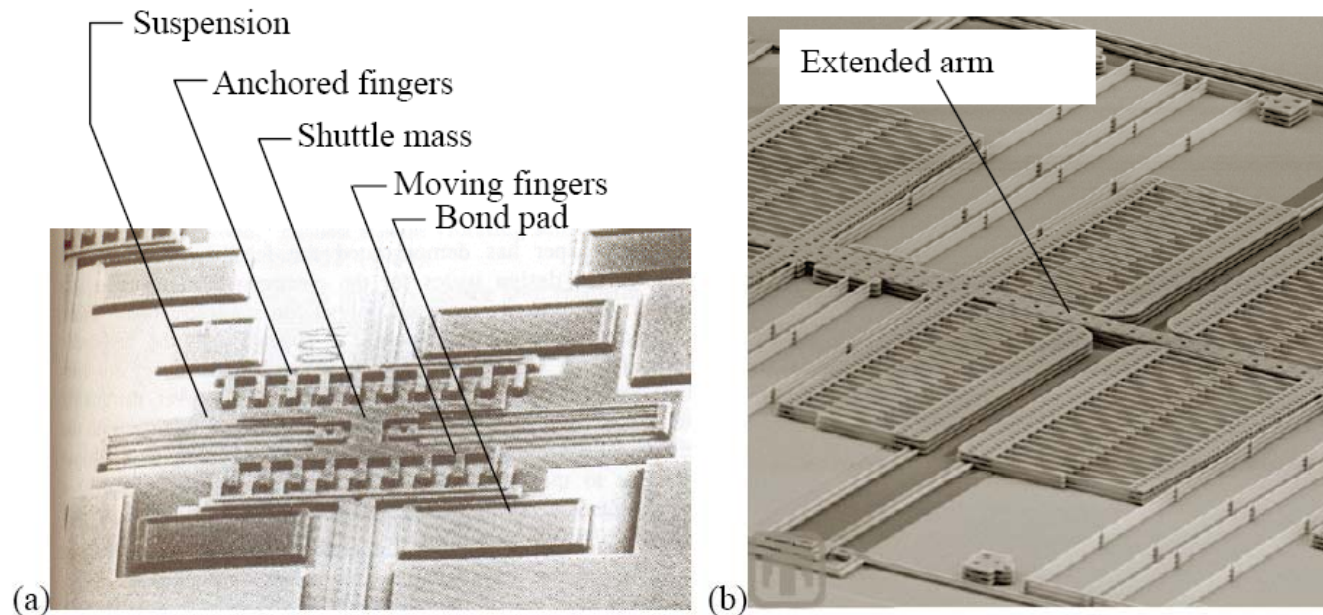
Stress gradients cause curling and wrinkling of even single layers.

At the micro scale, these effects are so significant that, it helps in conceiving novel sensors such as virus and single-molecule detectors.

Why is electrostatic force favorable at the micro scale?

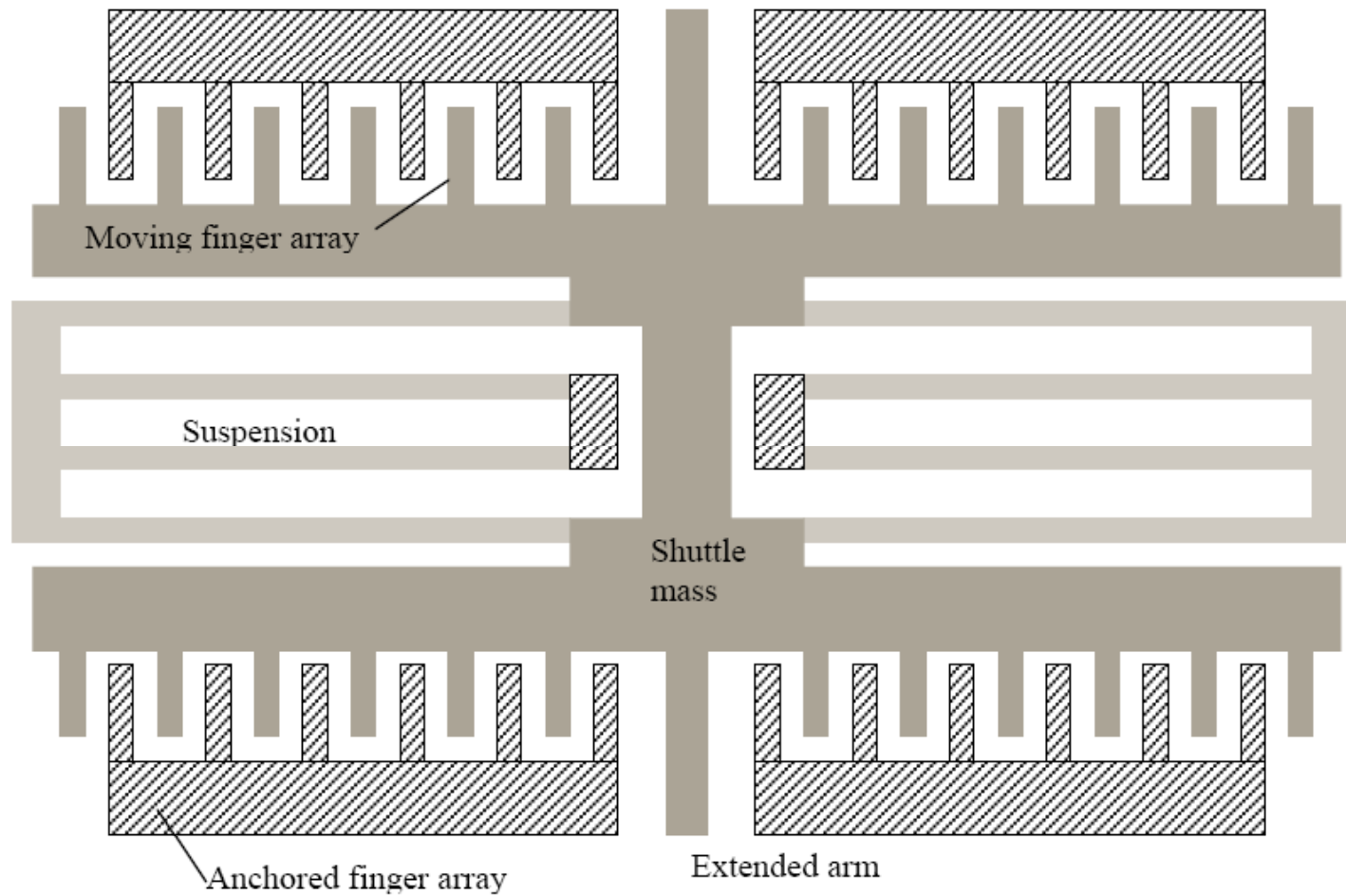


Comb-drive linear microactuator



- (a) Tang, W. C., Nguyen, C., Judy, M. W., and Howe, R. T., "Electrostatic Comb-Drive of Lateral Polysilicon Resonators," *Sensors and Actuators A*, 21 (1), 1990, pp. 328-331
- (b) <http://mems.sandia.gov/scripts/images.asp> (Sandia National Laboratories, New Mexico, USA)

Schematic of the comb-drive



Is an electrostatic comb-drive attractive at the micron scale?



E	Young's modulus	w	Thickness of beams	g	Gap between comb-fingers
t	Width of the suspension beams	l	Length of the suspension beams	N	Number of comb-pairs
				V	Voltage

$$k = \frac{2Et w^3}{l^3} \quad \text{Lumped mechanical stiffness of the suspension}$$

$$F_e = \frac{N \epsilon_0 t V^2}{2g} \quad \text{Electrostatic force}$$

$$\delta = \frac{F_e}{k} = \left(\frac{N \epsilon_0 V^2}{4E} \right) \frac{l^3}{g w^3} \quad \text{Deflection of the shuttle}$$

$$\frac{\delta}{l} = \left(\frac{N \epsilon_0 V^2}{4E} \right) \frac{l^2}{g w^3} \propto L^{-2} \quad \text{Scaling of relative deflection with size for fixed voltage}$$

$$V = \sqrt{\left(\frac{\delta}{l} \right) \left(\frac{4E}{N \epsilon_0} \right) \left(\frac{g w^3}{l^2} \right)} \propto L \quad \text{Scaling of voltage for fixed relative deflection}$$



Isn't magnetic actuation favored in microsystems?

➤ Electromagnetic force between two coils

$$F = \frac{\mu_0}{2\pi} I_1 I_2 \frac{l}{d}$$

Case 1: Constant current density

$$J = \frac{I}{A_{cs}} = \text{constant} \Rightarrow I \propto L^2 \Rightarrow F \propto L^4 \quad \text{Very, very bad}$$

Case 2: Constant temperature rise

$$\rho J^2 A_s \propto k_{\Delta T} \Rightarrow J \propto \frac{1}{L} \Rightarrow I \propto L \Rightarrow F \propto L^2 \quad \text{Bad}$$

Magnetic actuation in microsystems



- Between a coil and a permanent magnet

$$\vec{F} = I \vec{l} \times \vec{B}$$

Case 1: Constant current density

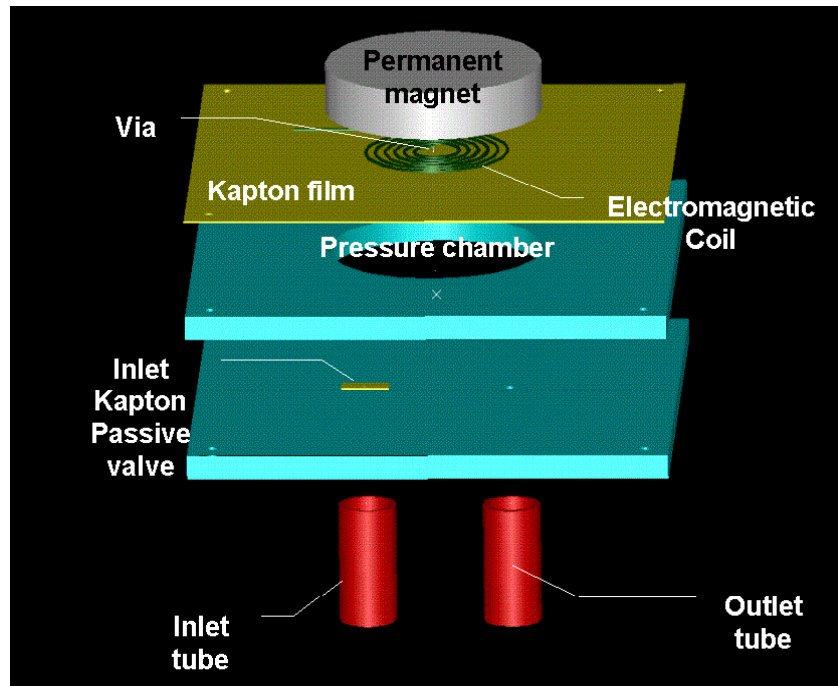
$$J = \frac{I}{A_{cs}} = \text{constant} \Rightarrow I \propto L^2 \Rightarrow F \propto L^3 \quad \text{Very bad}$$

Case 2: Constant temperature rise

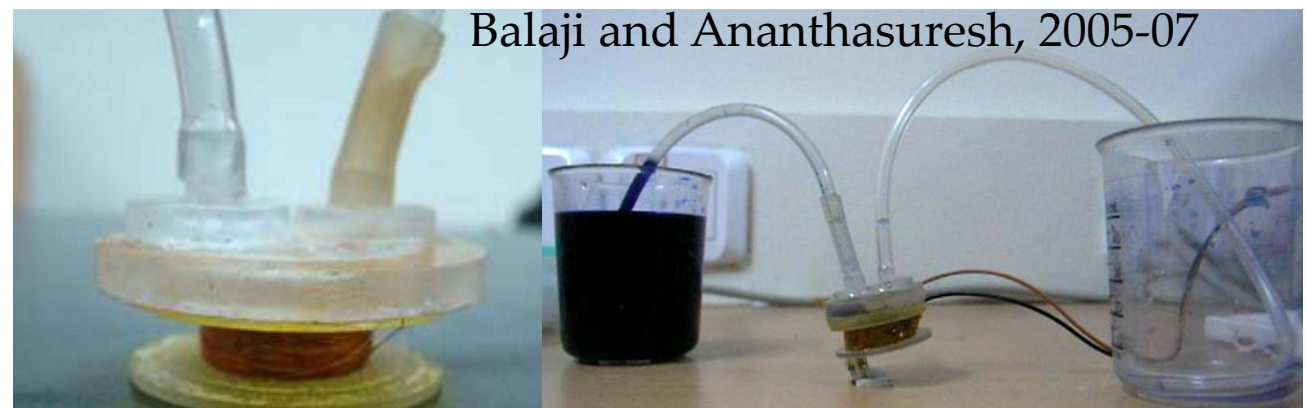
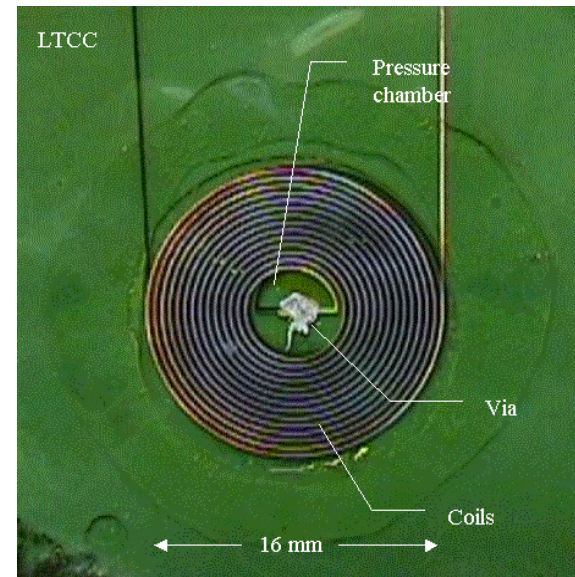
$$\rho J^2 A_s \propto k \Delta T \Rightarrow J \propto \frac{1}{L} \Rightarrow I \propto L \Rightarrow F \propto L^2 \quad \text{Still bad}$$

But with a powerful magnet, one can manage good magnitude of force.

Electromagnet-actuated minute pump



Kim, Ananthasuresh, and Bau, 2002.

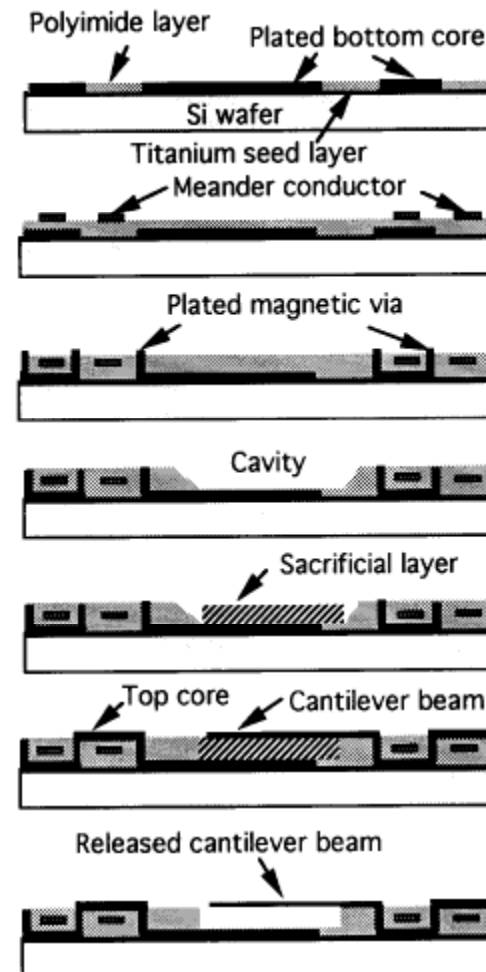
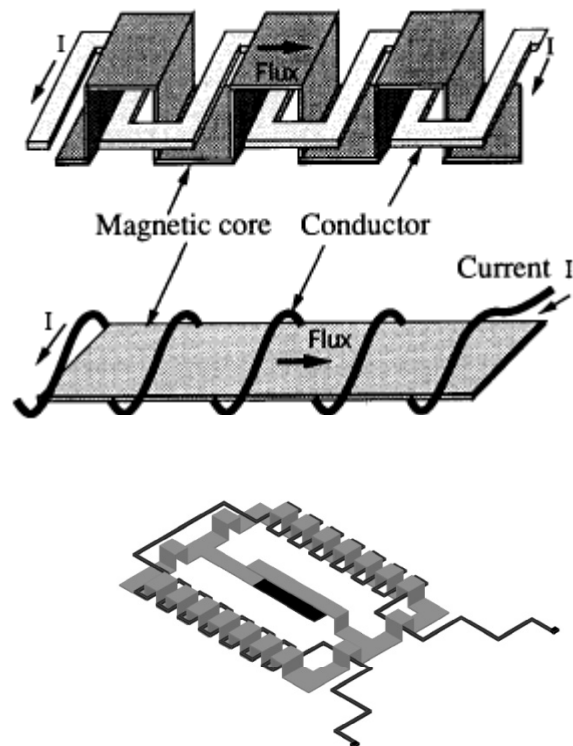


Balaji and Ananthasuresh, 2005-07

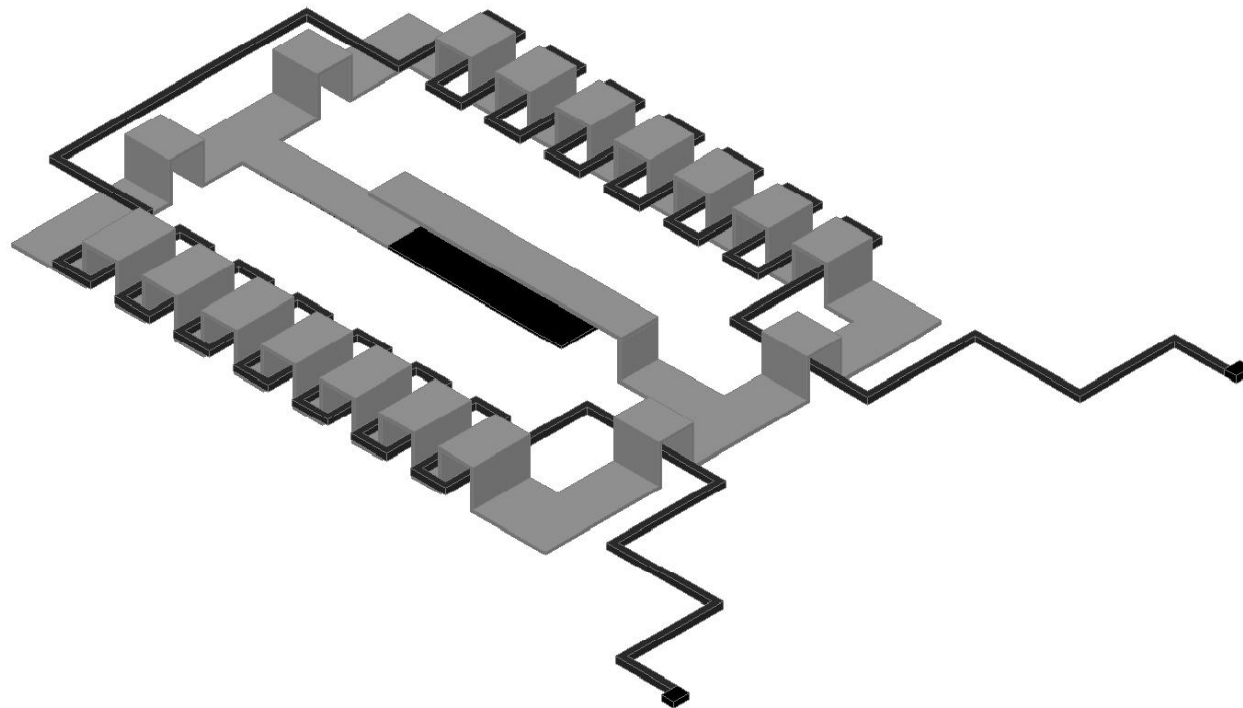
Practical issues in micro-magnetics

- Fabrication of micro coils is possible but it increases the process complexity.

Ahn and Allen, 1993, JMEMS



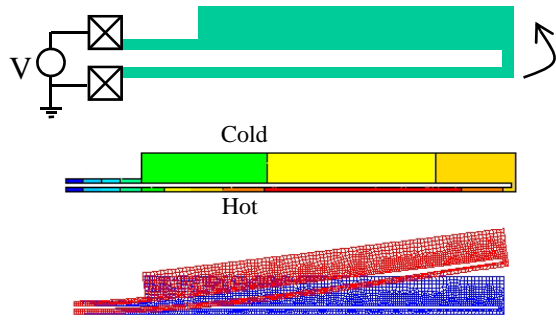
Magnetic actuator with coil on a chip



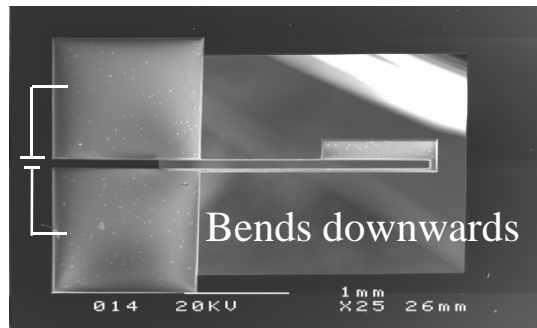
22 Is electro-thermal actuation favorable at the micro scale?



(Comtois and Bright's heater, 1994)

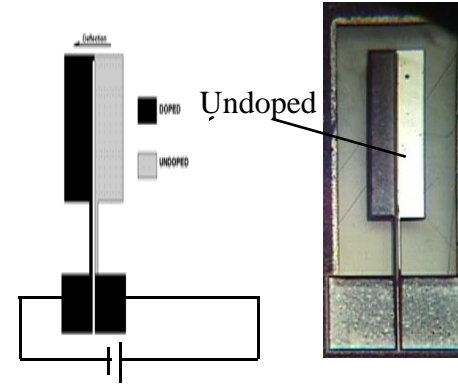


(Made with PennSOIL)

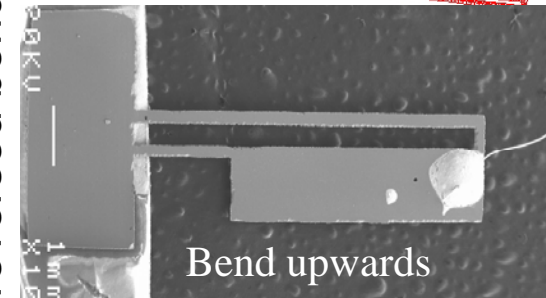
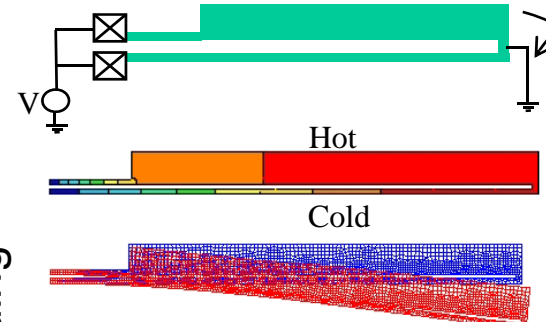


Many variations with shape, doping, electrical and thermal boundary conditions

Selective doping gives the same effect



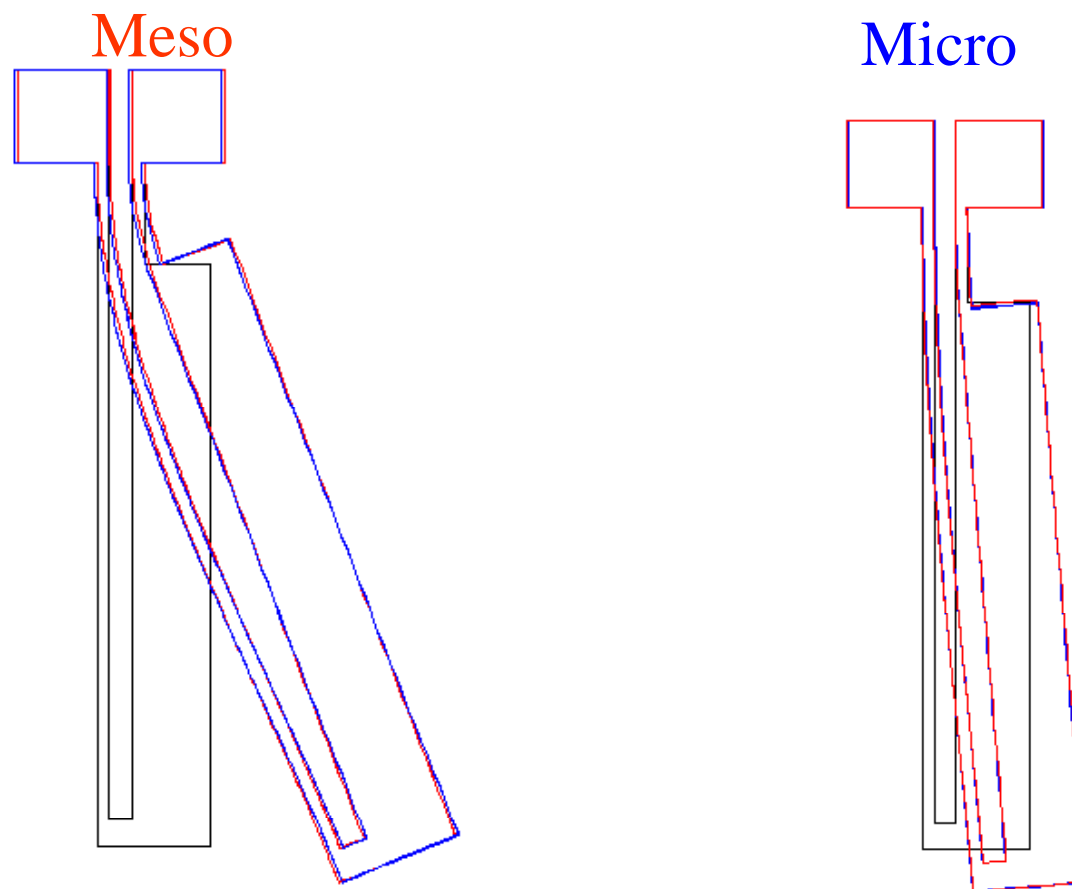
Parallel connection reversed bending





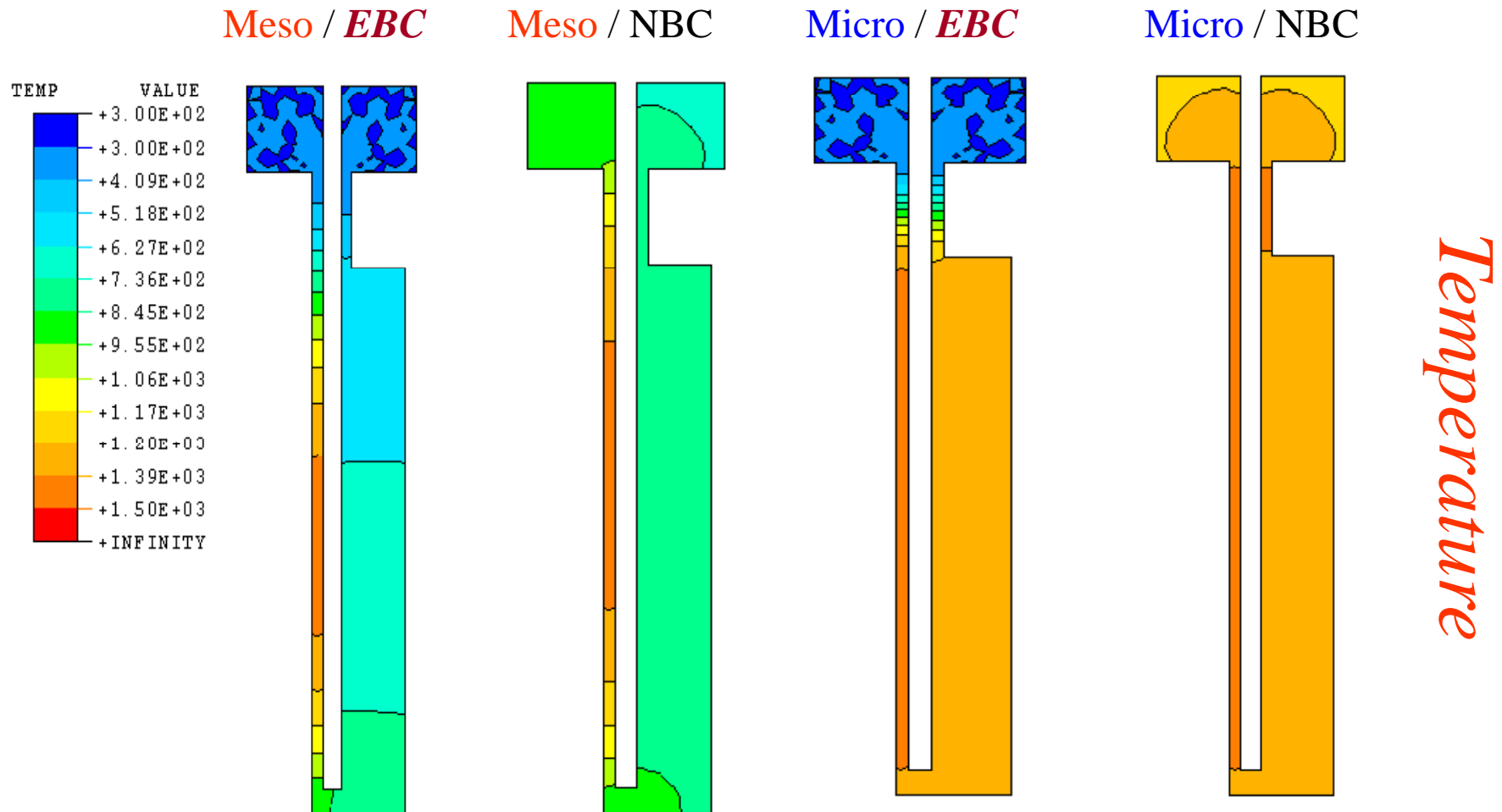
For the same maximum temperature...

10 times larger than micron scale seems to give better relative deflection.





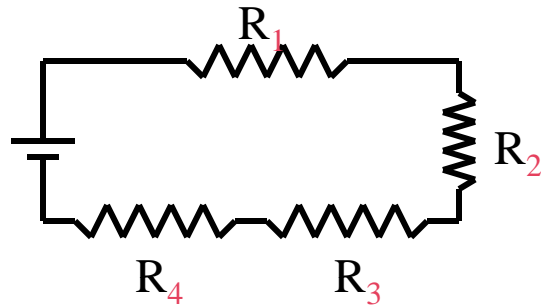
For the same maximum temperature...



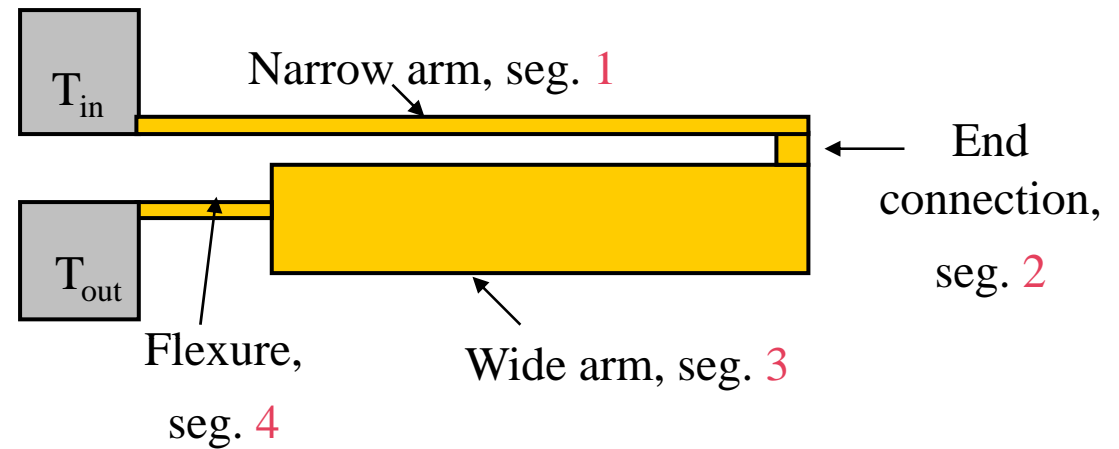
EBC: essential boundary condition; Dirichlet type
NBC: natural boundary condition; Neumann or mixed type



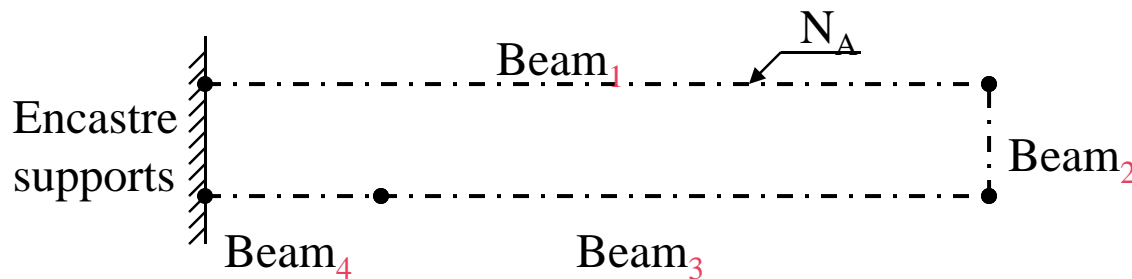
Simplified modeling



Electrical Model



Thermal Model



Elastic Model

(Maizel's theorem to compute the output deflection)



Why do elephants have large ears and dinosaurs fins?

Temperature raise due to metabolic heat produced in living things

$$\Delta T = PR_{th} = \frac{P}{hA} = \frac{pV}{hA} \propto \frac{p}{h} L$$

Power produced $\rightarrow P$ Specific power produced $\rightarrow p$ Volume $\rightarrow V$
 Thermal resistance $\rightarrow R_{th}$ Heat transfer coefficient $\rightarrow h$ Area $\rightarrow A$



➤ Heat transfer coefficient increases nonlinearly with size.

➤ Large animals do not get overheated by increasing the surface area or by decreasing the metabolic rate.

➤ Really large ones stay in water (e.g., whales) to have higher heat transfer coefficients.

➤ Small ones decrease heat transfer coefficients (e.g., feathers in birds) or increasing metabolic rates to stay sufficiently warm.



$$\text{Metabolic rate} \propto \text{Body mass}^{3/4}$$

Kleiber's allotropic law

Scaling in microfluidics



- How does Reynolds number vary with size?

$$\text{Re} = \text{Reynolds number} = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

$$\text{Re} = \frac{\rho UL}{\eta} = \frac{\rho(\alpha L)L}{\eta} \propto L^2$$

Reynolds number decreases rapidly with decreasing size.

Mixing is rather difficult in MEMS.
Small insects "swim" in the air rather than fly!

Scaling of diffusion



Diffusion coefficient

Boltzman constant

Temperature

$$D = \frac{kT}{6\pi\eta r}$$

Viscosity

Hydrodynamic radius

Diffusion distance in time t

$$x = \sqrt{2Dt}$$

Diffusion of a molecule over 10 μm is a million times faster than it is over 10 mm.

Thus, at very small sizes diffusion is enough for reagent molecules to mix whereas macro-sized entities need tubes and pumps to make this happen.



Scaling in microfluidics

- How about Knudsen number? What are its implications?

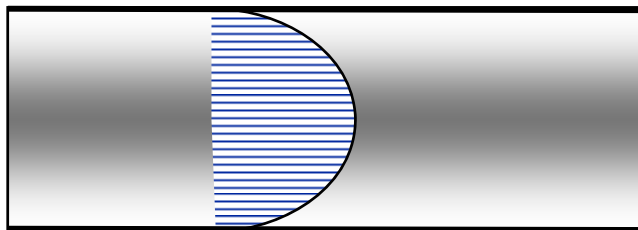
$$Kn = \text{Knudsen number} = \frac{\text{mean free path}}{\text{characteristic length}}$$

$Kn < 0.1 \rightarrow$ OK to use the no-slip boundary condition

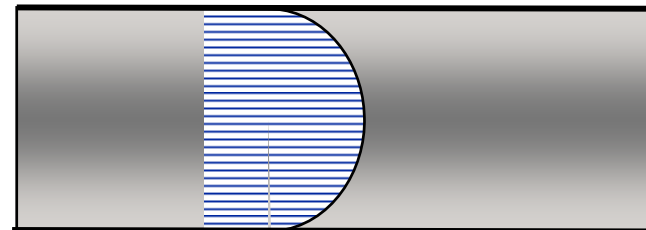
$Kn > 0.1 \rightarrow$ slip-flow regime

$Kn > 10 \rightarrow$ molecular dynamic (MD) simulation regime

No-slip flow



Slip flow



Surface tension at the micro scale



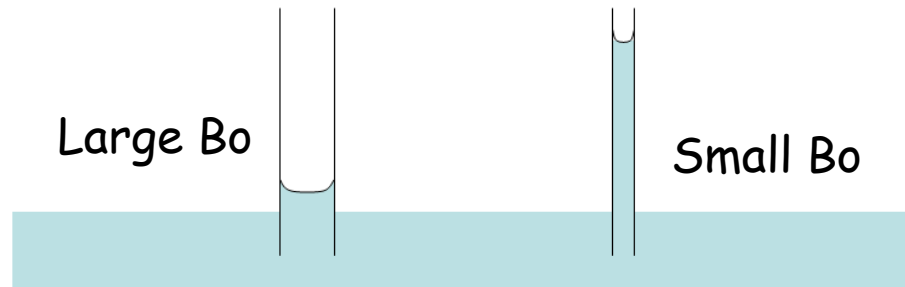
Surface tension scales with length! → It is an enormous force in the micro realm.



Water strider



Why does the liquid rise in a capillary?



Surface tension dominates gravity force at the smaller sizes.

The rise of a liquid in a small capillary as compared to a larger tube is a clear demonstration of this.

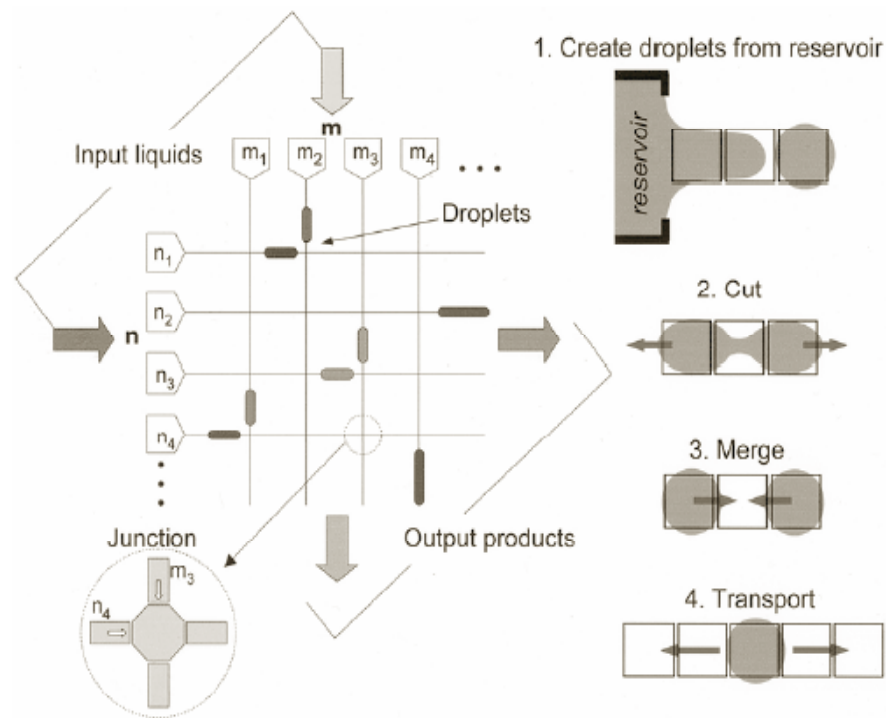
$$\text{Bo} = \text{Bond number} = \frac{\text{gravitational force}}{\text{surface tension}} \quad \text{Bo} = \frac{\rho g L^2}{\gamma}$$

How do you pick a mustard seed lying on the floor?

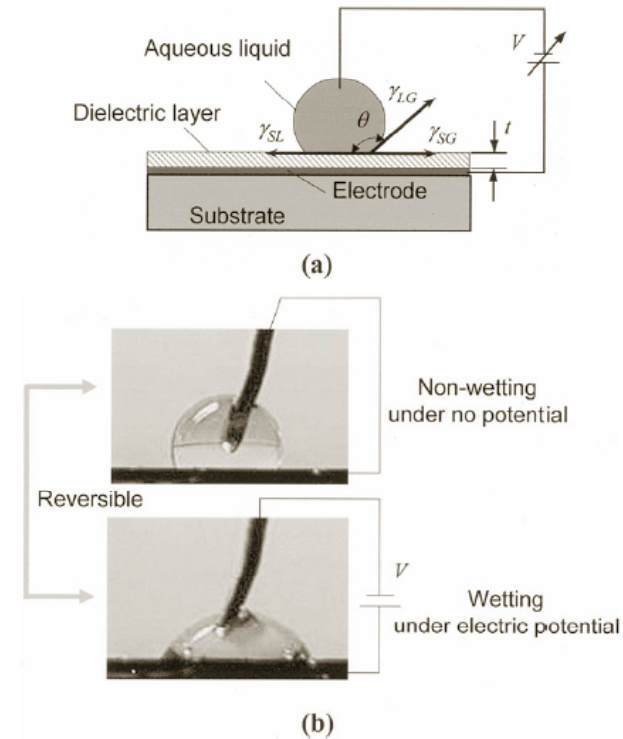
Surface tension at the micro scale



- Several actuation modes by changing surface tension: chemical, thermal, electrical, etc.

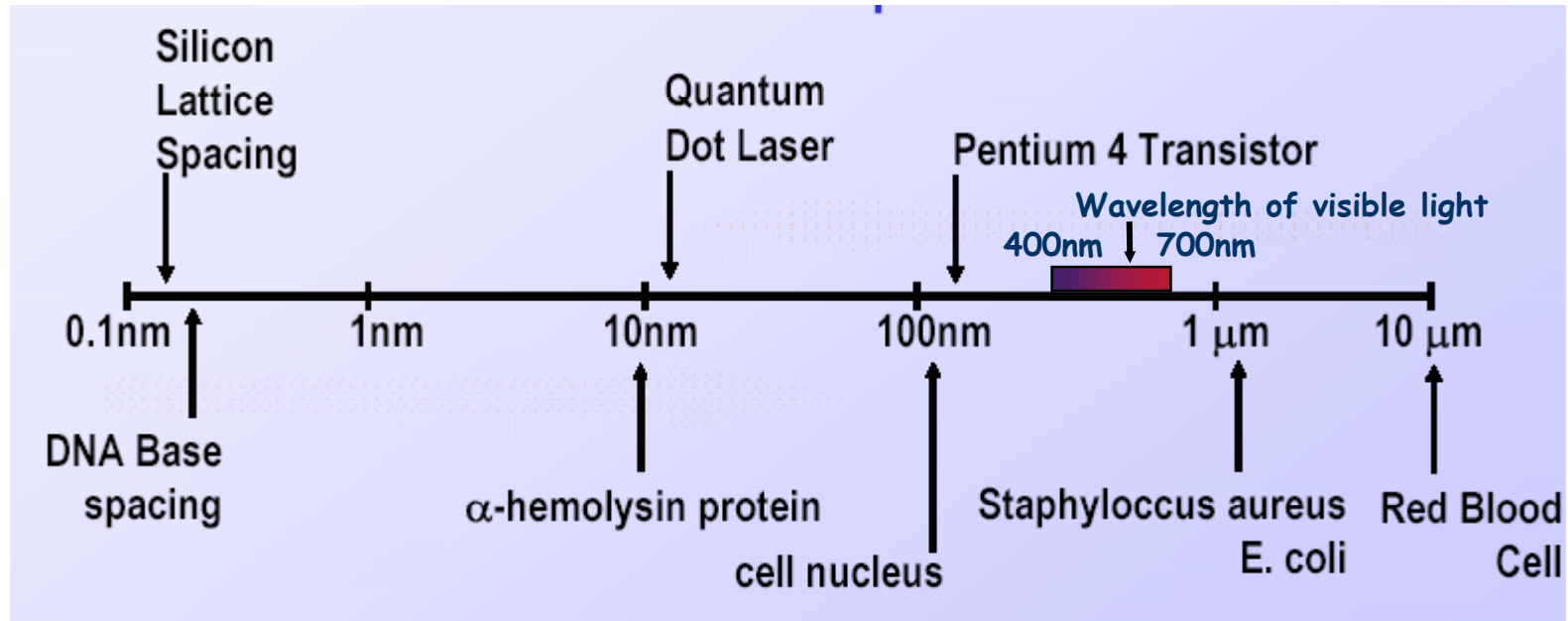


Cho, Moon, Kim, (JMEMS, 2003)



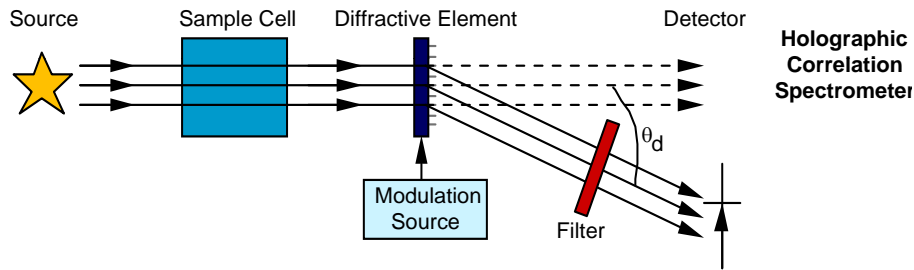
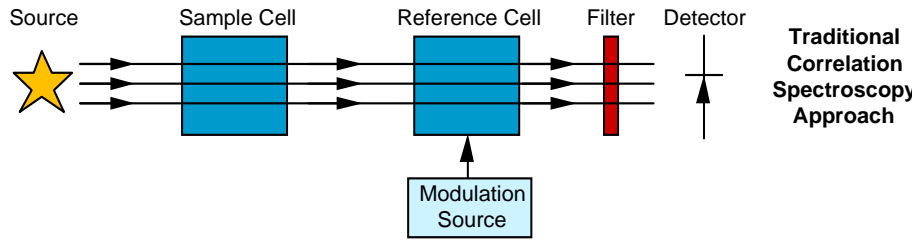
Digital Microfluidics by surface tension driven droplets

MEMS and optics

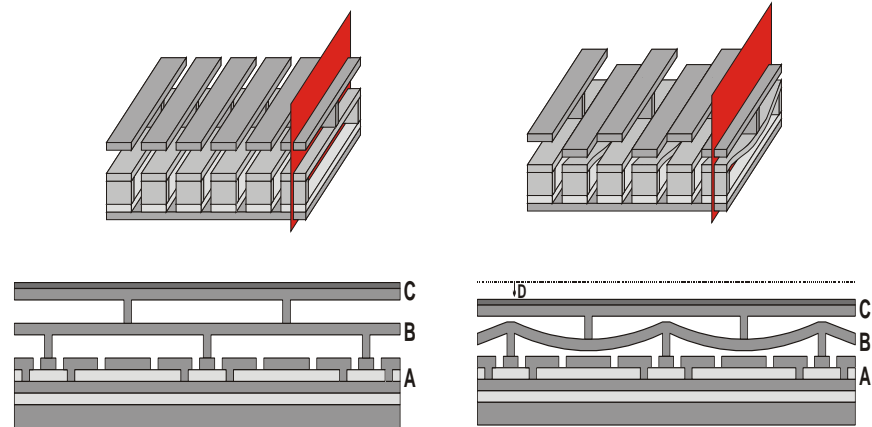
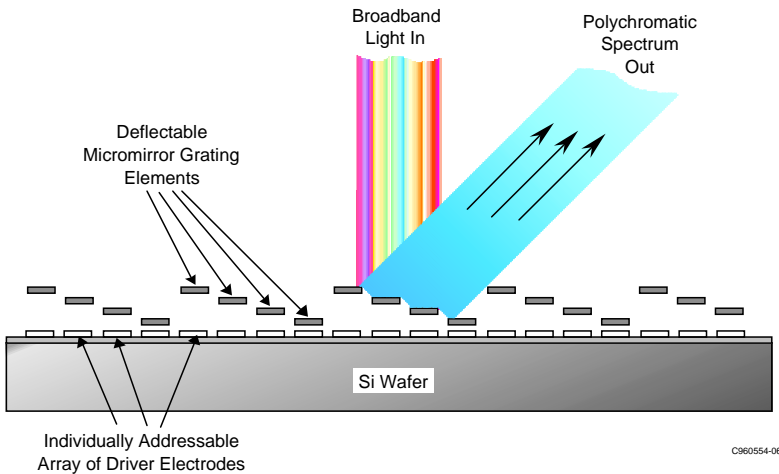


The motions of micro-mechanical devices overlap with the wavelength of the visible light and thus allowing us to “play with light” in interesting and useful ways.

One of many useful manipulations of light



Polychromator:
Honeywell-MIT-Sandia project



(Source for figures: Honeywell and S. D. Senturia)

35 When miniaturization of an optical system is not favorable?



When the light path needs to be large.

e.g., in infrared absorption spectrometer (Herriott cell)

$$w^2(z) = w_0^2 \left(1 + \frac{\lambda z}{\pi w_0^2} \right) \quad \text{Variation of spot size with distance travelled}$$

$$\lambda = 4.5 \mu\text{m} \quad \text{for CH}_4$$

With 100 μm spot, just over 1.6 cm, the size increases by 100%.

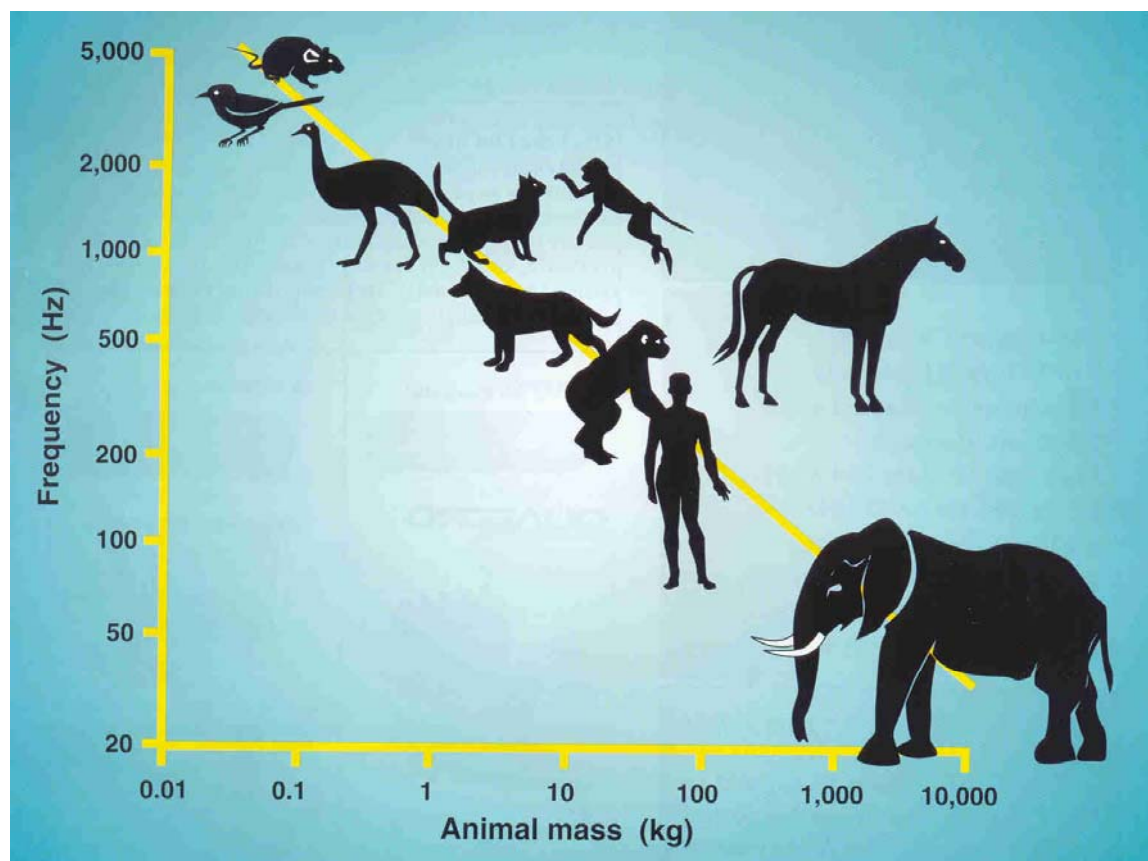
With 300 μm spot size of laser, it increases by only 3%.

Desta et al., 1995.

Scaling in acoustics in Nature



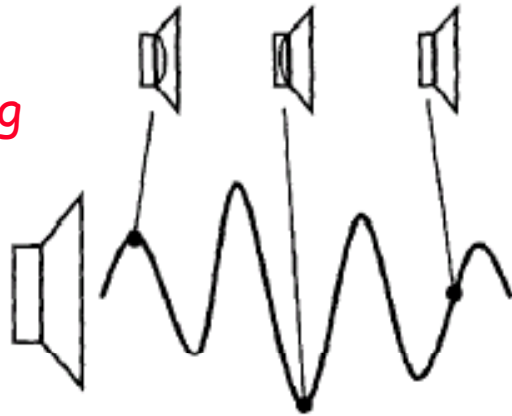
Frequency of sound produced vs. body mass



Fletcher, *Acoustic Australia*, Vol. 33, No. 3 pp. 83-88 (2005)

Scaling and scalability in micro acoustics

Analog



Digital reproduction of sound (DRS)

Advantages of DRS:

Large dynamic range is not necessary.

Nonlinearity can be controlled → distortion is minimized.

Fault tolerance.

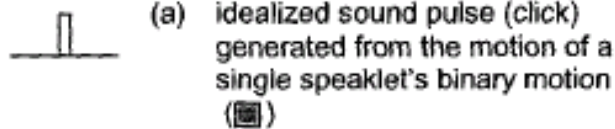
Intensity control.

Speaklets are combined to produce the sound effect.

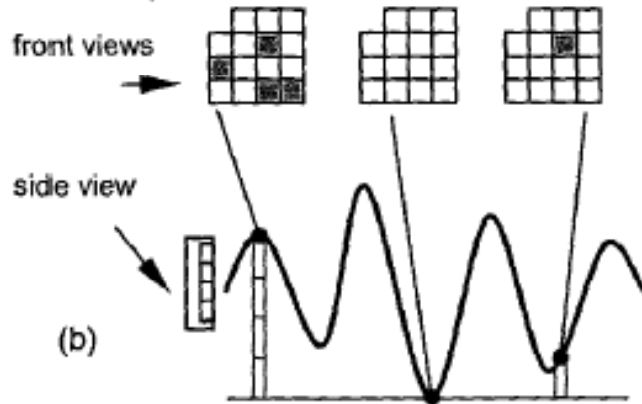
With low-pass filters, the sound is smoothened.

Diamond et al., 2003

Digital



DSR chip



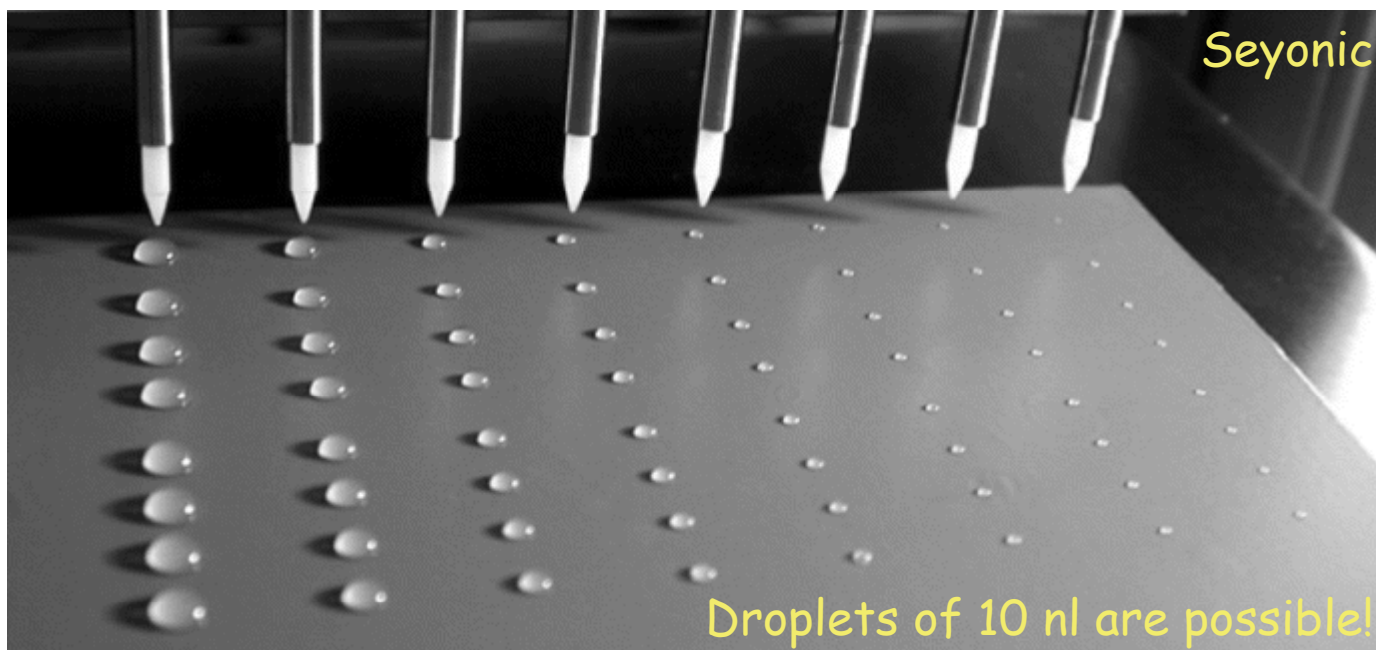
Bio and chemical microsystems



- Why is miniaturization useful and attractive for lab-on-a-chip?
 - Small sample sizes
 - Combinatorial analysis
 - Quick analysis
 - Disposable, cost-effective sensors
 - Controlled, implantable drug-delivery systems
 - Easy read-out and data-processing and storing
- Chemical reactors
 - Precise control over temperature, pressure, concentrations, etc.
 - More optimal use of chemicals



How small can the sample size be?



Too small a drop will evaporate too quickly.

Sample volumes are dictated by concentration as well.

Petersen et al., 1998

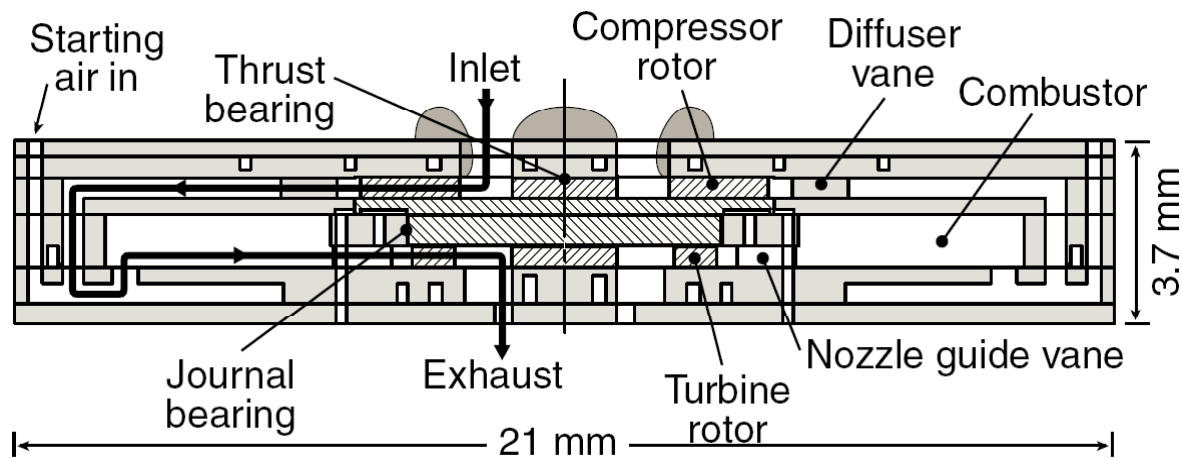
$$V_{\text{sample}} = \frac{1}{\eta N_A C}$$

Efficiency of the sensor $\rightarrow \eta$
 Avagadro's number $\rightarrow N_A$
 Concentration $\rightarrow C$

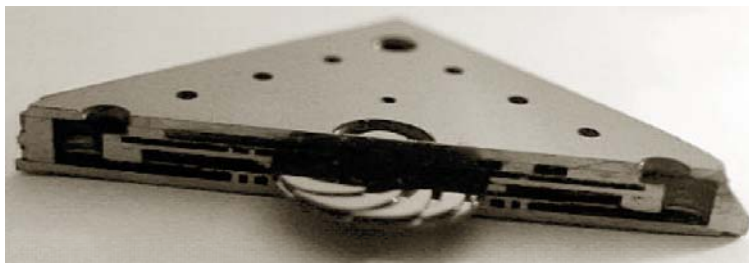
Scaling in micro power generators

A microengine can be made with the same power density as that of a big gas turbine engine.

MIT Microengine (Source: Epstein, 2003)



Power on a chip



Demo engine with H₂ fuel



Turbine-compressor test

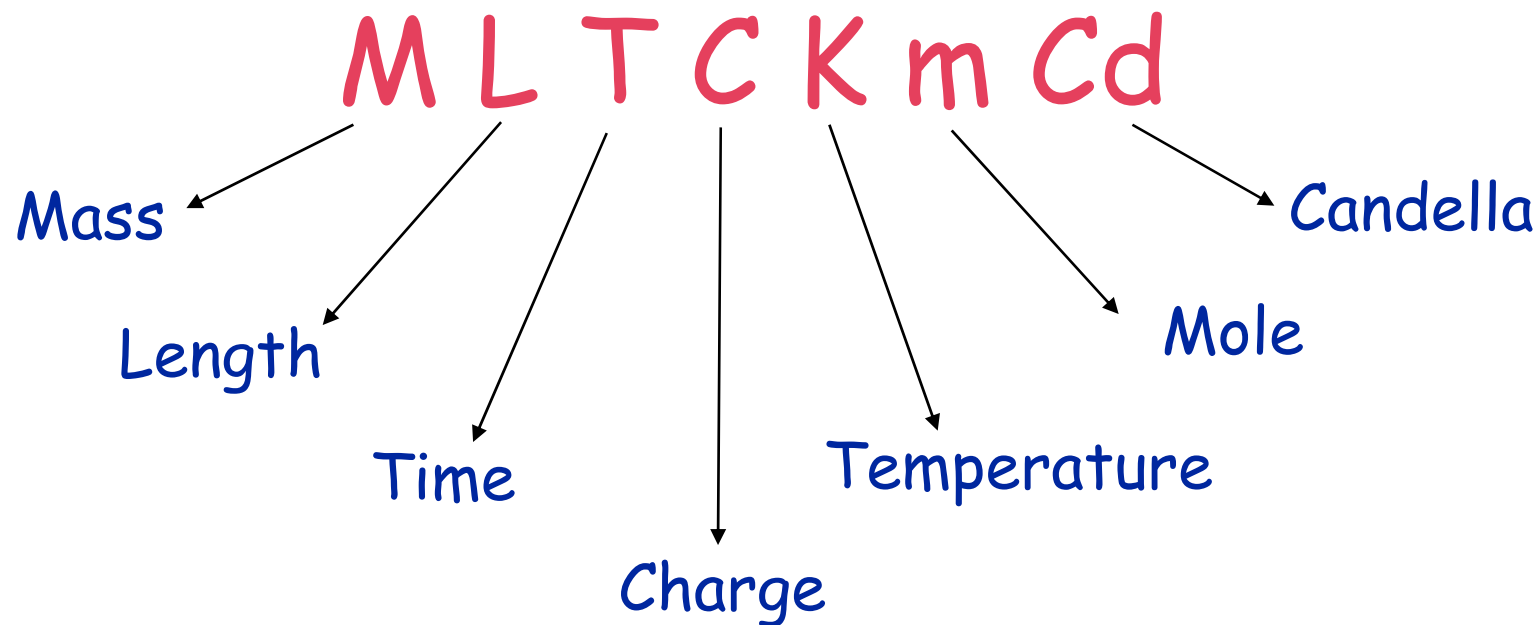


Detail of the DRIE-etched blades



A note about units and dimensions

- Too small/large a number creates numerical problems in computation. How do we overcome it?



Any quantity can be expressed in these dimensional quantities.



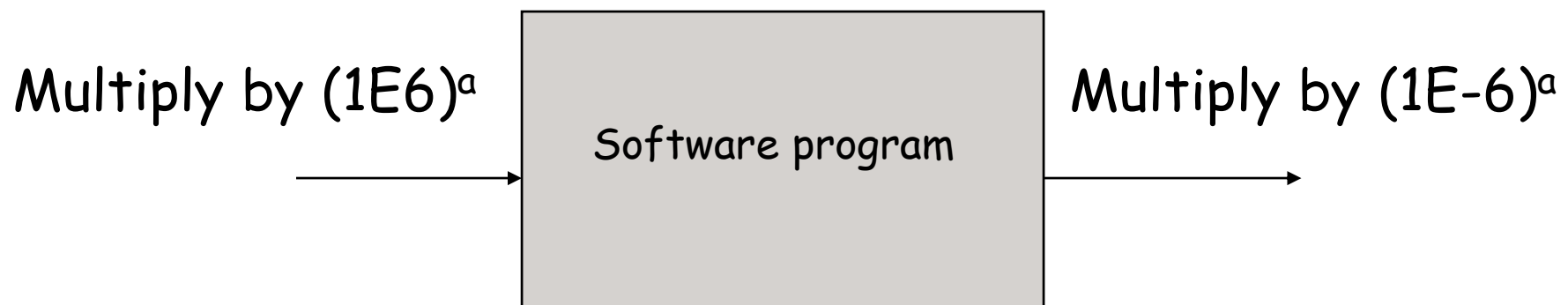
Dealing with units in a software

$$F = [MLT^{-2}] = \frac{q_1 q_2}{4\pi\epsilon_0 d^2} = \frac{[C][C]}{[?][L^2]} \Rightarrow [M^{-1}L^{-3}T^2C^2] \quad \text{Permittivity}$$

$$[ML^2T^{-3}] = VI = [?][T^{-1}C] \Rightarrow [ML^2T^{-2}C^{-1}] \quad \text{Voltage}$$

$$[ML^2T^{-3}] = I^2R = [C^2T^{-2}][?] \Rightarrow [ML^2T^{-1}C^{-2}] \quad \text{Resistance}$$

If we use micron units for length, for a quantity with L^a



Dimensions of some common quantities



QUANTITY	DIMENSIONS	Scale factor
Density	ML^{-3}	10^{-18}
Young's modulus	$ML^{-1}T^{-1}$	10^{-6}
Residual stress	$ML^{-1}T^{-1}$	10^{-6}
Resistance	$ML^2T^{-1}C^{-2}$	10^{12}
Resistivity	$ML^3T^{-1}C^{-2}$	10^{18}
Permittivity	$M^{-1}L^{-3}T^2C^2$	10^{-18}
Permeability	$ML^{-1}T^{-1}C^{-1}$	10^{-6}
Voltage	$ML^2T^{-2}C^{-1}$	10^{12}
Magnetic field	$MT^{-1}C^{-1}$	10^0

Main points



- Scaling decides the performance of micro devices.
- Some micro devices do not function at larger scales.
- Scaling effects provide an excellent way to understand and teach the micro/nano systems technology.
- Simple scaling analysis based on simple modeling helps us assess the suitability of an application to miniaturization.

For further reading

