ME 237: Mechanics of Microsystems : Lecture

Modeling Squeeze Film Effects in MEMS

Anish Roychowdhury

Adviser : Prof Rudra Pratap

Department of Mechanical Engineering
and
Centre for Nano Science and Engineering

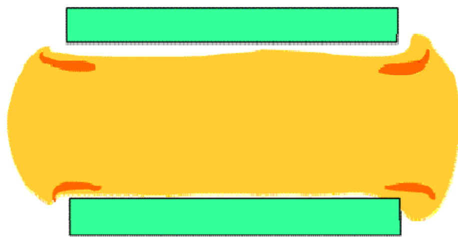
Indian Institute of Science, Bangalore, India

Outline

- Introduction : What is Squeeze Film and when does it occur?
- Motivation: Why is it important to study?
- Various MEMS devices showing Squeeze Film Effects
- Modelling : The Reynolds Equation
- Solution Methods
- The Eigen-expansion Method
- Understanding the Results
- Summary

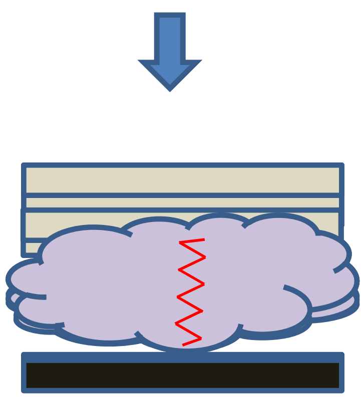
Introduction : What is Squeeze Film Effect

Schematic of Squeeze flow*

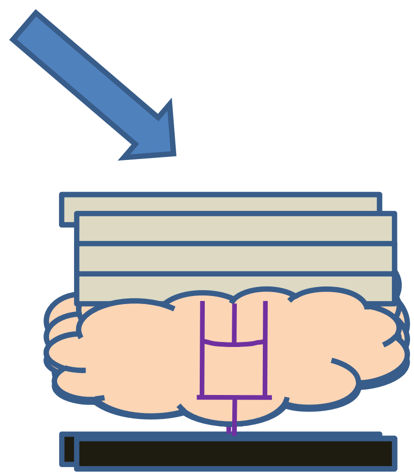


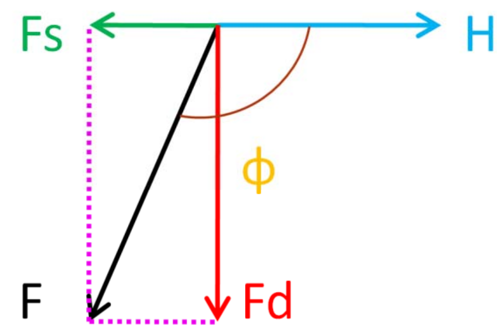
- $F=\int P dA$
- $F=\|F\| \exp(i(\omega t-\phi))$
- $F_s=\|F\|\cos(\phi)$
- $F_d=\|F\|\sin(\phi)$
- $K_{sq}=F_s/\delta h$
- $C_{sq}=F_s/\delta h \omega$

Stiffness effect



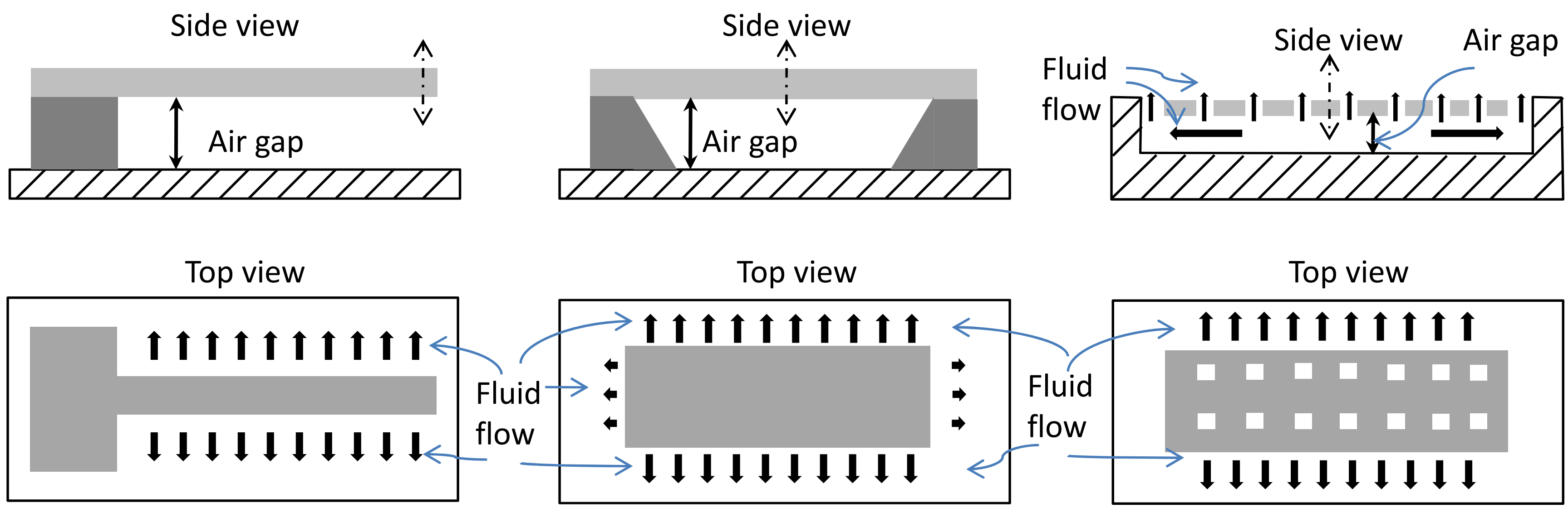
Damping effect





* Animation courtesy : Siddartha Patra, ME 2011, Mechanical Engineering, IISC Bangalore

Introduction : Squeeze Flow



Introduction : Relevant Conditions for Squeeze Film Effects

Continuum Flow regime

Vibration normal to a fixed substrate*

Si (High Q) based MEMS devices

L,W >> h

Geometry aspect

Materials aspect

Knudsen Number (Kn)

$$K_n = \frac{\lambda}{h}$$

λ = mean free path
h = characteristic flow length

Flow Regime Division

Range of Kn	Flow regime
$Kn < 0.01$	Continuum flow regime
$0.01 < Kn < 0.1$	Slip flow regime

Surface

$Q = 10^4 - 10^5$

Support Losses

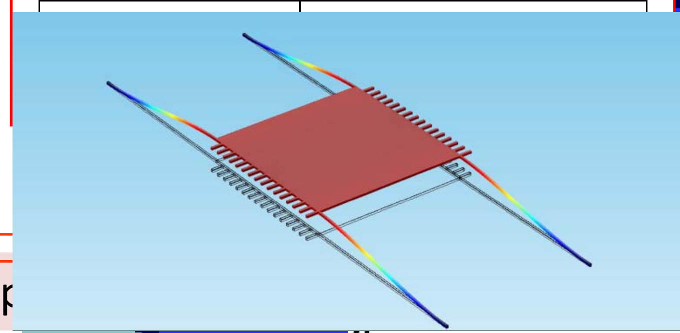
…eeze film
…mping
= 1- 10)

Others

$Q > 10^7$

h

Typ

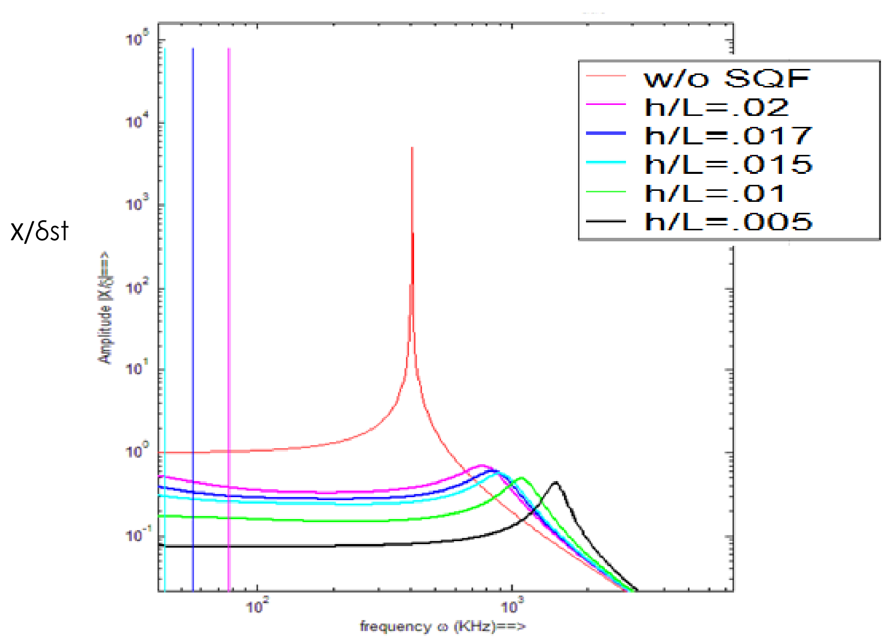


* Animation (MEMS Tuning Gyro)Courtesy JP Reddy, CeNSE , IISC

Motivation: Importance of squeeze film

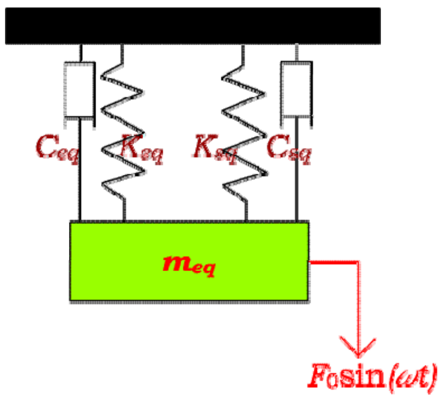
Gap to length ratio*

Shift of resonant peak at different h/L ratios



Change in effective stiffness and Damping *

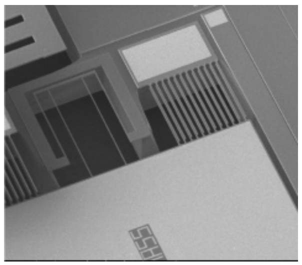
$$m_{eq}\ddot{x} + (C_{eq} + C_{sq})\dot{x} + (K_{eq} + K_{sq})x = F_0 \sin(\omega t)$$



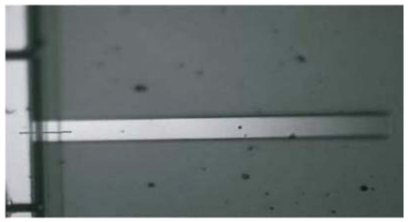
* ME Thesis, S Patra, Indian Institute of Science, Bangalore, 2011

Squeeze Film in MEMS devices

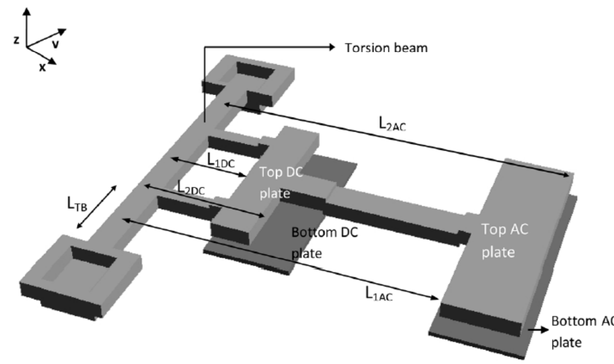
Accelerometer [1]



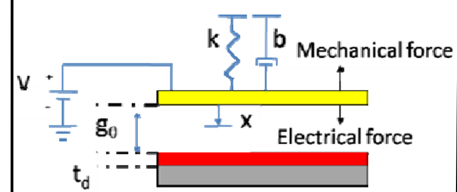
Cantilever resonator [3]



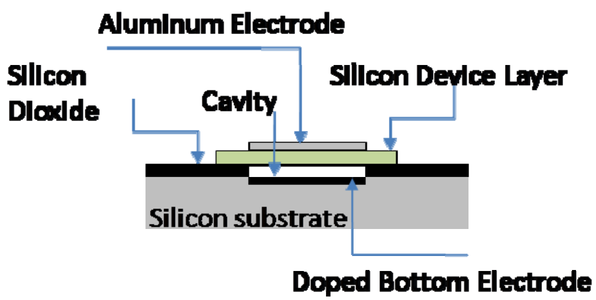
Varactor [5]



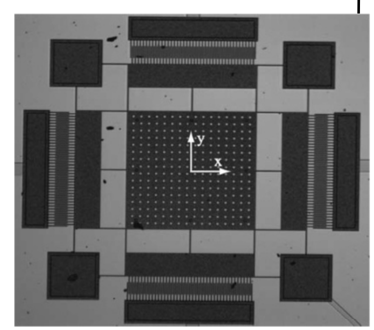
RF Switch [7]



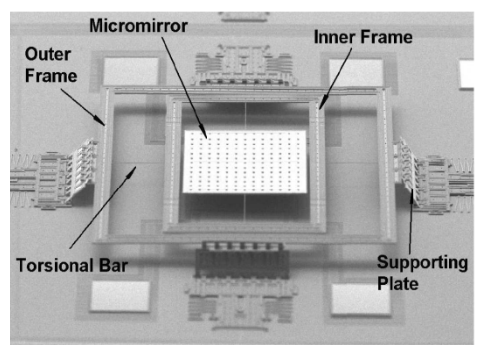
CMUT [4]



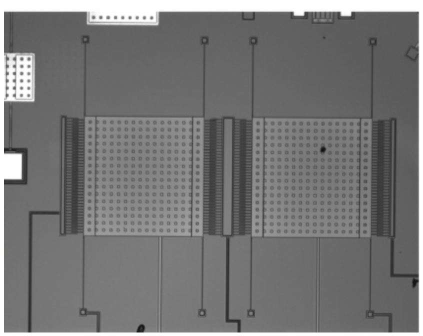
Yaw Rate sensor [8]



Torsion Mirror [6]



Gyroscope [2]



Getting to the Reynolds Equation

Under continuum hypothesis fluid flow is modelled using the Navier Stoke's Equation

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho g_i + \boxed{\frac{\partial}{\partial x_j}\left[2\mu e_{ij} - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\delta_{ij}\right]} \qquad (1)$$

Assuming small temperature differences within fluid 'μ' can be treated as constant and we can rewrite (1) as

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho g_i + \mu\left[\nabla^2 u_i + \frac{1}{3}\frac{\partial}{\partial x_i}(\nabla \cdot \mathbf{u})\right] \qquad (2)$$

Assuming **incompressible** flow we get from equation (2)

$$\boxed{\nabla \cdot \mathbf{u} = 0,}$$

$$\boxed{\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu\, \nabla^2 \mathbf{u}.} \qquad (3)$$

Contd.

Key equations

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{u}) = 0$$

Expanded Navier Stokes

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

Isothermal

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Contd.

Navier Stokes

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Neglect Body forces

Neglect fluid inertia

Thin Film thickness

Small am.p of Osc.

Fully Developed flow

Contd.

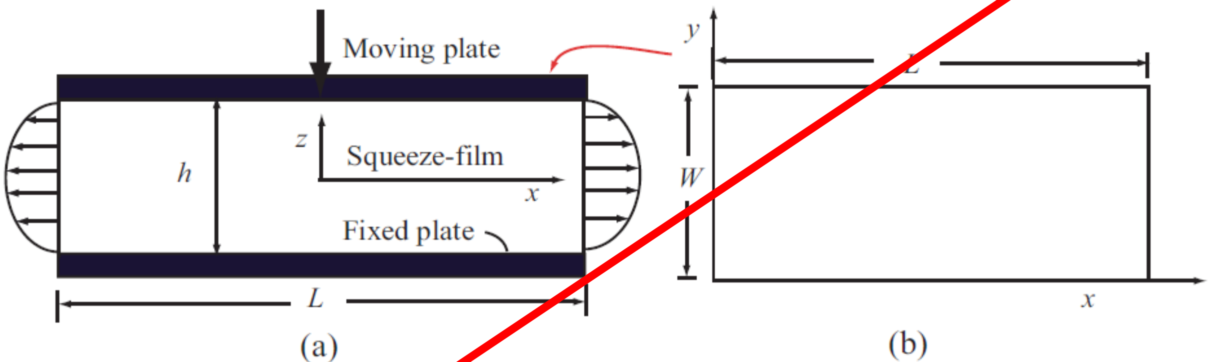
Thus we arrive at the 2D Reduced NS Eqn.

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right), \quad \text{and} \quad \frac{\partial p}{\partial y} = \frac{\partial}{\partial z}\left(\mu \frac{\partial v}{\partial z}\right)$$

No Slip – BC

$$u = 0,\ v = 0 \text{ at } z = \pm h/2$$

a) SFD Flow - normal motion b) plate Top view *



Velocities

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(z^2 - h^2/4)$$

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial y}(z^2 - h^2/4)$$

Flow rates

$$q_x = -\frac{h^3}{12\mu}\frac{\partial p}{\partial x}, \quad \text{and} \quad q_y = -\frac{h^3}{12\mu}\frac{\partial p}{\partial y}.$$

* Source [2]

Contd.

Using Velocities

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(z^2 - h^2/4)$$

&

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial y}(z^2 - h^2/4)$$

$$\frac{\partial}{\partial x}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial y}\right) = 12\frac{\partial (ph)}{\partial t}$$

$$q_x = -\frac{h^3}{12\mu}\frac{\partial p}{\partial x}, \quad \text{and} \quad q_y = -\frac{h^3}{12\mu}\frac{\partial p}{\partial y}.$$

&

Isothermal Flow

$$p \propto \rho$$

$$\frac{\partial}{\partial x}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial y}\right) = 12\frac{\partial (ph)}{\partial t}.$$

The Reynolds Equation

$$\frac{\partial}{\partial x}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial y}\right)=12\frac{\partial(ph)}{\partial t}.$$

Nonlinear compressible Reynolds Equation

$$\left[\frac{\partial^2\hat{p}}{\partial x^2}+\frac{\partial^2\hat{p}}{\partial y^2}\right]=\frac{12\mu}{P_a h_a^3}\left[h_a\frac{\partial\hat{p}}{\partial t}+P_a\frac{\partial\hat{h}}{\partial t}\right]$$

Linearized compressible Reynolds Equation

$$\left[\frac{\partial^2\Phi}{\partial X^2}+\frac{\partial^2\Phi}{\partial Y^2}\right]=\sigma\left[\frac{\partial\Phi}{\partial\tau}+\frac{\partial\epsilon}{\partial\tau}\right]$$

Linearized Non dimensional compressible Reynolds Equation

Squeeze Number

The Squeeze number

$$\sigma = 12\mu_{\text{eff}} \frac{1}{p_o} \left(\frac{L}{h}\right)^2 \omega$$

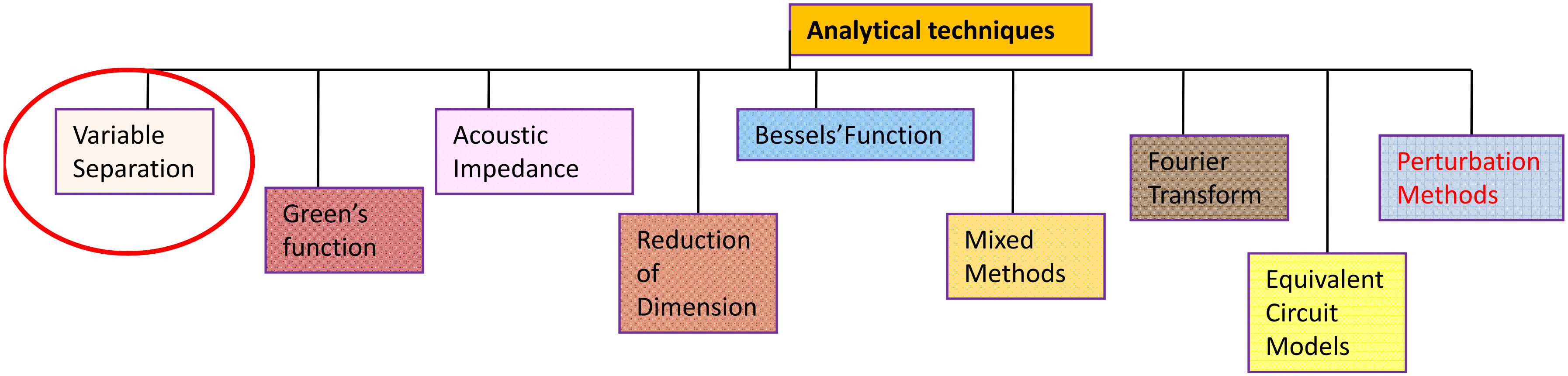
Rarefaction

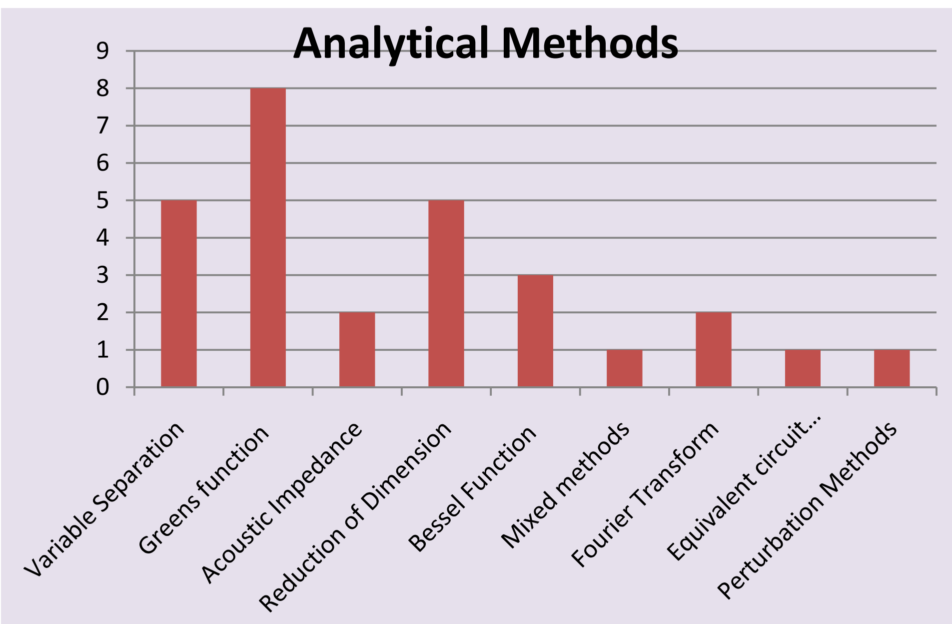
Ambient pressure

Length to air gap ratio

Oscillation frequency

Analytical Techniques for solution of Reynolds Equation



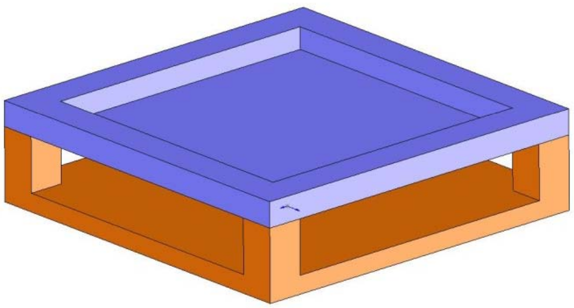


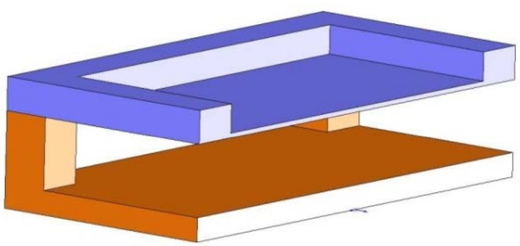
Based on – **28 papers surveyed**

Problem deifinition : Variable flow boundary: All sides clamped plate

Structural BC – all sides clamped

Rectangular plate – all sides open





Geometry – flow B.C.'s

OOOO

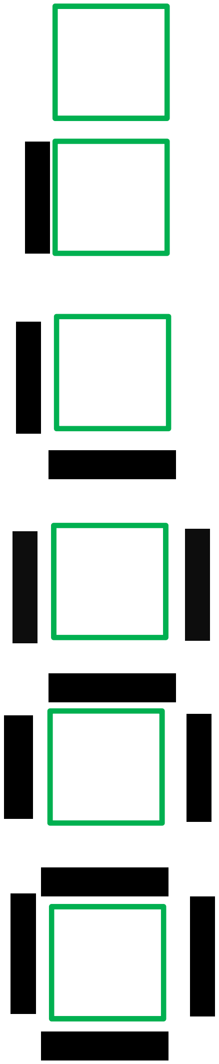
OOOC

OOCC

OCOC

OCCC

CCCC



Separation of Variables : Eigen Expansion

We are solving linearized Reynolds eqn.

$$\nabla^2 P - \alpha^2 \frac{\partial P}{\partial t} = \alpha^2 \frac{\partial H}{\partial t} \quad \text{——} \quad (1)$$

Consider homogeneous form of (1)

$$\nabla^2 P - \alpha^2 \frac{\partial P}{\partial t} = 0 \quad \text{——} \quad (2)$$

Using eigen expansion and separation of variables we assume

$$P = u(x, y, t) \quad \text{——} \quad (3)$$

Where

$$u = \sum_{m,n} f(x, y)_{mn} T(t) \quad \text{——} \quad (4)$$

Thus from (2), (3), and 4 we get

$$\frac{\nabla^2 f}{f} = \frac{\alpha^2 \frac{\partial T}{\partial t}}{T} = -k_{mn}^2 \quad \text{——} \quad (5)$$

Now assume the following

$$k_{mn}^2 = k_m^2 + k_n^2 \quad \text{——} \quad (6)$$

$$f_{mn} = a_{mn} \psi_{mn}(x, y) \quad \text{——} \quad (7)$$

Solution: Contd.

Thus we get

$$P = \sum_{m,n} a_{mn}\psi_{mn}(x,y)T(t) \quad \text{——} \quad (8)$$

Assuming harmonic source term

$$H = \delta\phi e^{i\omega t} \quad \text{——} \quad (9)$$

Thus choosing

$$T(t) = e^{i\omega t} \quad \text{——} \quad (10)$$

Using (8),(9),(10) and Reynolds Eqn

$$\sum_{m,n} -a_{mn}k_{mn}^2\psi_{mn}(x,y)e^{i\omega t} - a_{mn}\alpha^2\psi_{mn}(x,y)i\omega e^{i\omega t} = \delta\phi e^{i\omega t} \quad \text{——} \quad (11)$$

Multiplying both sides of (11) by $\psi_{pq}(x,y)$ —— (12)

And using orthogonality

$$\int\int \psi_{mn}(x,y)\psi_{pq}(x,y)dxdy = \delta_{mn}\delta_{pq} \quad \text{——} \quad (13)$$

Solution: Contd.

We get

$$a_{mn} = \delta \frac{\int_0^L \int_0^W i\omega\alpha^2 \phi \psi_{mn} dy dx}{\int_0^L \int_0^W [-k_{mn}^2 - i\omega\alpha^2]\psi_{mn}^2 dy dx} \quad \text{— (14)}$$

Kinematic BCs are satisfied

Now the first mode shape of the plate can be approximated as

$$\phi(x,y) = \sin\left(\frac{\pi x}{L}\right)^2 \sin\left(\frac{\pi y}{W}\right)^2 \quad \text{— (15)}$$

Thus the solution is given by (8)

$$P = \sum_{m,n} a_{mn}\psi_{mn}(x,y)T(t)$$

Where $\psi_{mn}(x,y)$ Is an admissible eigen function

Open Boundary → $P = 0$

Closed Boundary → $\frac{\partial P}{\partial n} = 0$

Kinematic BC's

Mode shape

$$\phi(x,y) = \sin\left(\frac{\pi x}{L}\right)^2 \sin\left(\frac{\pi y}{W}\right)^2$$

Satisfies

- No deflection at fixed edges → $\phi(x,y) = 0$ @ $x = L \; , \; y = W$
- Maximum deflection at center → @ $x = \frac{L}{2} \; , \; y = \frac{W}{2}$

$$\phi_x(x,y) = 0 \text{ and } \phi_y(x,y) = 0$$

$$\phi_{xx}(x,y)\phi_{yy}(x,y) - \phi_{xy}(x,y)^2 > 0$$

$$\phi_{xx}(x,y) < 0$$

Admissible Eigen Function s: OOOO

From separation of variables for P

$$\nabla^2\psi_{mn} + k_{mn}^2\psi_{mn} = 0$$

Assuming

$$\psi_{mn}(x,y) = X_m(x)Y_n(y)$$

$$k_{mn}^2 = k_m^2 + k_n^2$$

We get

$$X_m''Y_n + X_mY_n'' + k_{mn}^2X_mY_n = 0$$

Separating variables we get the x and y equations

$$X_m = A\sin(k_mx) + B\cos(k_mx)$$

$$Y_n = C\sin(k_nx) + D\cos(k_nx)$$

Applying zero pressure BCs, we get

$$\psi_{mn}(x,y) = \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{W}\right)$$

Solution for the All sides open case : OOOO

All sides open

Zero pressure B.C.'s

$$P(0,y) = P(L,y) = 0$$

$$P(x,0) = P(x,W) = 0$$

Admissible eigen function

$$\psi_{mn}(x,y) = \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{W}\right)$$

And using

$$\phi(x,y) = \sin\left(\frac{\pi x}{L}\right)^2 \sin\left(\frac{\pi y}{W}\right)^2$$

with

$$a_{mn} = \delta \frac{\int_0^L \int_0^W i\omega\alpha^2 \phi \psi_{mn} dy dx}{\int_0^L \int_0^W [-k_{mn}^2 - i\omega\alpha^2]\psi_{mn}^2 dy dx}$$

in

$$P = \sum_{m,n} a_{mn}\psi_{mn}(x,y)T(t)$$

with $T(t) = e^{i\omega t}$

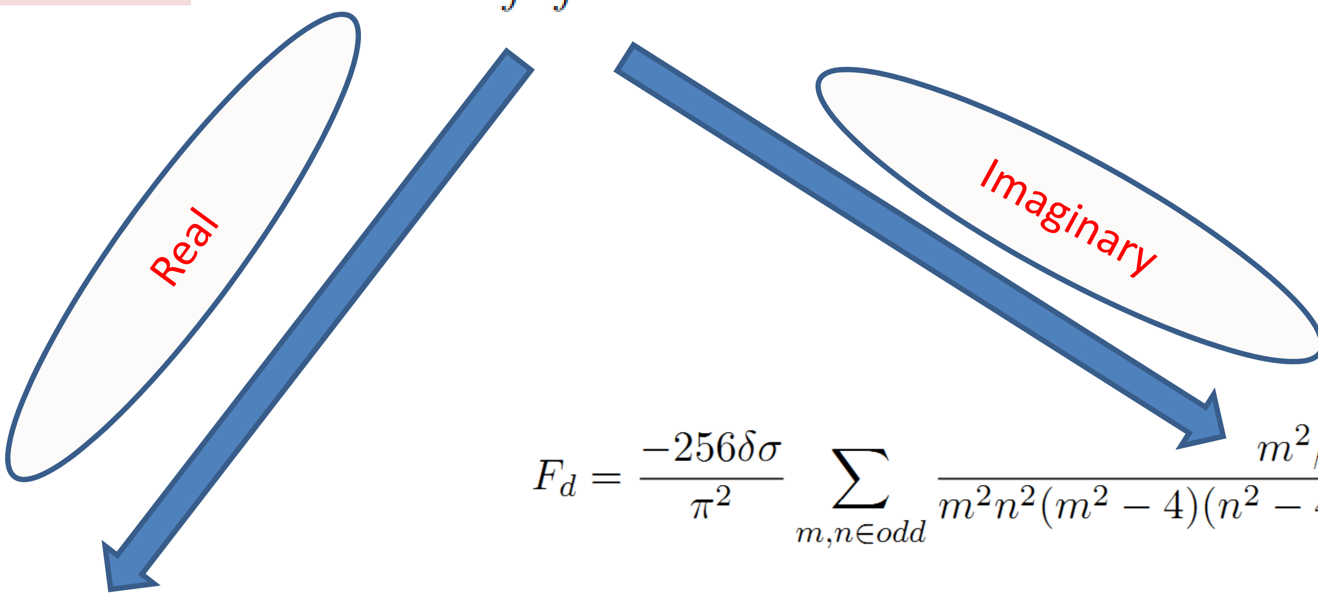
Solution for the All sides open case : OOOO

Non dimensional pressure

$$P = -64\delta \sum_{m,n \in odd} \frac{\left[i\pi^2\sigma(m^2\beta^2 + n^2) + \sigma^2\right] \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{W}\right)}{\left[\pi^4(m^2\beta^2 + n^2)^2 + \sigma^2\right] mn\pi^2(m^2 - 4)(n^2 - 4)}$$

Total Force

$$F_{tot} = \int\int P_a P(x, y, t)\, dx\, dy$$

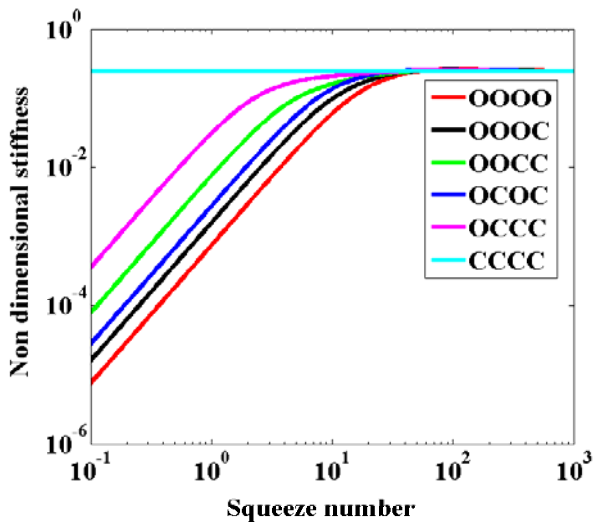


$$F_d = \frac{-256\delta\sigma}{\pi^2} \sum_{m,n \in odd} \frac{m^2\beta^2 + n^2}{m^2n^2(m^2 - 4)(n^2 - 4)\left[\pi^4(m^2\beta^2 + n^2)^2 + \sigma^2\right]}$$

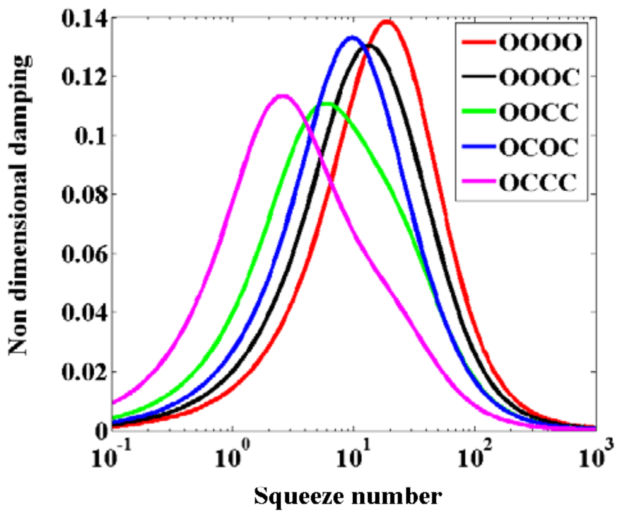
$$F_s = \frac{-256\delta\sigma^2}{\pi^4} \sum_{m,n \in odd} \frac{1}{m^2n^2(m^2 - 4)(n^2 - 4)\left[\pi^4(m^2\beta^2 + n^2)^2 + \sigma^2\right]}$$

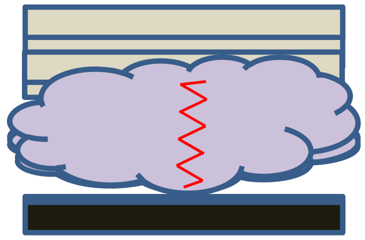
Results : Stiffness and Damping

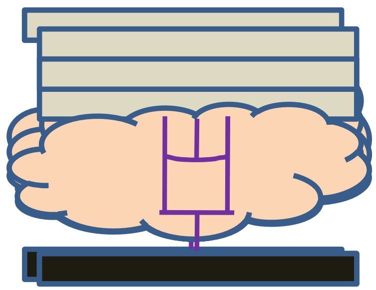
Stiffness



Damping

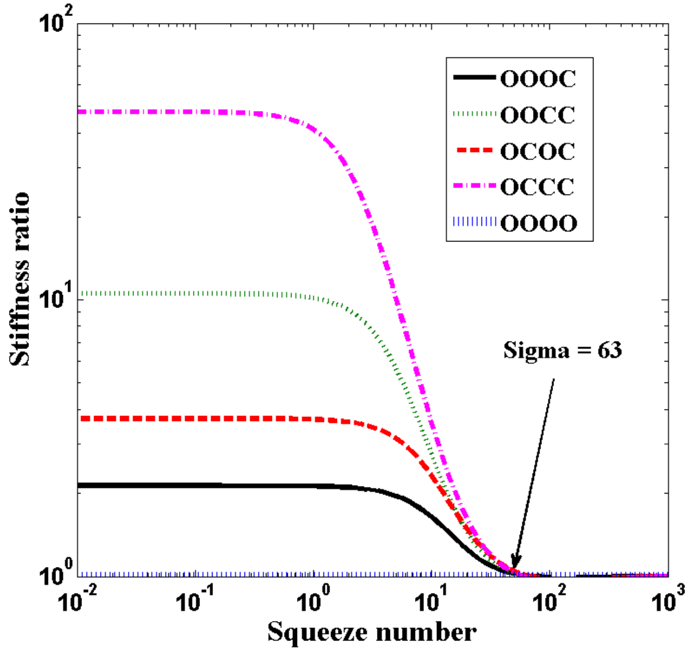




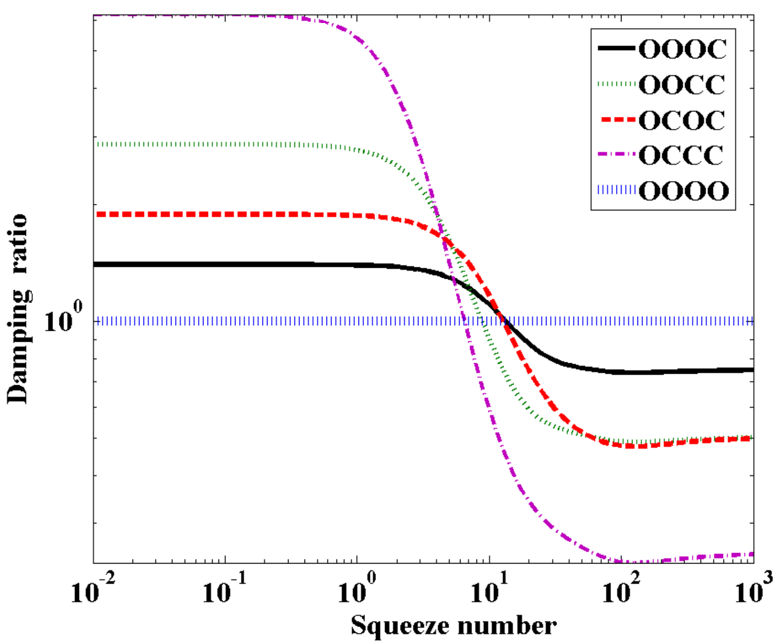


Results : Stiffness and Damping Relative comparison among flow BCs

Stiffness Ratios



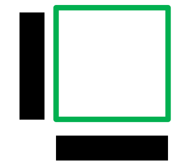
Damping Ratios

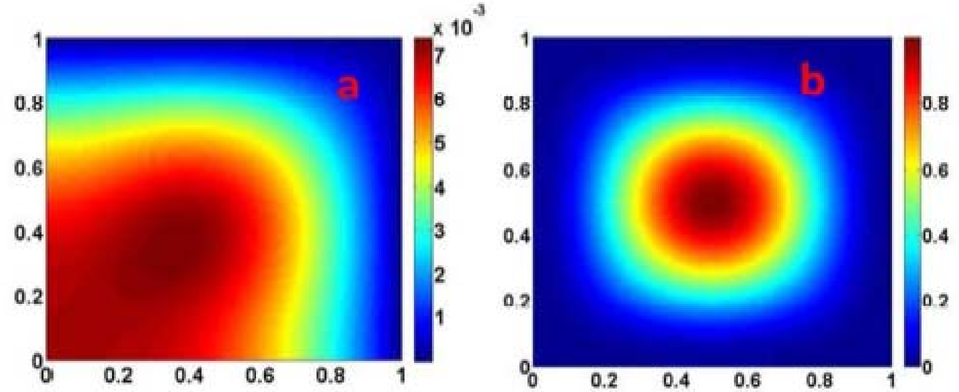


Results : Pressure and Phase variation: OOCC

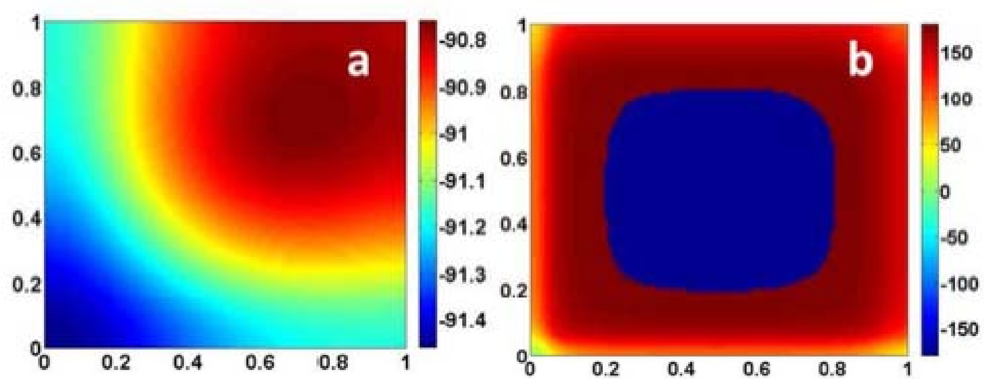
Pressure variation : a) σ = 0.1, b) σ = 1000

OOCC



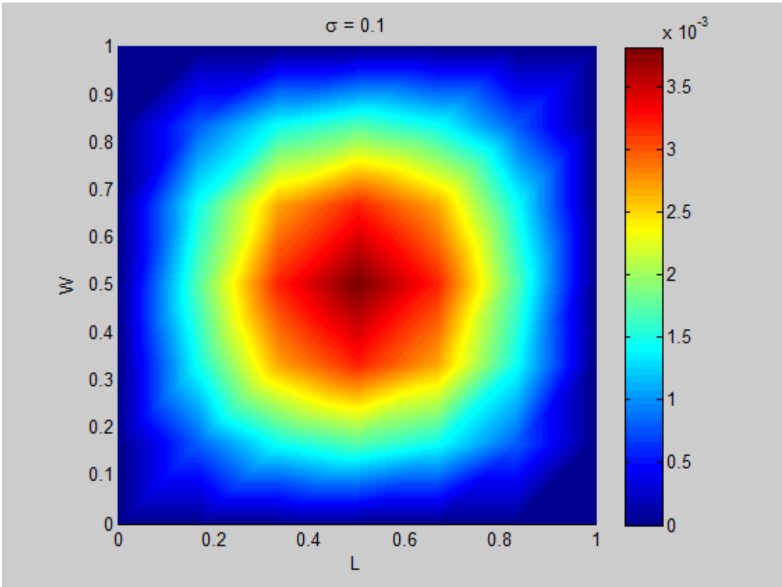


Phase variation : a) σ = 0.1, b) σ = 1000

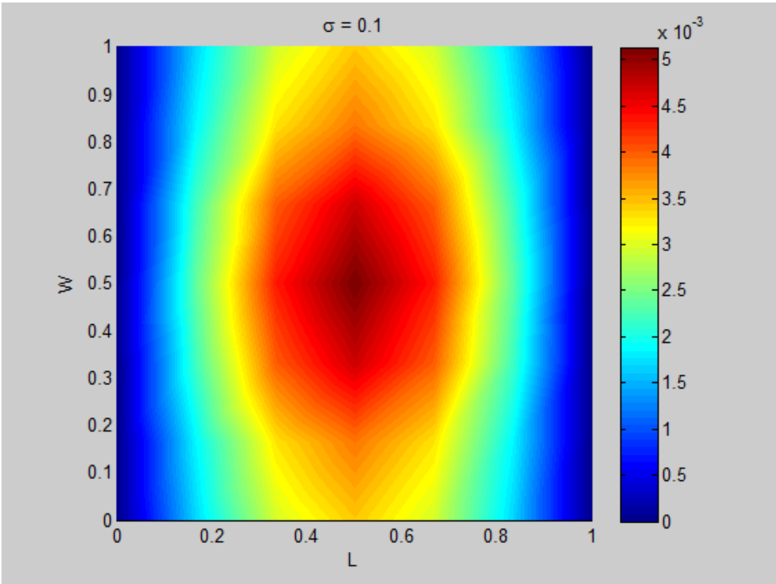


Pressure Profiles varying with Squeeze number

OOOO



OCOC



Summary

- We have seen when and where squeeze film effects occur.
- We discussed modeling aspects and solution methods.
- Variation of squeeze film parameters with squeeze number was shown.
- Effect of flow boundary conditions were discussed.
- Pressure and phase variation with squeeze number were shown.

Other aspects in squeeze film modeling

Complexities	Coupled Domains	Commercial Software for Modeling Squeeze Film
Rarefaction Compressibility Inertia Perforations Non trivial BC's Complex geometries	Structural Fluid Electrostatic	ANSYS COMSOL NISA

References

1. Khan S, Thejas and Bhat N (2009) Design and characterization of a micromachined accelerometer with mechanical amplifier for intrusion detection. 3rd National ISSS conference on MEMS, Smart Structures and Materials, Kolkata, India (Oct. 14-16, 2009).

2. Patil N (2006) Design and analysis of MEMS angular rate sensors. Master's Thesis: Indian Institute of Science, Bangalore, India.

3. Pandey AK, Venkatesh KP and Pratap R (2009) Effect of metal coating and residual stress on the resonant frequency of MEMS resonators. Sãdhãna 34(4):651-661

4. Ahmad B and Pratap R (2011) Analytical evaluation of squeeze film forces in a CMUT with sealed air filled cavity. IEEE Sensors 11(10): 2426-2431.

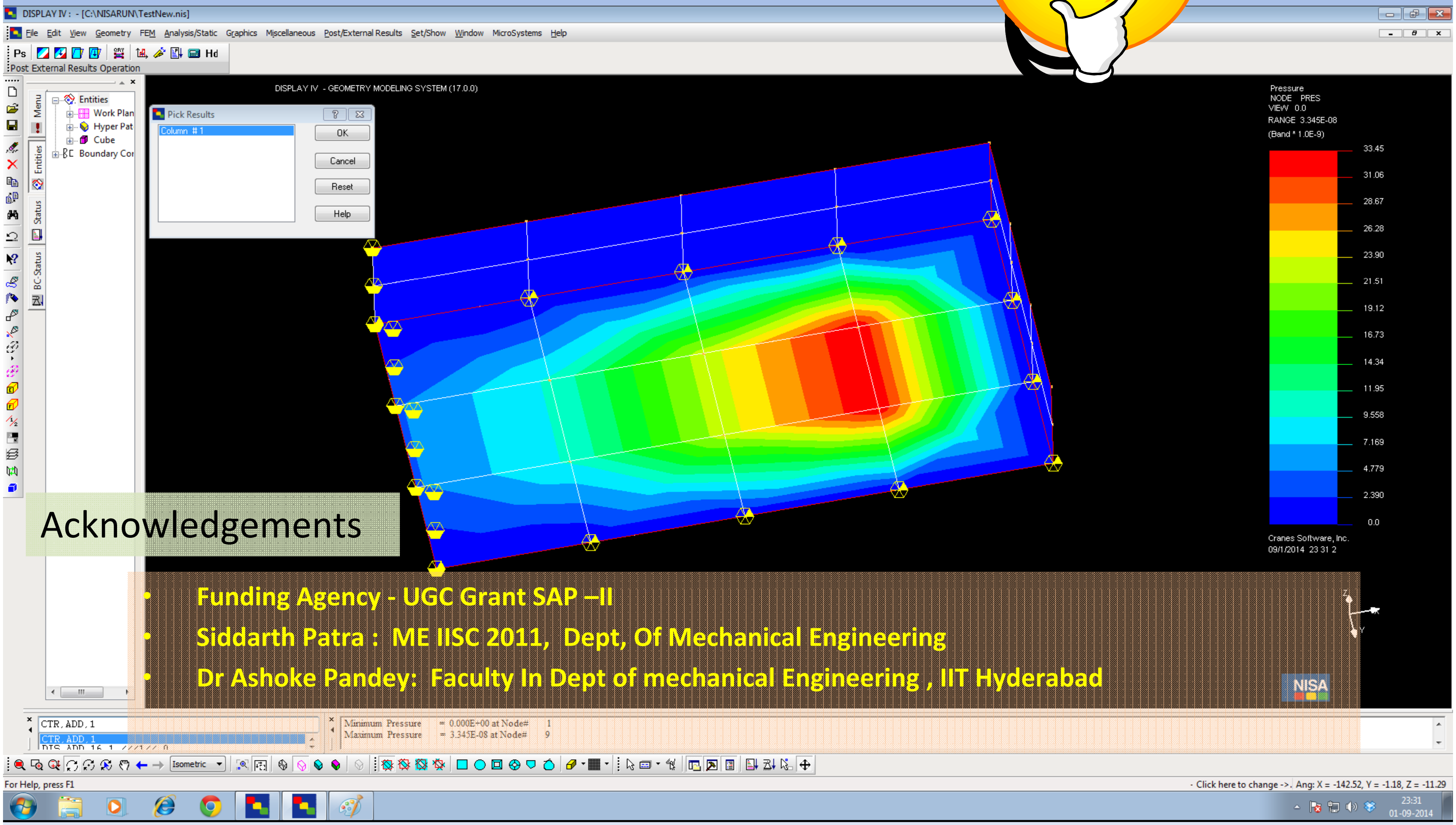
5. Venkatesh C, Bhat N, Vinoy K J, et al.(2012) Microelectromechanical torsional varactors with low parasitic capacitances and high dynamic range. J. Micro/Nanolith. MEMS MOEMS 11(1):013006.

6. Pandey A (2007) Analytical, Numerical, and Experimental Studies of Fluid Damping in MEMS Devices. PhD Thesis: Indian Institute of Science, Bangalore, India.

7. Shekhar S, Vinoy K J and Ananthasuresh G K (2011) Switching and release time analysis of electrostatically actuated capacitive RF MEMS switches. Sensors & Transducers Journal 130(7):77-90.

8. Venkatesh K P, Patil N, Pandey A K, et al. (2009) Design and characterization of in-plane MEMS yaw rate sensor. Sãdhãna 34(4):633-634.

Thank You!

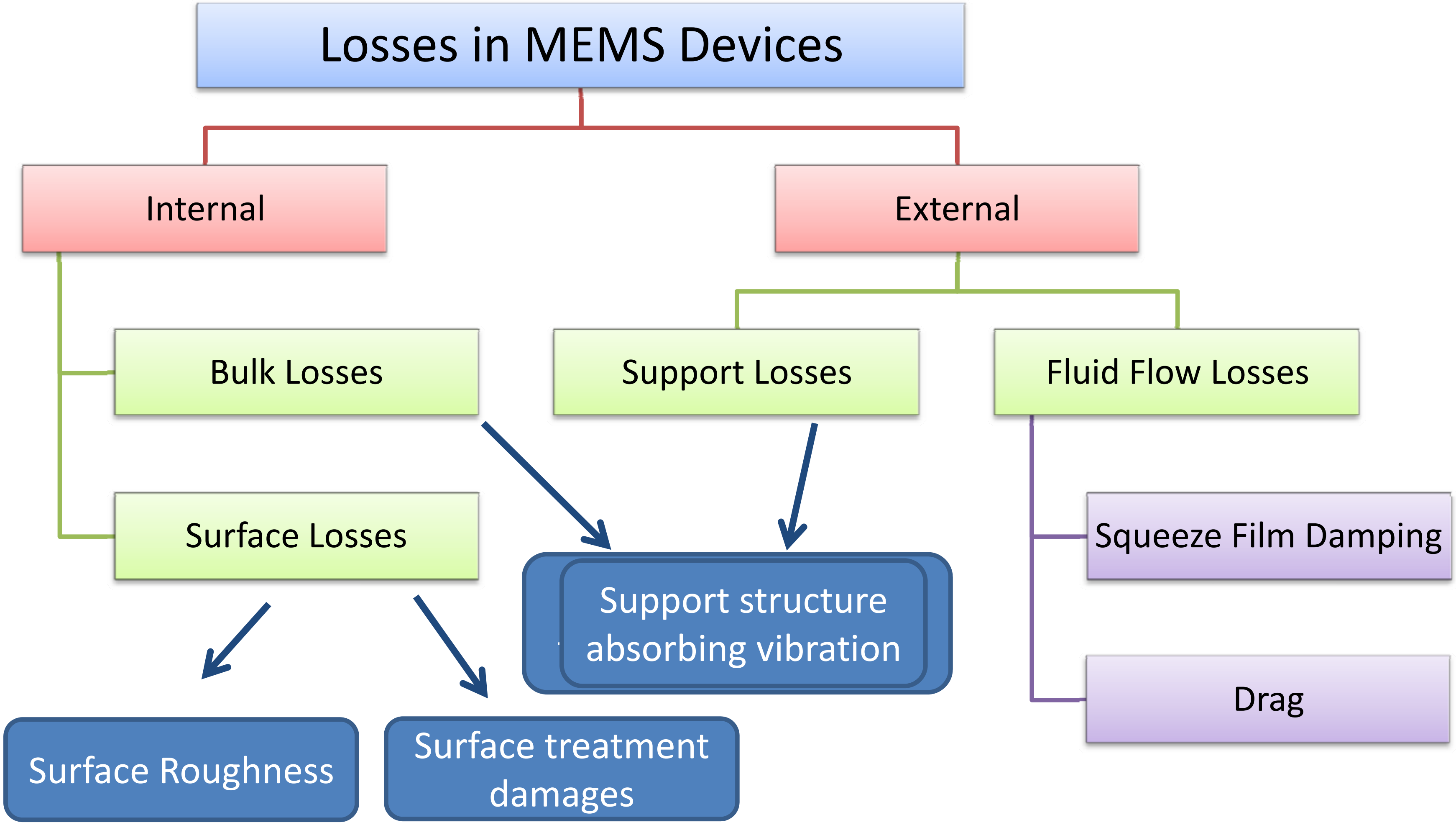


Acknowledgements

- Funding Agency - UGC Grant SAP –II
- Siddarth Patra : ME IISC 2011, Dept, Of Mechanical Engineering
- Dr Ashoke Pandey: Faculty In Dept of mechanical Engineering , IIT Hyderabad

Help Slides

Losses in MEMS devices



Total Quality factor for a system is given as :

$$\frac{1}{Q_{total}} = \sum_i \frac{1}{Q_i}$$

Definitions and Basic Terminology

Number Density of molecules (n)

$$n = \frac{P}{k_B T}$$

P = Pressure
K_b (Boltzmann Const) = 1.3805 x 10^{23}
T = Temperature

Under standard conditions

P = 1.013 x 10^5 Pa (1 atm)
T = 273.15 K
n = 2.69x 10^{25} x m^{-3}

Mean Free Path λ

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Average distance travelled by a molecule between two successive collisions

Under ambient conditions

T = 300 K
d~ 3.7 x 10^{-10} m
n = 2.41x 10^{20} x P m^{-3}
λ = 0.0068/P and for P = 1.013 x 10^5 Pa (1 atm) λ = 67 nm

Molecular diameter(d)

Under STP using Hard sphere model
d~ 3.7 x 10^{-10} m

Characteristic flow length (h)

Characteristic length of the flow channel in case of Squeeze film it is the gap thickness

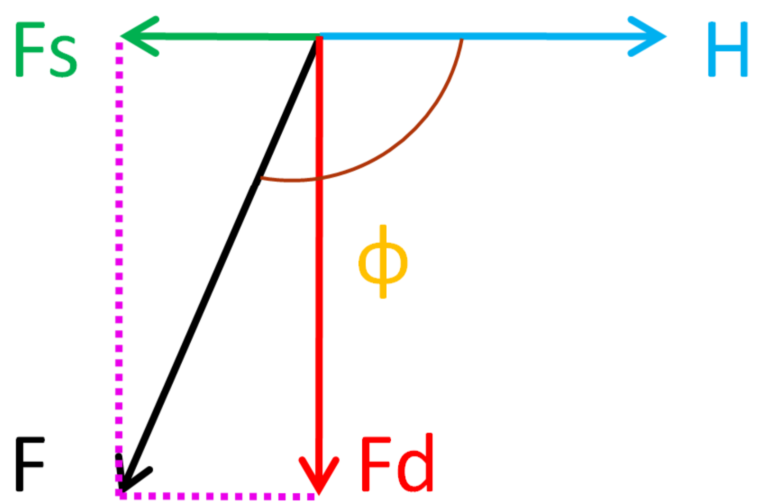
Effective Viscosity*

Author	Reference	Effective Viscosity (μ^*)	Derived from
Burgdorfer	1959 [17]	$\frac{\mu}{1+6K_n}$	Navier-Stokes equation
Hsia et al.	1983 [18]	$\frac{\mu}{1+6K_n+6K_n^2}$	Experimental data fitting
Fukui et al.	1988 [19]	$\frac{D}{6Q(D)}$; $D=\frac{\sqrt{\pi}}{2K_n}$	Boltzmann equation
Seidel et al.	1993 [6]	$\frac{0.7\mu}{K_n}$	Experimemtal data fitting
Mitsuya	1993 [20]	$\frac{\mu}{1+6\left(\frac{2-\alpha}{\alpha}\right)K_n+\frac{8}{3}K_n^2}$	Navier-Stokes equation
Veijola et al.	1995 [8]	$\frac{\mu}{1+9.638K_n^{1.159}}$	Approximation of Fukui's model
C.-L Chen	1996 [21]	$\frac{\mu}{1+6\alpha K_n}$	Navier-Stokes equation

* MS Thesis, J Young, Massachusetts Institute of Technology, 1998

Vector Diagram – Force and Displacement*

- $F=\int PdA$
- $F=\|F\|\exp(i\,(\omega t-\phi))$
- $Fs=\|F\|\cos(\phi)$
- $Fd=\|F\|\sin(\phi)$
- $Ksq=Fs/\delta h$
- $Csq=Fs/\delta h/\ \omega$



* Slide courtesy S Patra

Discussion – Analytical

We have

$$\nabla^2 P - \alpha^2 \frac{\partial P}{\partial t} = \alpha^2 \frac{\partial H}{\partial t}$$

$$P = \sum_{m,n} a_{mn} \psi_{mn}(x,y) T(t)$$

$$P = \tilde{P} e^{i\omega t}$$

$$\nabla^2_{X,Y} \tilde{P} - i\sigma \tilde{P} = \frac{\partial^2 \tilde{P}}{\partial X^2} + \frac{\partial^2 \tilde{P}}{\partial Y^2} - i\sigma \tilde{P} = i\sigma\delta\psi$$

For low **σ** $i\sigma \tilde{P}$

- Is negligible resulting in differential pressure to be small and out of phase 90° with displacement

For high **σ** $i\sigma \tilde{P}$

- Is dominant term and pressure profile approximately matches displacement profile and pressure profile is +- 180º out of phase with displacement