## Homework \#6 (and practice problems before the mid-term exam)

1. The figure below shows the suspension-spring portion of a micromachined accelerometer that uses piezoresistive sensing technique. The structure shown here can be considered as a cantilever of thickness $t_{1}$ over which a second (piezoresistive) layer of thickness $t_{2}$ is deposited towards the fixed end. The other relevant dimensions are shown in the figure. If there is tip-load of $100 \mu N$ due to the acceleration, determine the tip-deflection using $l_{1}=150 \mu \mathrm{~m}, l_{2}=50 \mu \mathrm{~m}, t_{1}=1 \mu \mathrm{~m}, t_{2}=0.6 \mu \mathrm{~m}$, uniform width of $w=15 \mu \mathrm{~m}$, and $E_{1}=165 G P a$ and $E_{2}=130 G P a$ being the Young's modulus of the materials of the two layers.

2. The figure below shows a bent-beam thermal actuator. If it is uniformly heated by a temperature $\Delta T$, what is the vertical deflection at the bend? Assume uniform crosssection area of $A$ and moment of inertia of $I$, Young's modulus of $E$, and thermal expansion coefficient of $\alpha$.

3. What is the vertical deflection in the above bent-beam structure if there is a compressive/tensile residual stress $\sigma_{0}$ induced by the microfabrication process? Assume in this case that the structure is not heated.
4. A compliant orthoplanar micromachined spring is shown in the next figure. Compute the vertical (perpendicular to the substrate) and the rotational (torsional) stiffness constants of the central circular platform while the outer circular edge is clamped as shown in the figure.


Compliant orthoplanar spring

5. In the magnetic actuator shown above, get the deflection $z$ as a function of the applied DC voltage $V$. If there is going to be a critical voltage, find an expression for it. Use the
same symbols that were used in the example solved in the class where the armature moves horizontally in the gap.
6. A research group has found that adding a capacitor in series with the voltage source and the spring-restrained parallel-plate capacitor can extend the stable region throughout the gap even in the voltage-control mode. Analyze this situation and find what condition on the value of this external capacitor can provide this benefit. Also, comment on what drawback results with this modification (there is no free lunch, right?).
7. The figure below shows the schematic of the micro-mirror used in the optical Lucent's optical cross-connect. It is a two-axis mirror that can reflect off the light rays incident on it to a desired location. There are four electrodes underneath the mirror separated by a gap, g. The details of the serpentine spring are shown below. The width of the beams that make up the serpentine structure is b. The four springs enable the mirror to rotate about the two axes. Any of the four actuators can be individually activated to apply electrostatic force on the mirror. The thickness of the structural layer (with which the mirror and springs are made) is t .
o First, obtain equivalent torsional (rotational) spring constant of the serpentine spring using either analytical calculation or FEMLAB.
o Using the equivalent spring constant, write down the lumped-model dynamic equations with electrostatic force computed using the parallel-plate approximation.
o Simulate the dynamics of the lumped model using Matlab. Take the case of applying voltage V on the two electrodes on the right side for five time periods corresponding to the lowest natural frequency. Then, turn them off for five time periods. Then, actuate the top two electrodes for five time periods by applying the same voltage V.

Use the following data:
d = 500 um; c = 20 um; a = 200 um;
$\mathrm{t}=3 \mathrm{um} ; \mathrm{g}=25 \mathrm{um}$;
$\mathrm{V}=200 \mathrm{~V}$ (if it is too high, that is beyond pull-in, try a smaller value);
p = $25 \mathrm{um} ; ~ \mathrm{q}=2 \mathrm{um} ; \mathrm{b}=1 \mathrm{um}$;
Young's modulus $=150 \mathrm{GPa} ;$ Poisson's ratio $=0.25 ;$ density $=2800 \mathrm{~kg} / \mathrm{m}^{3}$


Details of the serpentine spring


