Solution to homework #1

Question 1
You were all very impressed by Feynman's vision. Each of you identifies an aspect of his vision that has come true or is yet to come true. I am sure you will be certainly interested in what Professor Stephen D. Senturia had to say about this. Please look at his paper "Feynman Revisited" published in the 1994 IEEE MEMS Symposum in Oiso, Japan. It is posted as a pdf file.

Problem 2
The dimensions of the magnetic flux density B (unit Tesla) can be obtained in more than one way. Either of the following known formulae could be used.

i. $F = I\vec{B}\times\vec{l}$ is the force acting on a current carrying conductor aligned in the $\vec{l}$ direction in a magnetic field. So, $[MLT^{-2}] = [CT^{-1}][?][L] \Rightarrow [?] = [MT^{-1}C^{-1}]$.

ii. $B = \dfrac{\text{magnetic flux}}{\text{area}} = \dfrac{\phi}{A} = \dfrac{(\text{EMF})}{(\text{reluctance})A}$; $\text{reluctance} = \dfrac{L}{\mu A}; \text{EMF} = NI = (\text{number of turns})(\text{current})$. This second approach needs the dimensions of μ, the magnetic permeability, to be found first, which is done next.

The dimensions of μ_0 too can be found in several ways. One way is to use Ampere's law of force between two current carrying conductors, which is given by

$$\vec{F}_{12} = \frac{\mu_0}{4\pi}\oint_{C_1}\oint_{C_2}\frac{\left(I_2 d\vec{l}_2\right)\times\left(I_1 d\vec{l}_1\times\hat{r}\right)}{R^2}$$

where $\vec{F}_{12}$ is the force acting on conductor 2 by conductor 1, $\vec{l}_1$ and $\vec{l}_2$ are lengths along the conductors, R is the magnitude of the vector from conductor 1 to conductor 2, and $\hat{r}$ is the unit vector along $\vec{R}$. Thus, the dimensions of μ_0 can be obtained as follows.

$$\vec{F}_{12} = \frac{\mu_0}{4\pi}\oint_{C_1}\oint_{C_2}\frac{\left(I_2 d\vec{l}_2\right)\times\left(I_1 d\vec{l}_1\times\hat{r}\right)}{R^2}$$

$$[MLT^{-2}] = [?]\frac{\left([CT^{-1}L]\right)\left([CT^{-1}L]\right)}{[L^2]} \Rightarrow [?] = \dim[\mu_0]) = [MLC^{-2}]$$

Problem 3
The fundamental natural frequency of a thin circular clamped plate is given by

$$\omega_1 = 11.84\sqrt{\frac{Et^3}{\rho_s d^4(1-\nu^2)}}$$

where

ω_1 is the first natural frequency in Hz
E is the Young's modulus of the material
t is the thickness of the plate
ρ_s is the surface density (mass per unit surface area)
d is the diameter of the plate
ν is the Poisson's ratio, which is dimensionless.

Note that ρ_s is the surface density. It means that is is equal to ρt whre ρ is the volume density (kg/m^3).. Since it is a plate of uniform thickness, it is written this way for convenience. So, we get

$$\omega_1 = 11.84\sqrt{\frac{Et^2}{\rho d^4(1-\nu^2)}} \propto \frac{1}{L} \text{ (since material properties are fixed in scaling analysis.)}$$

It is useful to check the dimensions of the entire formula as is done below.

$$\omega_1 = 11.84\sqrt{\frac{Et^2}{\rho d^4(1-\nu^2)}}$$

$$[T^{-1}] = \sqrt{\frac{[ML^{-1}T^{-2}][L^2]}{[ML^{-3}][L^4]}}$$