Lecture Notes in Mechanical Engineering

Dibakar Sen
Santhakumar Mohan
Gondi Kondaiah Ananthasuresh *Editors*

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In Search of Fundamentally Robust and Intrinsically Complex Designs in Pursuit of High-Performance Mechanisms and Machine Design

With an eye toward improving performance and achieving simplicity in machine and mechanism design, one must delve into intrinsically complex processes. I will describe six topics from my past research, which may be cohesively connected with the common theme of finding simplicity amid complexity. I would like to begin with the following quote:

> Conventional wisdom can be like concrete; it is good to walk on, and it gets you places fast. But nothing beautiful, rare or worthwhile will grow in it. However, if it is formed to cup a seedling in a nurturing environment, growth is but certain and its fruits are sure to follow.

This was something I had written when I was in my late forties; I continue to believe in it till this day. Over the years, I am pleased to see many younger colleagues subscribe to it and grow in their careers. At this point, I wish to recall a friend, a mentor, and an accomplished engineer, Mr. Dimitry Grabbe (1928–2011). He was the Director of Applied Technology at AMP Inc., having to his credit more than 500 patents in areas such as machine design, semiconductor packaging, optoelectronic connector design, and printed-circuit-board technology. Among his awards were ASME Leonardo da Vinci and IEEE Components, Packaging, and Manufacturing Technology Awards. He had been an integral part of the US space exploration and had founded the Maine Research Corporation. My association with him greatly influenced my thinking, and thus shaping my early career.

Let me begin with my first topic, the discovery and development of the primal equations in designing four-bar crank-rocker linkages. Much work exists on this topic. Extensively reported in the master's thesis of my student, Mark J. Erickson, these equations offer profound simplicity and elegance, providing a lot of insights, while revealing families of solutions for a whole host of mobility synthesis problems:

$$sin\left(\frac{\Delta\theta_4}{2}\right)sin(\mu_m) = \frac{R_2}{R_4} \quad \& \quad sin(\theta_{4m})sin\left(\frac{\Delta\mu}{2}\right) = \frac{R_2}{R_4}$$
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$$sin\left(\frac{\Delta\theta_4}{2}\right)sin(\mu_{3m}) = \frac{R_2}{R_1} \quad \& \quad sin(\theta_{4m})sin\left(\frac{\Delta\theta_3}{2}\right) = \frac{R_2}{R_1}$$

These equations involve 10 variables: four link lengths, $R_1$to R_4, the mean and range values of three angles, i.e., the coupler angle, θ_3, the rocker angle, θ_4, and the transmission angle, μ. One of the equations was found using symbolic mathematics, and then the rest were subsequently derived. With four free variables to choose, there exist 210 combinations, with a few eliminated as being redundant or not satisfying Grashof's criterion. The equations made it easy to rule out impossible user specifications and also to obtain possible solutions. They also led to a graphical method to locate the ground pivots. There are other interpretations and uses of these equations, e.g., obtaining a simple harmonic motion of the output member when the input crank is considered to be relatively much smaller.

Let me move on to my second topic: the mechanical advantage of Single-Input Port and Multiple-Output Port (SIPMOP) mechanisms. For example, a hand-held device, called the *elastrator* [1], is used to stretch a rather stiff elastic band into a nearly square-shaped opening. This means that the force applied by the hand stretches the band in two orthogonal directions simultaneously. Therefore, the device requires one input force and exerts two output forces. This system is analogous to two springs in series acted upon by a single force. We found that its net Mechanical Advantage (MA) is governed by an equation [1], where MA1 and MA2 are component mechanical advantages evaluated in orthogonal directions 1 and 2, respectively, assuming only a single output force in action with a single input force:

$$\frac{1}{MA} = \frac{1}{MA_1} + \frac{1}{MA_2} \Rightarrow MA = \frac{MA_1 MA_2}{MA_1 + MA_2}$$

Two fundamental observations emerge: (i) MA will be lower than even the lower of MA_1 and MA_2, and (ii) it will converge to the lower value only if the higher value tends to infinity. Having recognized this, we were able to effectively design the elastrator (and other similar mechanisms) for a high mechanical advantage. Furthermore, the use of a SIPMOP allows us to synthesize a mechanism with a nonlinear output function. For example, a single-degree-of-freedom SIPMOP mechanism, symmetric about two planes, with two linear springs in orthogonal directions, is studied for a mechanical advantage as well as energy absorption. The energy vs. displacement profiles can be nonlinear concave or convex, or even linear. The linear energy case is very interesting in that it produces a nonlinear, constant-force mechanism wherein the force remains a constant over a finite displacement. Such a constant-force mechanism was used to design an accelerated pavement testing machine, to support the research of a civil engineering faculty member, Dr. Thomas White, at Purdue University.

Compliant mechanisms, which derive some or all of their functionality from the deformation of their flexible segments, are often multiple-output mechanisms. So, the SIPMOP concepts we discussed apply to them as well. The third topic we will consider is a compliant mechanism, again from the viewpoint of mechanical advantage. A fully compliant crimping mechanism was developed at AMP Inc. It was desired that its mechanical advantage increases monotonically with displacement. But even with a dozen variant prototypes tested, the mechanical advantage showed a declining trend with displacement. The subject has been thoroughly analyzed and the following conclusions are drawn: two significant factors affecting mechanical advantage are (i) the structural configuration of the Pseudo-Rigid-Body Model (PRBM) [2] and (ii) the energy stored in compliant elements of the mechanism. Several case studies examined suggest that minimizing the latter contribution relative to that of an optimized structural configuration will improve the mechanical advantage of a compliant mechanism. Therefore, its effect cannot be neglected. This crimping mechanism became an obsession and, in part, paved the way for research on compliant mechanisms design.

Let me dwell a little longer on constant-force mechanisms, as the fourth topic I want to discuss. A notable development with John Jenuwine and Jeff Albers, master's students at Purdue University, was the design of a constant-force accelerated pavement testing machine. Morgan Murphy and Larry Howell, Ph.D. students at Purdue, and I considered a variety of compliant mechanism configurations for generating constant force [3]. We utilized various segment types, from rigid to compliant, and joint types from revolute and prismatic to flexures to obtain a wide range of them. Some used cam-like surfaces in a vein to reduce the number of segments involved. The concept of generating constant force has led to a number of interesting applications. Today, we can accomplish it much more easily using simple beam segments. What looked complex at the beginning became simple as we continued to explore various aspects of the problem.

The fifth topic I wish to touch upon is the design of compliant mechanisms for generating arbitrary force–displacement profiles. Since the early days of the AMP Inc. crimping mechanism, we have come a long way in our understanding about compliant mechanisms. An important development has been the ability to represent compliant mechanisms with Pseudo-Rigid-Body Models (PRBM). The classical work of Bishopp and Drucker [4] considers the analysis of large elastic deformation of a cantilever beam with an end load, using elliptic integral solutions. Using this as a benchmark, a striking observation here has been that the locus of the tip can be approximated using a PRBM, with a pseudo-rigid-body angle in the order of 75° corresponding to a beam end location relative error less than 0.5%. This is true even for very large variations in the ratio of the transverse to the axial beam end loads. It enabled us to define terms such as characteristic pivot, radius, and radius factor, and allowing very interesting, hitherto unknown revelations regarding large-deflection beam behavior and properties. For instance, a nearly linear relation was found between the beam end force and pseudo-rigid-body angle. The PRBM has been found to be effective and accurate for analysis and design of compliant mechanisms, from simple hand tools and exercisers to more complex applications in MEMS, biomedical devices, and more. Among those who have been actively engaged in compliant mechanisms research, the works of Larry Howell and G. K. Ananthasuresh have been momentous.

I will finish with a brief discussion of the sixth topic: minimization of indeterminacy in array-like supporting structures using compliant mechanisms. Imagine long beams on multiple supports. A mattress with multiple springs is a good practical example. Many similar applications are statically indeterminate structures, and pose problems with their analysis and design. There is a nice way to alleviate this indeterminacy, if we were to replace a large number of these supports as constant-force supports made possible by compliant mechanisms. Imagine several supports on a single girder, all but two are constant-force compliant supports, designed for known force. Then, the unknown reaction forces at the two "rigid" supports may be quickly evaluated from the loading. In such a setting, I believe the accuracy of the positioning of the intermediate constant-force supports, and their engagement with the girder may not quite be as critical, as long as a preload is maintained, allowing much freedom in arranging the supports with resulting cost benefits. I leave you with these thoughts to imagine many other interesting possibilities.

As you saw, the six intrinsically complex topics we dealt with aimed at achieving simplicity. A major theme that emerged from these is that of compliant mechanisms. Let me define a compliant mechanism, while contrasting it with elastic or flexible mechanisms.

> ...elastic or flexible mechanisms are necessarily those that undergo elastic deformation. Their response may or may not be vibratory. In this context, a high-speed mechanism may be regarded as elastic or flexible. The Webster's definition of the word 'compliant', on the other hand is 'readily disposed to comply. SUBMISSIVE'...

V. K. Stokes of General Electric made a Visiting Fellow presentation, entitled "Thermoplastics in Load-bearing Applications: New Opportunities for Engineering Science" in the School of Chemical Engineering at Purdue University in 1987. He said, "...time has arrived for more challenging uses for thermoplastic materials in load-bearing applications ..." He cited weight reduction, ease of fabrication of complex shapes, and cost reduction resulting from functional integration as advantages. He added, "Lack of development of the design methodology was currently a barrier in their application." He foresaw such developments as new opportunities for engineering science. Today, "compliant mechanism design" has met that challenge, with momentous, unending applications in the aerospace industry, the automotive industry, the healthcare industry, and many more. Clearly, there is no limit to the application areas, when they offer numerous and lucrative benefits, and the complex problems are rendered simpler to solve, and easier to understand, apply, and design.

Departing from the "conventional wisdom" and leaning heavily toward inclusion of compliance in design, provided the "seedling in a nurturing environment," and the "fruits" are now beginning to be evident.

Ashok Midha
Mechanical and Aerospace Engineering, Missouri
University of Science and Technology, Rolla, Missouri, USA
midha@mst.edu

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