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CASE STUDIES AND A NOTE ON THE DEGREES-OF-FREEDOM IN COMPLIANT MECHANISMS

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ABSTRACT

The presence of compliant members in a mechanism makes the determination of its degrees-of-freedom out of reach of the traditional Grübler's formula, as this formula does not account for the additional mobility allowed by compliance. Recent research efforts have led to a generalization of Grübler's formula that encompasses compliant mechanisms as well. The objective of this work is to apply the formula to a set of practical compliant devices, and further clarify and simplify the approach. This is accomplished as follows: i) The concept of "virtual rigid segments" is introduced to facilitate the identification of the segment compliance of distributed segments with forces applied to them. ii) The motion of compliant devices is interpreted by identifying the inputs equal in number to the calculated possible degrees of freedom. iii) A wide variety of case studies are presented to illustrate the method of application and the value of the approach.

INTRODUCTION

It is well known that the constrained motion of all the members of a mechanism, and thus the behavior of the entire mechanism, is completely determined by a minimal set of independent driving inputs called the *degrees-of-freedom* (dof). To adequately predict and control the motion, the number of actuators in a mechanism should be equal to its number of dof. The concept of dof is useful not only in evaluating the capability and controllability of an existing mechanism, but also in conceiving a new design for a mechanism. Traditional mechanisms are comprised of rigid links and joints that permit only rigid body motions, and their dof can be determined by geometry based arguments. Grübler's formula is often used to determine the dof of such mechanisms. There is another class of mechanisms, called *compliant mechanisms*, which consist of intrinsically flexible members. When applied to these mechanisms, Grübler's formula yields a dof that is less than or equal to zero, thus characterizing them as "structures". However, these are mechanisms, because they transfer energy from an input to an output. The determination of dof for compliant mechanisms falls outside the realm of Grübler's formula, as this formula does not account for the additional mobility allowed by compliance.

Midha and his co-researchers (Her and Midha, 1987; Midha et al., 1994; Murphy et al., 1994; and Howell and Midha, 1995) developed several new concepts, notations, and analysis techniques to account for additional dof engendered of compliance in the mechanism. A fairly comprehensive discussion of the mobility in compliant mechanisms was first presented by Her and Midha (1987). They defined the dof as the sum of two components each of which accounts separately for the mobility due to the rigid body dof and compliant dof. Midha et al. (1994) developed a consistent notation, nomenclature, and classification for the components of compliant mechanisms. Murphy et al. (1994a, b, and c) developed a mathematical model for the representation of compliant mechanisms to perform mobility analysis and type synthesis. Combining all these developments, Murphy et al. (1994c) presented a "dof formula" which is a generalization of the Grübler's formula for compliant mechanisms. The planar version of this formula is given below.

$$dof = 3(n_{seg} - 1) - \sum_{j=1}^{2} (3-j)n_{Kj} - 3n_{fix} - \sum_{j=1}^{2} (3-j)n_{Cj} + \sum_{j=1}^{q} (j)n_{scj} \quad (1)$$

Note 1: Replace 3 with 6, and 2 with 5 to obtain the dof formula for the 3-D case.

where

n_{seg} = total number of segments

Note 2: Segments in a mechanism can be identified based upon the connections (kinematic joints or compliant connections), the flexibility (rigid or compliant), forces acting on compliant members, etc.

n_{fix} = the number of fixed connections (as in a clamped connection between a rigid segment and a compliant segment)

n_{Kj} = the number of kinematic joints with j dof

n_{Cj} = the number of compliant connections with j dof (e.g., one-axis flexural hinge is of n_{C1} type)

n_{scj} = the number of segments with a *segment compliance* of j

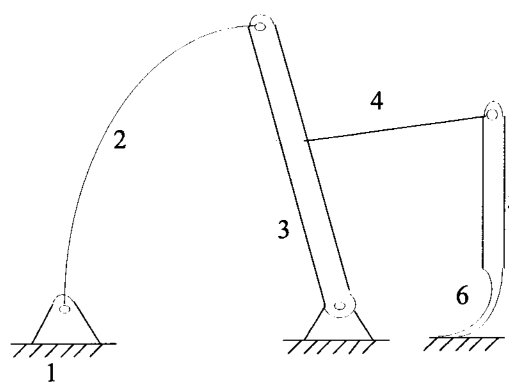
q = highest value of the segment compliance present in the mechanism

Note 3: The segment compliance (*sc*) is a measure of flexibility of a segment (e.g., for a rigid segment, $sc = 0$; for a segment that can only stretch/contract, $sc = 1$)

In this paper, we consider several case studies to apply this formula. Our case studies show that the formula correctly predicts the dof for mechanisms containing compliant connections, and compliant segments for which segment compliance (*sc*) can be clearly identified. In general, for a planar segment connected with k other segments (*k-nary*) *sc* can have any integer value from 0 to $3(k-1)$ (Murphy et al., 1994a). Unfortunately, it is not always straightforward to determine the *sc* of a segment which may depend on a segment's structure, function, and boundary conditions. In view of this, in addition to illustrating the application of Equation (1) to many practical compliant devices, a method is presented here to make the designation of segment compliant easier. The method also pays attention to the interpretation of multiple dof in terms of the inputs to the mechanism, and to the intuitive understanding of the kinematic motion of the compliant devices.

SEGMENTS AND SEGMENT COMPLIANCE

Compliant mechanisms consist of rigid and compliant segments joined together by kinematic joints as well as compliant connections (Midha et al., 1994). The identification of rigid segments in a compliant mechanism is easy, whereas defining the adequate number of compliant segments is not as straightforward. The compliant segments are distinguished a) structurally by distinct motion characteristics or discontinuities of material or cross-sectional properties, and b) functionally by points of application of force or displacement boundary conditions (Howell and Midha, 1995). Definition of more segments may also be required when one segment connects to another at more than one location (Murphy et al., 1995b). After identifying all the segments in the mechanism, the next task in the dof analysis is to select the segment compliance (*sc*) for all the segments. The segment compliance is a measure of flexibility. Naturally, it is zero for rigid segments. For a spring with one flexural mode, *sc* is equal to one. For other types of compliant segments, the segment compliance depends on the segment's structure, function, and boundary conditions. As stated earlier, the maximum *sc* for a k-nary segment is $3(k-1)$. Consider, for example, the mechanism shown in Figure 1. For this mechanism, using the method of "pseudo-rigid-body models", Howell and Midha (1995) treated segments 2, 4, and 6 as mono-compliant, i.e., $sc = 1$, and obtained a value of 1 for the dof of the mechanism. One could interpret this dof as the rotation input at the kinematic joint connecting segments 3 and 1. This mechanism can also have more than 1 dof if different *sc*'s are assumed to account for other inputs on the mechanism.



Segment	Type	*sc*
1	Ground	0
3, 5	Rigid	0
2, 4, 6	Compliant	1

Figure 1 An example of a compliant mechanism (Howell and Midha, 1995)

The problem of identifying *sc* becomes more involved when there are many distributed compliant members that are joined with each other or with rigid segments of the mechanism. Monolithically constructed fully compliant mechanisms also pose difficulties in the identification of the segments and their *sc*'s for different loading conditions. Figure 2 shows three instances of the distributed type, fully compliant mechanisms (Ananthasuresh et al., 1994). The determination of *sc* for these mechanisms requires the knowledge of deformation modes of the flexible continuum under the applied loads. In general, it becomes difficult to identify the segment compliance if there are segments with distributed compliance, and there are forces acting on the compliant segments.

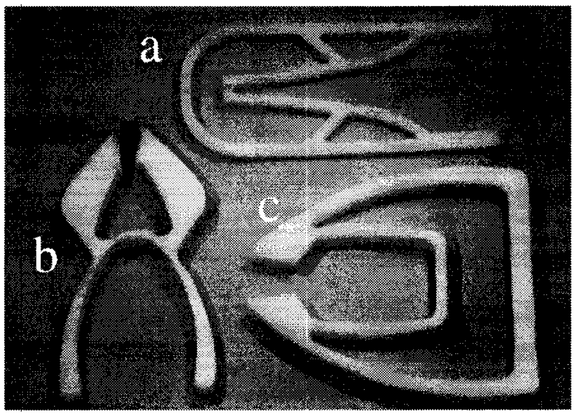


Figure 2 Compliant mechanisms with distributed compliance: a. crimper, b. clamp (holding an object), and c. gripper

Some methods have been proposed to address the issues of segments and segment compliance. Her and Midha (1987) used "pseudo joints" to incorporate the forces on the compliant members. Murphy et al. (1994b) suggested that a compliant segment with a force applied somewhere other than the ends, be divided into two segments joined by a fixed connection. Recently, Howell and Midha (1995) presented a useful analysis method for many types of compliant mechanisms with known input and output locations. They used pseudo-rigid-body models to determine the *sc* for a number of common types of compliant segments. During the course of our case studies, we encountered some examples which could not be addressed by the direct application of these methods. In some of these instances, even though the dof could be calculated, the interpretation of the dof in terms of the inputs to the mechanism was not very straightforward. Hence, this paper introduces "virtual-rigid-segment" concept to add further insight to the problem of distributed compliant segments with forces. This new concept enables the designer to assume the maximum *sc* in ambiguous situations. If this results in multiple dof, there is a provision in the method to interpret all the dof and also obtain an intuitive appreciation for the kinematic motion of the mechanism. The method and its application are explained in a later section.

INTERPRETATION OF MULTIPLE DEGREES-OF-FREEDOM

The degrees of freedom pertain to the number of independent inputs that can be applied on the mechanism. For the sake of discussion, without the loss of generality, it is more convenient to consider displacement inputs rather than force inputs. One advantage of this is the ability to perform the kinematic analysis, i.e., investigating the motion capabilities of the mechanism without considering the forces that caused the motion. The main implication of the concept of dof is that if inputs equal in number to the dof are applied on the mechanism, then the motion of every point in the mechanism should be fully determined. Conversely, fewer inputs fail to completely determine the motion of the entire mechanism. This is true with rigid link mechanisms in general. On the contrary, for many compliant mechanisms it is possible to obtain fully determined (analytically predictable) motion for fewer inputs than the maximum number of independent inputs which the mechanism is capable of accepting. This reveals the subtle difference between the dof for traditional "rigid link-rigid joint" mechanisms and compliant mechanisms. The difference is that a rigid link mechanism does not give rise to geometrically deterministic (or analytically predictable) motion if subjected to fewer inputs than the maximum made possible by dof. For example, in the five-bar rigid link mechanism ($dof = 2$) shown in Figure 3, specifying θ or ϕ alone is not sufficient to determine the positions of all the links; both the angles need to be specified to completely determine the motion of every link.

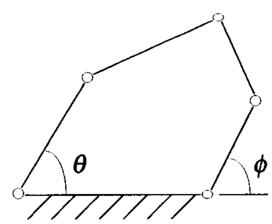


Figure 3 Five-bar rigid link mechanism

Unlike rigid link mechanisms, in general, a compliant mechanism can give analytically predictable motion even when fewer inputs are applied. Hence, the dof as given by Equation (1) needs an interpretation that is different from that of the rigid link mechanisms. Murphy et al. (1994b) interpreted the dof given by Equation (1) as the maximum number of inputs that can be specified for the compliant mechanism. To determine the minimum number of inputs, it is necessary to apply Grübler's formula to the mechanism and determine the rigid body dof only without accounting for compliance. Then, the minimum number of inputs that can give analytically predictable motion is equal to the rigid body dof, or one if rigid body dof is less than one (Murphy et al., 1994b).

In general, compliant mechanisms have multiple dof. This may not be obvious unless we compute the dof as outlined above. Notwithstanding the fact that compliant mechanisms can give deterministic motion even when fewer inputs are applied, it could lead to erroneous performance if the mechanism is not restrained with all the possible input actuations. One consequence is that the motion of the mechanism might be different from the intended motion due to forces that are not under control. Therefore, it is useful to gain an intuitive understanding of the motion capability of the mechanism and also identify all the inputs that can be applied on the mechanism to fully constrain its motion. Murphy et al. (1994b) studied this aspect as part of the topological analysis of compliant mechanisms. They interpreted the inputs as forces. Since our purpose here is to verify and interpret the dof kinematically, we view inputs as displacements specified on various segments. Although, forces play the key role in providing mobility to compliant mechanisms, kinematic interpretation is still possible. This is explained through an example below and it is the underlying theme in all the case studies considered in this paper.

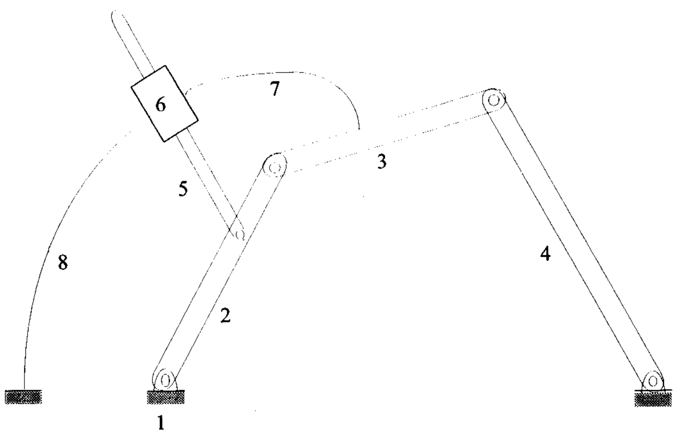


Figure 4 An example for the kinematic interpretation of motion in compliant mechanisms

Consider the mechanism shown in Figure 4. Using Equation (1), and an *sc* of 3 for both the compliant segments, the dof is found to be three. Even without the aid of Grübler's formula, it is clear that the rigid body dof of this mechanism is non-positive. Thus, the number of inputs to this mechanism is between one and three. A rotation input to the rigid segment 4, for instance, can give analytically predictable motion for the entire mechanism. After providing this input, if two additional inputs for the relative rotation between segments 2 and 5, and the translation of the sliding segment 6 are also specified, the resulting motion is not only analytically predictable, but is also complete. It is complete in the sense that no further displacement inputs are possible without overriding the previously specified displacement inputs. It is important to note here that the motion of every point including the compliant segments is determined here. Thus, for completely restrained motion this mechanism should be given three inputs.

One limitation of the above interpretation is that compliant segments can still take additional inputs, and thus there is really no upper limit on the dof. Thus, the dof for compliant mechanisms remains undetermined if the inputs acting on the compliant members are not known. If we know the forces acting on the compliant members *a priori*, then it is possible to determine dof correctly. As a verification for the computed dof, it is beneficial to interpret the kinematic motion as it was done above for the mechanism shown in Figure 4. The method presented in the next section deals with the issues of computing the dof and its verification through kinematic interpretation of the motion.

DETERMINATION AND INTERPRETATION OF THE DOF USING VIRTUAL RIGID SEGMENTS

The motivation for proposing a new concept for representing compliant mechanisms is twofold as described in the last two sections. First, the determination of *sc* should be made more straightforward, and second, the representation should facilitate easy interpretation of the kinematic motion under the applied inputs to verify the computed dof. To achieve these objectives, we use *virtual rigid segments* (VRS's) in the schematic representation of the mechanism. The VRS's are used in the following cases:

1. The locations where there are forces acting on the compliant segments.
2. The locations where a compliant segment is connected with another compliant segment or a rigid segment. The connection can be either a kinematic joint or a compliant connection.

The type of connection between a VRS and a compliant segment is a clamped connection. In providing a means to incorporate the forces applied on the compliant segments, VRS's serve the same purpose as "pseudo joints" proposed by Her and Midha (1987). The concept of VRS is also consistent with a more recent work of Murphy et al. (1994b) which recommended a split of the segment into two and a clamped connection between them at the point of application of the force. The VRS makes this more explicit by representing a rigid segment in the schematic diagram. Clearly, this rigid segment is non-existent, and is therefore called a "virtual rigid segment".

In addition to making way for the inclusion of forces applied on compliant segments, VRS's are also useful in interpreting the dof. Using VRS's as described in the second condition above is aimed to serve this latter purpose. It should be recalled that permitting inputs on compliant segments leads to undetermined dof. Therefore, it is preferable to have the inputs applied to rigid segments only. Since the verification of dof is done through geometrical visualization of the kinematic motion of the mechanism, the inputs need to be displacements rather than forces. Several examples in our case studies required, contrary to our premise, one or more inputs to be applied on compliant segments to fulfill the maximum number of inputs as per the dof. The VRS's were introduced to circumvent this problem.

The VRS's are found to serve another useful purpose for the following reason. In order to lessen the ambiguity with regard to choosing *sc*, we recommend that maximum possible *sc*, i.e., $3(k-1)$ for a k-nary segment, be assumed for those segments for which *sc* cannot be clearly identified. This leads to a large number of degrees of freedom and that makes it necessary to interpret all of those kinematically. The introduction of VRS's facilitate that interpretation as can be seen in all the case studies considered later.

For the purpose of illustrating the concept of the VRS, consider an example from Figure 2a. This is a one-piece crimping mechanism which is a distributed type, fully compliant mechanism. The simplified schematic representation of its symmetric half is shown in Figure 5. The VRS's are used here to incorporate the force and the sliding boundary condition. Using the dof formula given in Equation (1),

$$n_{seg} = 5;\; n_{fix} = 4;\; n_{KI} = 1;$$
$$dof = 3(5-1) - 3(4) - 2(1) + n_{sc1} + 2\,n_{sc2} + 3\,n_{sc3}$$
$$= -2 + n_{sc1} + 2\,n_{sc2} + 3\,n_{sc3}$$

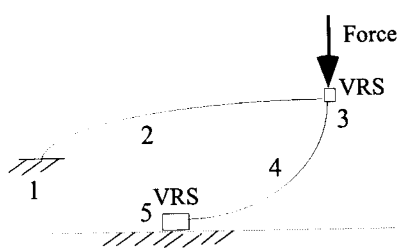
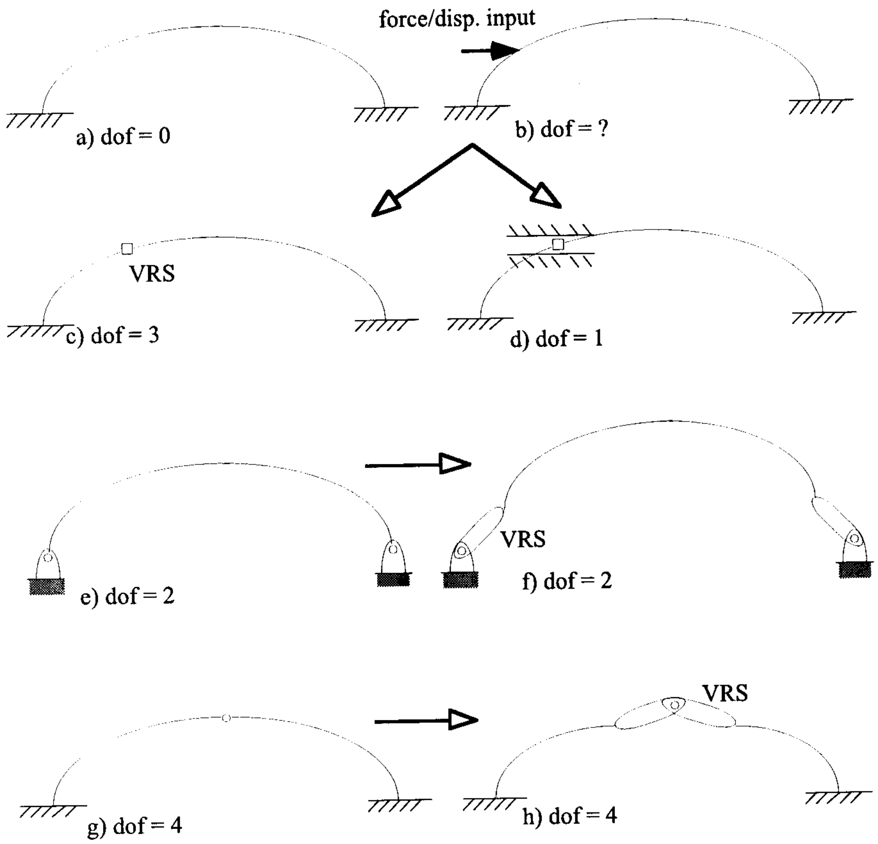


Figure 5 Simplified schematic of the symmetric half of the compliant crimping mechanism

From the operation of this mechanism under the indicated force, we expect it to have one dof. To obtain this desired value of dof the cumulative sum of *sc*'s should be equal to 3. One way to make this happen is by assuming *sc* of 1 for segment 2 (which seems to be bending only at the connection with the fixed segment), and *sc* of 2 for segment 4. Although it may be possible to guess *sc*'s in this manner based on the function of the device, it is not useful to do it this way when our objective is to determine dof for a given mechanism. Our objective here is to make the determination of *sc* unambiguous, and the application of dof formula straightforward.

To ease such a situation, we assume maximum possible *sc* for a given segment whenever the determination of *sc* is not fully clear. The maximum value of *sc* for a planar k-nary segment can have is $3(k-1)$. Therefore, binary segments 2 and 4 have an *sc* of 3 each. Consequently, the dof for the mechanism becomes 4. Although it may seem inappropriate to find four dof when it is clear that the device should have one dof, the virtual rigid segments (VRS's) facilitate a meaningful interpretation for the extra dof. In this case, the VRS in segment 3 can take three inputs viz. x and y translational inputs and a rotation about the z-axis. The VRS in segment 5 can take one x translational input. One could also interpret the inputs in terms of forces and moments/torques on rigid segments. If these four inputs are given, the motion of all the members in the mechanism is completely determined, i.e., not only the rigid segments, but also every point on the compliant segments.

As a second illustrative example, consider a flexible curved beam with different boundary conditions and forces shown in Figure 6. A doubly clamped beam (case a) has zero dof when there are no forces acting on it. If there is an input as shown in case b, we can introduce a VRS and arrive at three dof (case c). The result is not surprising, because now the VRS can take three inputs that will completely determine the deformation of the entire flexible beam. On the other hand, if the input is a displacement in the horizontal direction only, we can introduce a kinematic prismatic joint to incorporate this (case d). Then, we get one dof which is correct. Cases e and g, and their interpretations with VRS's (cases f and h) illustrate what types of motion inputs can be given to the mechanism. In case f, clearly the two inputs are one rotation each to the two VRS's. In case h, to locate and orient the two 'floating' VRS's connected with a revolute joint, four displacements are necessary which is in agreement with the computed dof. Thus, multiple dof can easily be interpreted with the help of VRS's.



case	*nseg*	*nfix*	*nK1*	*nsc3*	*dof*
a	2	2	0	1	0
b	2	2	0	1	?
c	4	4	0	2	3
d	4	4	1	2	1
e	2	0	2	1	2
f	4	2	2	1	2
g	3	2	1	2	4
h	5	4	1	2	4

Figure 6 A simple example to illustrate the dof computation using VRS's

The following are the steps involved in determining and interpreting the dof of a compliant mechanism.

1. Identify all rigid and compliant segments; kinematic joints; fixed and compliant connections.

2. If there are forces acting on a compliant segment, split that into two and introduce a VRS with clamped connections between them.

3. Introduce VRS's in places where kinematic joints involve compliant segments.

4. If two compliant segments connect with each other with a clamped connection (e.g., compliant segments with multiple branches) introduce a VRS appropriately. This may not always be necessary as k-nary segments are permitted by Equation (1).

5. Upon introducing the necessary number of VRS's, ensure proper connections (kinematic or compliant) among all the segments including the VRS's.

6. If the segment compliance of a compliant member is not obvious, assume maximum *sc* *(3(k-1)* for a k-nary segment).

7. Using the dof formula given in Equation (1), calculate the dof.

8. Identify a set of displacement inputs on rigid segments, equal in number to the dof, and visualize the motion kinematically so that the motion of every point in the mechanism is completely determined, and no further displacement is possible without overriding the displacement inputs that are already assigned. This serves as a verification of the sc's assumed in computing the dof using the formula.

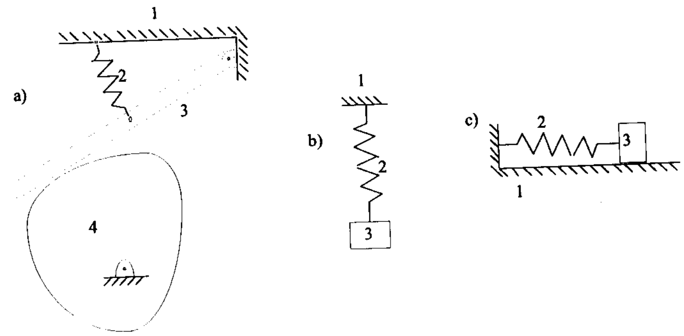
The degrees-of-freedom analysis of a number of practical compliant devices is considered in the next section. All of these elucidate the ideas described in the earlier sections. A few very simple examples are presented first to illustrate the method and then some practical devices are considered. Even though the case studies are presented after explaining the concepts, it should be emphasized that some of these examples led to the very concepts described earlier in this paper.

CASE STUDIES

1. Spring loaded cam-follower and mass-spring systems

Using the dof formula given in Equation (1), it is possible to include force-closure springs also in the dof analysis. In the simple example of a spring loaded cam-follower (Figure 7a), the force-closure spring is also counted as a segment. This compliant segment has an *sc* of 1 as it can only stretch or contract. It can be seen in the table below that the dof is calculated correctly.

The dof analysis of the suspended mass-spring system (Figure 7b) is trivial, but the spring loaded translating block illustrates an important point. It can be seen in Figure 7c that this system has only one dof, but if we assume an *sc* of 1 for the compliant spring segment, a misleading result of -1 dof is obtained. This is because the sliding joint between segments 1 and 3 is redundant if an *sc* of 1 is assumed for the spring segment to permit only translation. The ambiguity can be resolved in two ways as shown in the last two entries in the table. The point to note here is that if *sc*'s less than the maximum possible are assumed, it is important to propagate its effect throughout the mechanism to avoid misleading results.

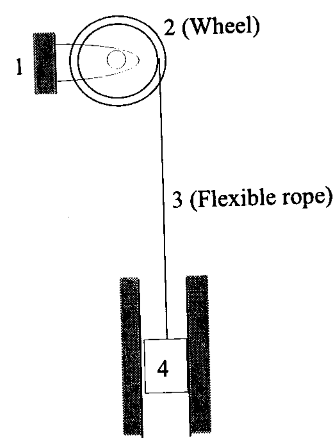


Case	*nseg*	*nfix*	*nK1*	*nK2*	*nsc1*	*nsc3*	*dof*
a	4	0	4	1	1	0	1
b	3	2	0	0	1	0	1
c	3	2	1	0	1	0	-1
c	3	2	1	0	0	1	1
c	3	2	0	0	1	0	1

Figure 7 Degree of freedom analysis of spring loaded mechanisms

2. Winch mechanism

The traditional winch mechanism is a compliant mechanism as its functionality depends on the flexible rope. The rope in this mechanism can be treated as a compliant segment with an *sc* of 3, which leads to two degrees of freedom. The rotation of the wheel is obviously one input that can be given to the mechanism. The second input is due to the extensibility of the rope even when a brake is applied on the wheel. Thus, with two displacement inputs, the motion of every point in this mechanism is fully determined and no other independent displacement input is possible. In the absence of the prismatic joint constraining the block to translate with respect to the fixed frame, the dof will increase by two. These additional dof can be viewed as the two extra freedoms for the block in locating and orienting it in the plane of the mechanism.

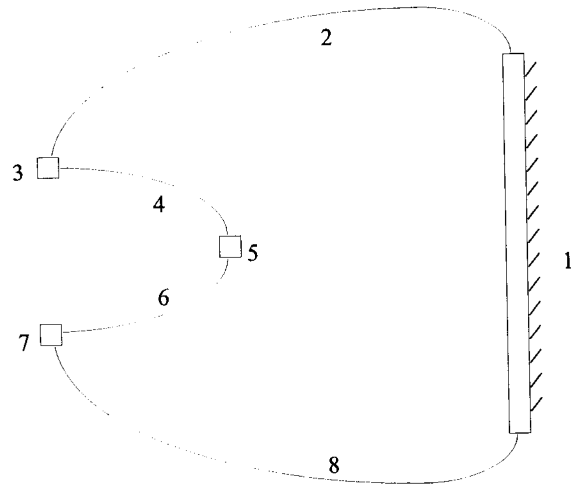


Case	*nseg*	*nfix*	*nK1*	*nsc3*	*dof*
Winch	4	2	2	1	2

Figure 8 Winch mechanism

3. Monolithic compliant gripper

The schematic representation for the one-piece compliant gripper (Figure 2c) is shown in Figure 9 below. Three VRS's are used here to incorporate the forces at the input and output ports. If we assume maximum *sc* for all compliant segments, the dof is 9. Since there are three VRS's all connected with the compliant segments, three inputs on each of them fulfill the requirements of dof and the complete description of the deformation of the mechanism. On the other hand, if we can judge that functionally the dominant mode of deformation of the compliant segments is only unimodal bending, an *sc* of one can be chosen for all four compliant segments. This would lead to one dof and the input can be on segment 5 along the line of symmetry. However, the assumption of an *sc* of 1 makes no provision for the output forces.



Ground segment: 1

VRS's: 3, 5, 7

Compliant segments: 2, 4, 6, 8

Case	*nseg*	*nfix*	*nsc1*	*nsc3*	*dof*
a	8	8	0	4	9
b	8	8	4	0	1

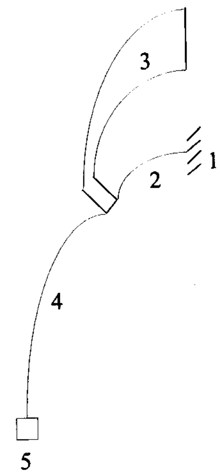
Figure 9 Monolithic compliant gripper

4. Monolithic compliant hand-clamp

The schematic representation in Figure 10 shows the symmetric half of the compliant, hand-held clamp from Figure 2b. The device opens its jaws when force is applied and clamps the object upon the removal of the applied forces. The VRS, segment 5, is introduced to account for the actuation force. Assuming maximum *sc* for segments 2 and 4 leads to 6 degrees of freedom. The six inputs can be evenly divided between the VRS 5 and the rigid segment 3. If segments 2 and 4 are treated as mono-compliant, we get 2 degrees of freedom which account for inputs at the input and output ports.

5. Monolithic compliant stapler

Figure 11a shows the lengthwise sectional view of the monolithic compliant stapler designed and fabricated by Ananthasuresh and Saggere (1994). The operating principle of this device is exactly the same as that of the conventional multi-part stapler, but it uses two compliant pivots and a compliant spring. The schematic version is shown in Figure 11b. As can be seen in the table below, this device has three degrees of freedom: two to enable the staple-plunger (segment 2) and the staple-trough (segment 5) to pivot, and the third to locate segment 4 which serves the dual purpose of holding the staples securely and enabling loading the staples into the track. The dof analysis is very useful in generating conceptual designs of this type of devices.



Ground segment: 1

Rigid segment: 3

VRS: 5

Compliant segments: 2, 4

Case	*nseg*	*nfix*	*nsc1*	*nsc3*	*dof*
a	5	4	0	2	6
b	5	4	2	0	2

Figure 10 Schematic representation of the symmetric half of the compliant clamp using a VRS

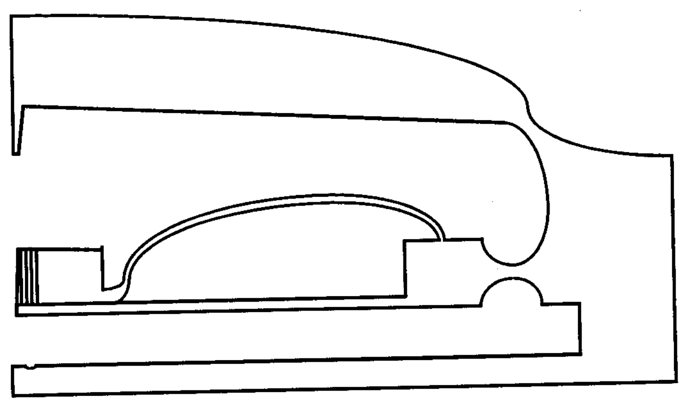
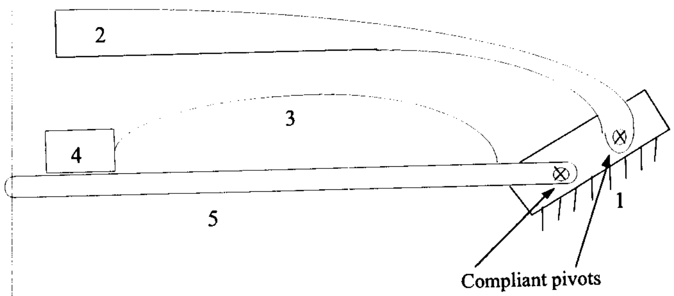


Figure 11a Sectional view of a monolithic compliant stapler (Ananthasuresh & Saggere, 1994)



Case	nseg	nfix	nK1	nC1	nsc3	dof
Stapler	5	2	1	2	1	3

Figure 11b Schematic representation of the monolithic compliant stapler as a compliant mechanism

6. Nail-clipper

The nail-clipper (Figure 12a) is a compliant mechanism because it contains a compliant member which is the crucial part of this simple device. Referring to the schematic representation in Figure 12b, an *sc* of 3 for the compliant segment yields three degrees of freedom. One input goes to the pivoting of segment 2. The rigid segment 3 can take the remaining two inputs (the segment 3 cannot take three inputs independent of segment 2 because it has to maintain contact with segment 2). If the *sc* is chosen as one, which is plausible since the loading is constrained and predictable, the obvious result of one degree of freedom is obtained.

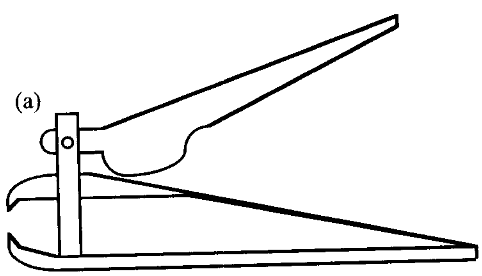
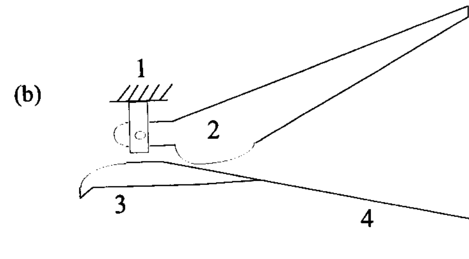


Figure 12a Sketch of a nail-clipper



Case	nseg	nfix	nK1	nK2	nsc1	nsc3	dof
a	4	2	1	1	0	1	3
b	4	2	1	1	1	0	1

Figure 12b Schematic representation of a nail-clipper as a compliant mechanism

7. Elasticarm: a truly flexible robotic arm

Unlike the usual "flexible" robotic manipulators which are flexible only because in actual working conditions they cannot be made to behave perfectly rigid, Catto and Moon (1995) reported their work on a truly flexible robotic arm shown schematically in Figure 13a. It consists of a compliant beam which is designed to be flexible, and an attached cable that is wound over a motor driven spool. The beam itself can be driven by a motor at its connection to the fixed frame. In its schematic representation as a compliant mechanism, one VRS is introduced at the connection of the beam and the cable. Another VRS is used to make the direct rotational input to the beam explicit. This mechanism has five degrees of freedom. The table shows the computation of dof with and without the VRS's as cases a and b. Both give the same result for dof, but the advantage with the representation using VRS's is that the dof can be interpreted very easily. Two rotation inputs go to segments 2 and 6. The VRS, segment 4, can take the three additional inputs. In fact, these inputs account for the reactions that come from the "hand" attached at the end of the beam (Catto and Moon, 1995).

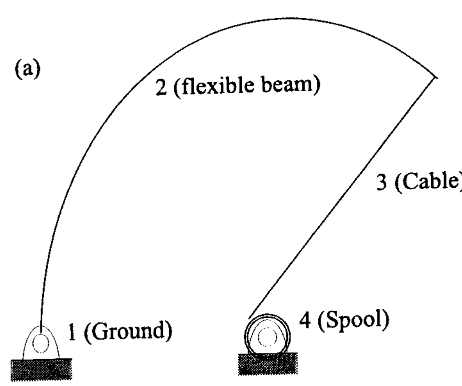
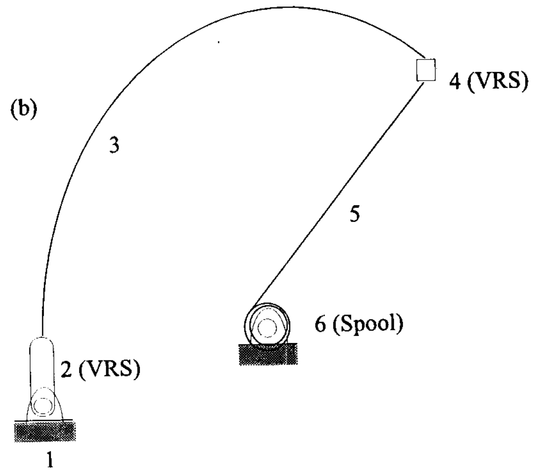


Figure 13a Elasticarm (Catto and Moon, 1995)



Case	nseg	nfix	nK1	nsc3	dof
a	4	2	2	2	5
b	6	4	2	2	5

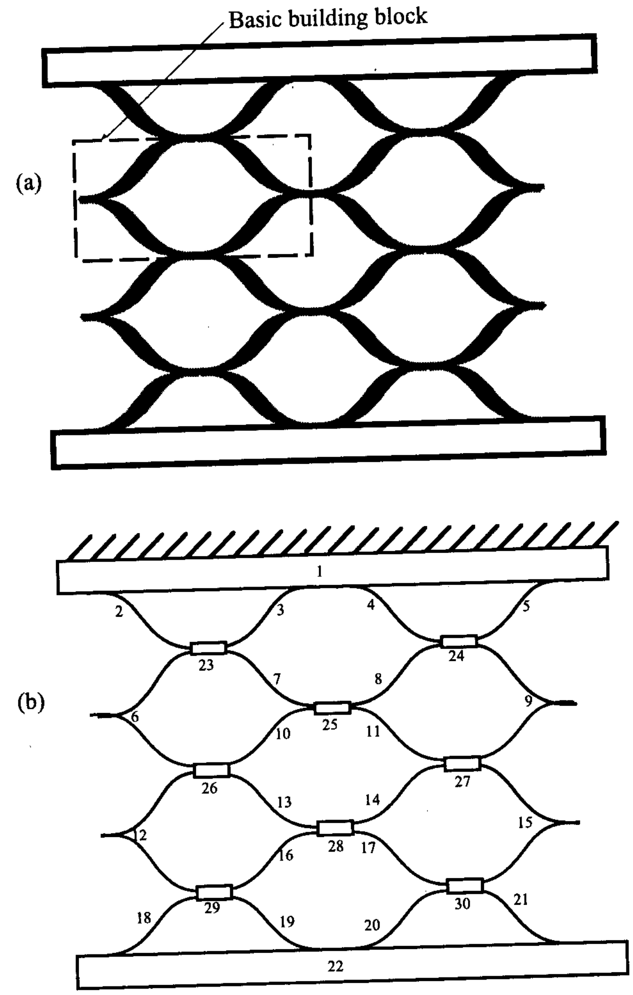
Figure 13b Representation of elasticarm with VRS's

8. Distributed electrostatic microactuator

Yamaguchi and Kawamura (1993) designed and fabricated a distributed electrostatic microactuator (DEMA) which is a repeated array of one building block (Figure 14a). The flexible beams in this design are composed of one conducting layer and an inner dielectric insulating layer. Upon application of the voltage the electrostatic force tends to pull the beams together, which then contact and pull together on the insulator side. Although the dof analysis is not essential for a design such as this, one can still gain some insight by studying its motion capability. A combined use of 20 tri-compliant (sc = 3) compliant segments, 1 rigid segment, 8 VRS's, and a fixed segment, the DEMA is represented schematically in Figure 14b. The VRS's are used to indicate the lumping of the distributed electrostatic force at a single location. In case a, 27 dof refer to three inputs each to the 8 VRS's and the rigid segment 22. If we make use of repetitive and symmetric structure of the device and constrain it further with an additional 9 prismatic joints so that the rigid segment and 8 VRS's can only translate in the downward direction, the dof reduces to 9. Now the rigid segment 22 and 8 VRS's can take only the downward translating input which accounts for the 9 inputs.

9. Insect flight mechanism

Nature's designs are replete with structures and mechanisms that are compliant (Vogel, 1995; Ananthasuresh and Kota, 1995). One instance of nature's compliant design can be found in the flight mechanism of insects. A biologist's rendering of the compliant mechanism is shown in Figure 15a. The figure shows the up and down strokes of the insect's



Case	nseg	nfix	nK1	nsc1	nsc3	dof
a	30	40	0	0	20	27
b	30	40	9	0	20	9

Figure 14 (a) Distributed electrostatic micro actuator and (b) its schematic representation as a compliant mechanism with VRS's

wings as its various parts flex under the muscle action. One interpretation of this complex design is shown schematically in Figure 15b. As shown in the table, this abstraction is found to have eight degrees of freedom. These can be counted as follows. The VRS in segment 6 and the wing (interpreted as one rigid segment) can accept 6 inputs. The sliding VRS can be given one translational input and the compliant connection makes way for the remaining one input. Although this abstraction is not meant to be accurate, the large number of the dof indicates the complex motion of the wings of insects.

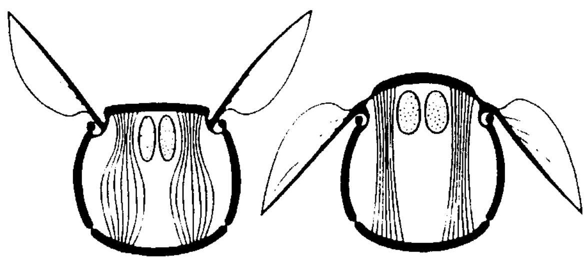
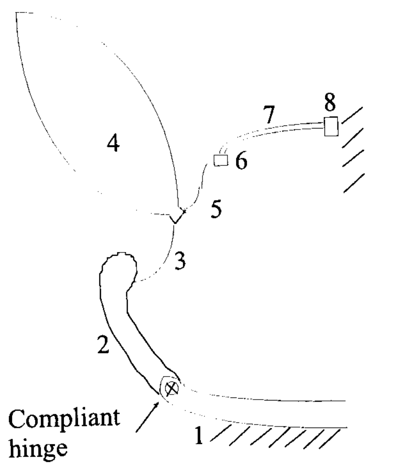


Figure 15a A biologist's rendering of the insect flight mechanism



Segment	Type	*sc*
1	Fixed	0
2, 4	Rigid	0
3, 5, 7	Compliant	3
6, 8	VRS	0

nseg	*nfix*	*nK1*	*nC1*	*nsc3*	*dof*
8	6	1	1	3	8

Figure 15b Schematic diagram of the compliant insect flight mechanism

CONCLUSIONS

An experienced designer can easily identify the dof and visualize the motion of the mechanism, but it may not always be obvious to everyone especially when compliant members are present in addition to the rigid members. It is demonstrated here that the degrees-of-freedom formula developed by Midha and his co-researchers can be successfully applied to a wide variety of compliant devices. To reduce the difficulties associated with the choice of segment compliance for the distributed type compliant segments with forces applied to them, a sequence of steps for the determination of degrees of freedom is proposed using the simple new concept of "virtual rigid segment". Even though the structural deformation due to the applied forces plays the crucial role in imparting mobility to compliant mechanisms, it is shown here that a kinematic interpretation of the motion capabilities of the mechanism is still possible. A useful intuitive procedure for interpreting the multiple degrees of freedom and understanding of the mobility of a wide variety of compliant devices is also presented here.

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REFERENCES

Ananthasuresh, G. K., and Kota, S., 1995, "Designing compliant Mechanisms," *Mechanical Engineering*, Vol. 117, No. 11, November, pp. 93-96.

Ananthasuresh, G. K., Kota, S., and Kikuchi, N., 1994, "Strategies for Systematic Synthesis of Compliant MEMS," *Proceedings of the 1994 ASME Winter Annual Meeting*, Nov. 6-11, Chicago, IL, Dynamic Systems and Control, DSC-Vol. 55-2, pp. 677-686.

Ananthasuresh, G. K., and Saggere, L., 1994, "A One-Piece Compliant Stapler," A design project entry that was placed first in the graduate division of *the 1994 ASME Student Mechanism Competition* at the *23rd Biennial Mechanisms Conference*, Minneapolis, Minnesota, Sep. 11-14. Available as a *Tech. Report*: UM-MEAM 95-20 from the University of Michigan, Ann Arbor, MI. (Advisor: S. Kota)

Catto, E. S., and Moon, F. C., 1995, "Robust Control of a Shape-Changing Flexible Robot Arm," *Proceedings of the SPIE Workshop on Smart Structures and Integrated Systems*, SPIE Proceeding Series, Vol. 2443, Ed. Indrajit Chopra, pp. 171-181.

Her, I., and Midha, A., 1987, "A Compliant Number Concept for Compliant Mechanisms and Type Synthesis," *Journal of Mechanisms, Transmissions, and Automation in Design*, Trans. ASME, Vol. 109, No. 3, pp. 348-355.

Howell, L. L., and Midha, A., 1995, "Determination of the Degrees of Freedom of Compliant Mechanisms Using the Pseudo-Rigid-Body Model Concept," *Proceedings of the IFToMM Ninth World Congress on the Theory of Machines and Mechanisms*, Milano, Italy, Vol. 2, pp. 1537-1541.

Midha, A., Norton, T. W., and Howell, L. L., 1994, "On the Nomenclature, Classification, and Abstractions of

Compliant Mechanisms," *ASME Journal of Mechanical Design*, Vol. 116, pp. 270-279.

Murphy, M. D., Midha, A., and Howell, L. L., 1994a, "Type Synthesis of Compliant Mechanisms Employing a Simplified Approach to Segment Type," *Proceedings of the 1994 ASME Mechanisms Conference*, Minneapolis, Minnesota, Sep. 11-14, DE-Vol. 70, pp. 51-60.

Murphy, M. D., Midha, A., and Howell, L. L., 1994b, "The Topological Analysis of Compliant Mechanisms," *Proceedings of the 1994 ASME Mechanisms Conference*, Minneapolis, Minnesota, Sep. 11-14, DE-Vol. 71, pp. 481-489.

Murphy, M. D., Midha, A., and Howell, L. L., 1994c, "On the Mobility of Compliant Mechanisms," *Proceedings of the 1994 ASME Mechanisms Conference*, Minneapolis, Minnesota, Sep. 11-14, DE-Vol. 71, pp. 475-479.

Vogel, S., 1995, "Better Bent Than Broken", *Discover*, May, 1995, pp. 62-67.

Yamaguchi, M., and Kawamura, K., 1993, "A Distributed Electrostatic Microactuator," *Proceedings of the International Workshop on Micro-Electro-Mechanical Systems*, MEMS 93, Fort Lauderdale, FL, pp. 18-23.