ME 254 Lecture 16

Kinematic approximation of the locus of the loaded tip of a cantilever beam

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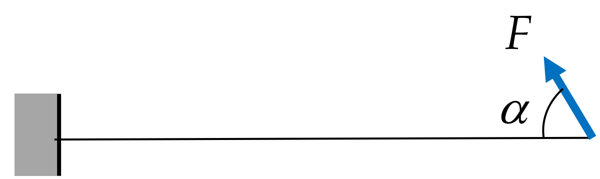
Elliptic integral solutions: the gist

$$\int_{\sin^{-1}\left(\frac{1}{p\sqrt{2}}\right)}^{\pi/2} \frac{d\phi}{\sqrt{\frac{F}{EI}}\sqrt{1-p^2\sin^2\phi}} = L$$

$$\int_{\sin^{-1}\left(\sqrt{\frac{1+\sin(\pi/2-\alpha)}{2p^2}}\right)}^{\pi/2} \frac{d\phi}{\sqrt{\frac{F}{EI}}\sqrt{1-p^2\sin^2\phi}} = L'$$



$$\int_{\sin^{-1}\left(\frac{1}{p\sqrt{2}}\right)}^{\cos^{-1}\left(\frac{M}{2p}\sqrt{\frac{1}{FEI}}\right)} \frac{d\phi}{\sqrt{\frac{F}{EI}}\sqrt{1-p^2\sin^2\phi}} = L$$

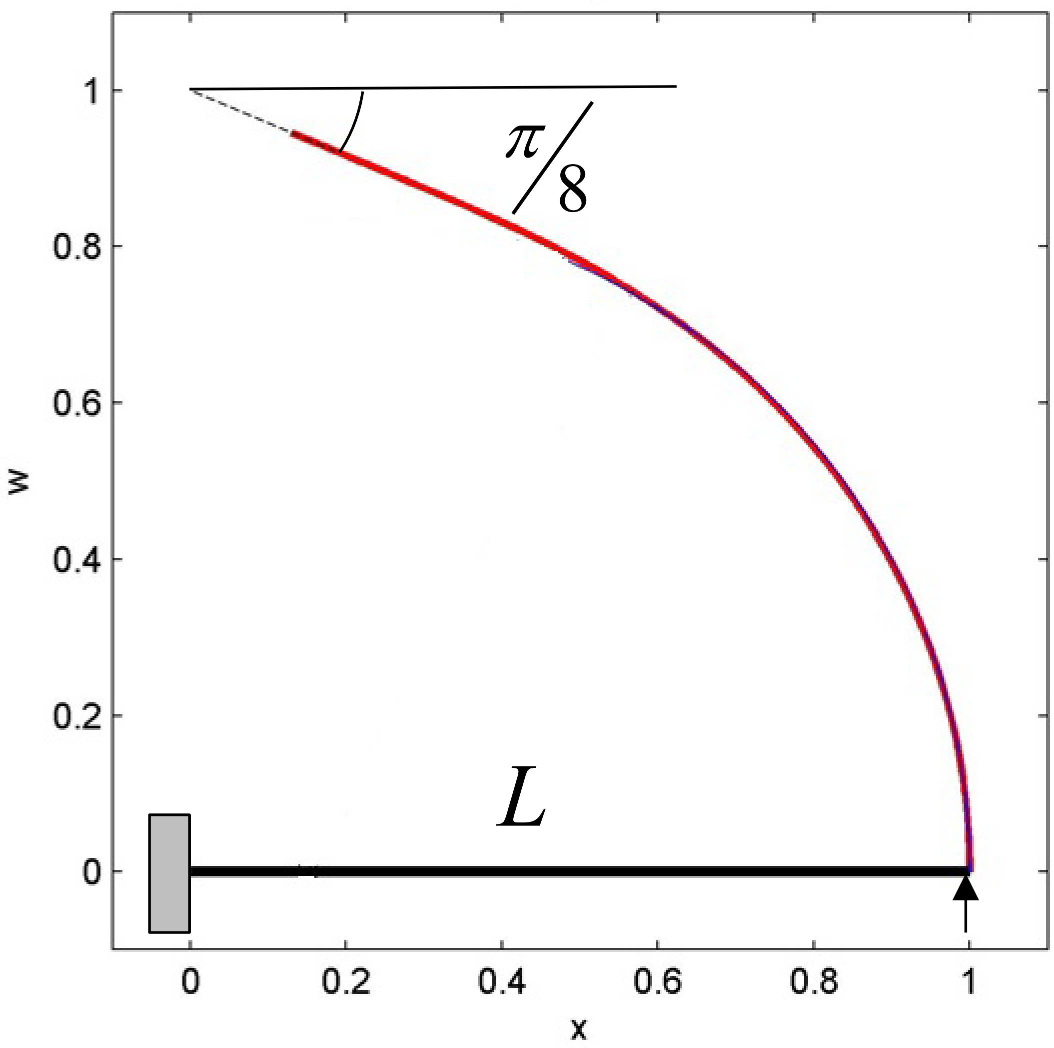
Coordinates of the loaded tip of a cantilever beam

$$\eta = \frac{FL^2}{EI} \qquad \phi_B = \sin^{-1}\left(\frac{1}{p\sqrt{2}}\right)$$

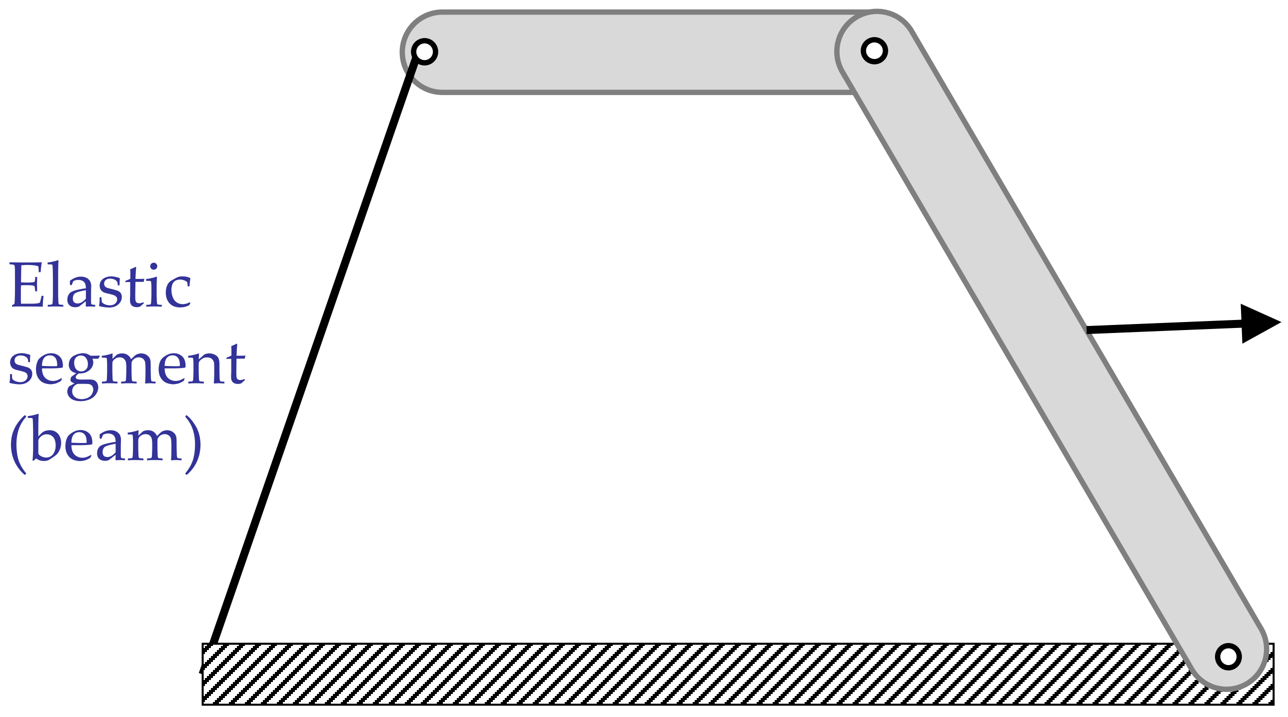
$$\frac{w}{L} = \sqrt{\frac{1}{\eta}}\left\{\mathbf{F}(\pi/2,\phi) - \mathbf{F}(p,\phi_B) - 2\mathbf{E}(p,\pi/2) + 2\mathbf{E}(p,\phi_B)\right\}$$

$$\frac{x}{L} = \frac{2p}{\sqrt{\eta}}\cos\phi_B$$

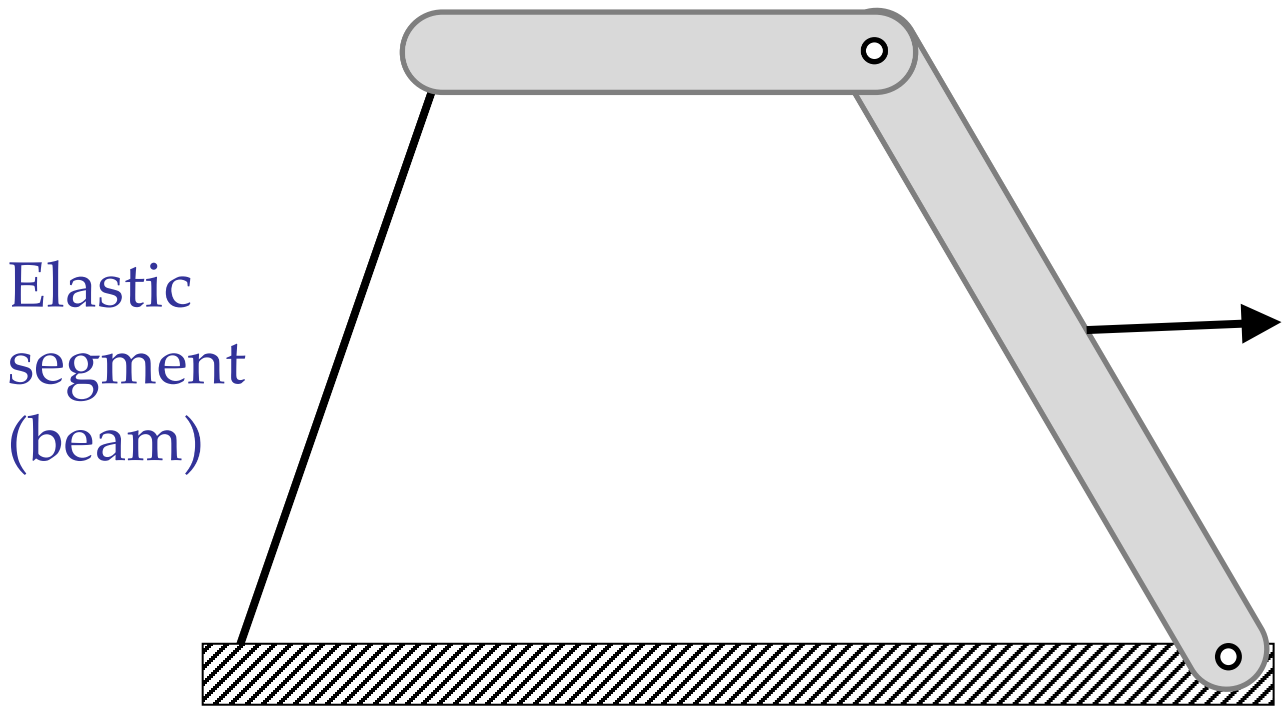
Locus of the loaded tip



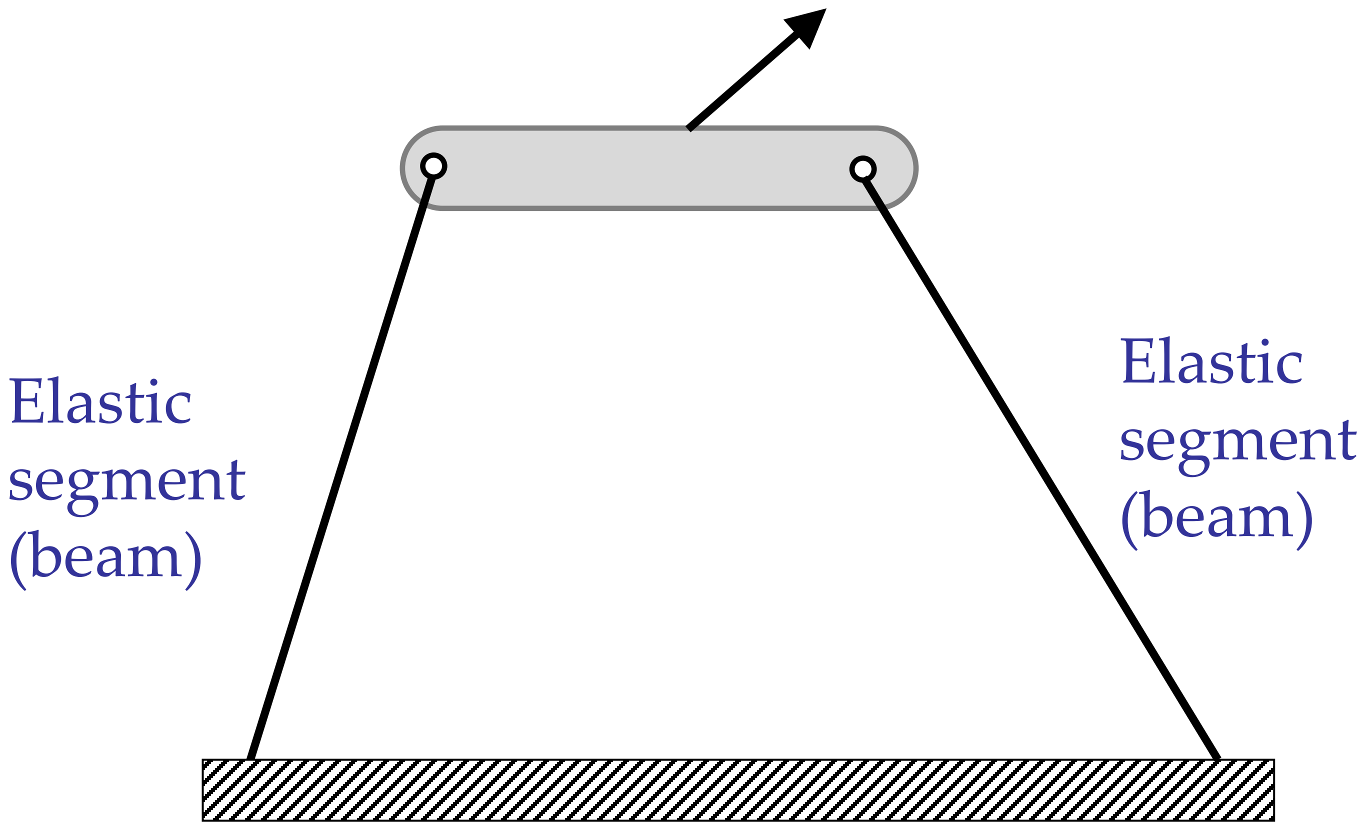
Can we solve this problem?



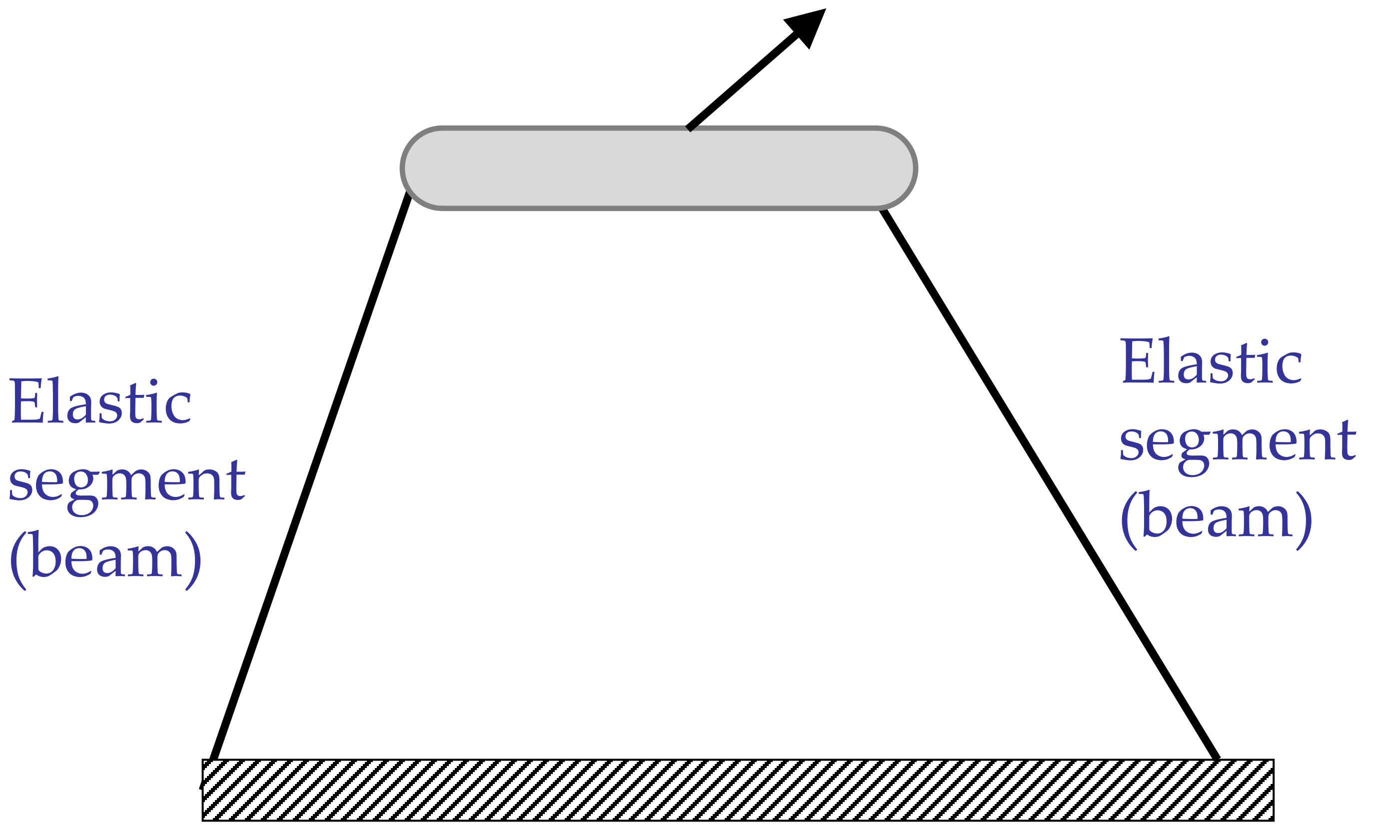
How about this problem?



And this one?



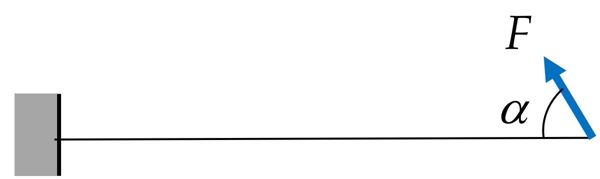
And this one?



We can solve them, but… we need to find *p* for each configuration.

$$\int_{\sin^{-1}\left(\frac{1}{p\sqrt{2}}\right)}^{\pi/2} \frac{d\phi}{\sqrt{\frac{F}{EI}}\sqrt{1-p^2\sin^2\phi}} = L$$

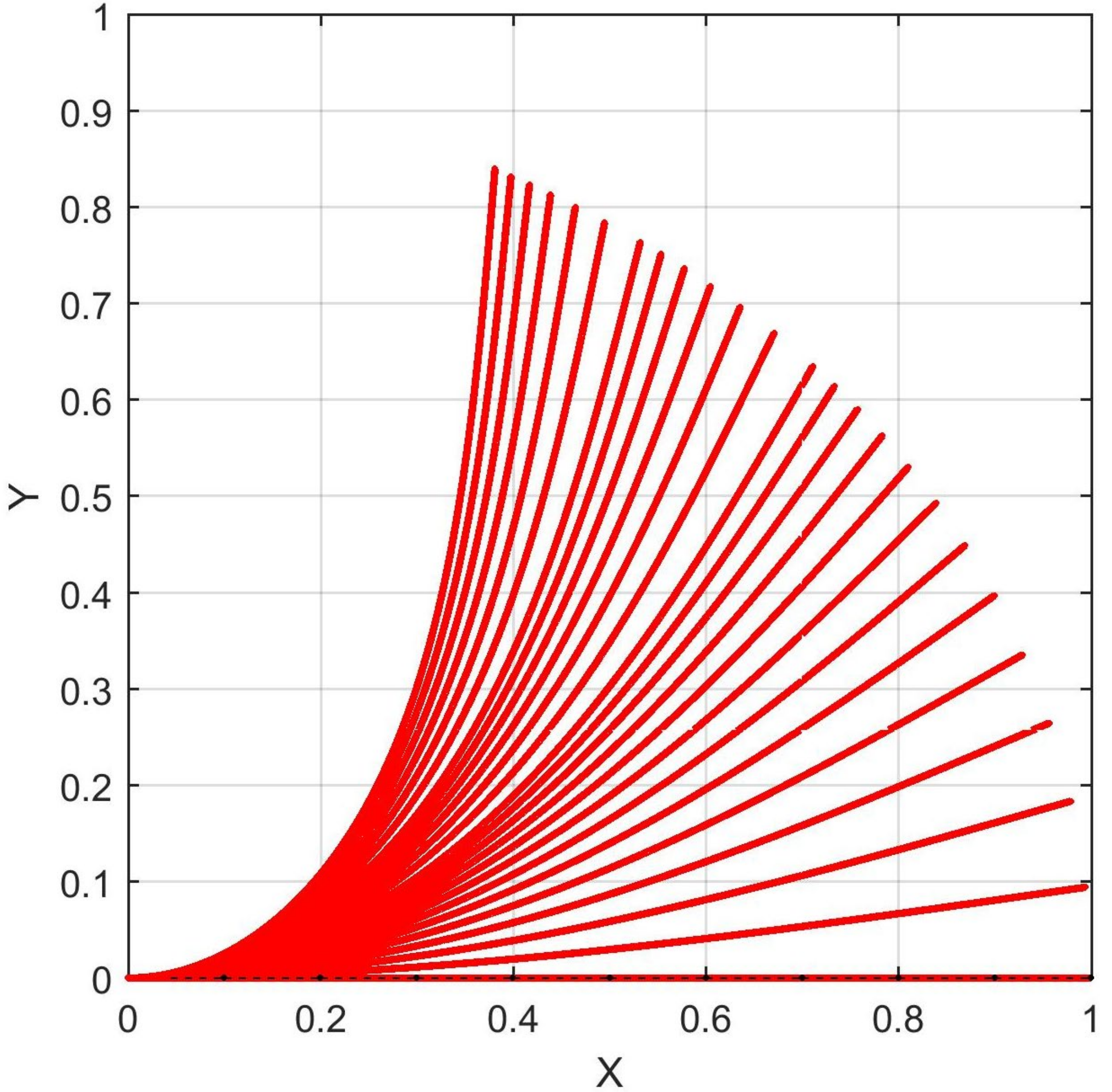
$$\int_{\sin^{-1}\left(\sqrt{\frac{1+\sin(\pi/2-\alpha)}{2p^2}}\right)}^{\pi/2} \frac{d\phi}{\sqrt{\frac{F}{EI}}\sqrt{1-p^2\sin^2\phi}} = L'$$



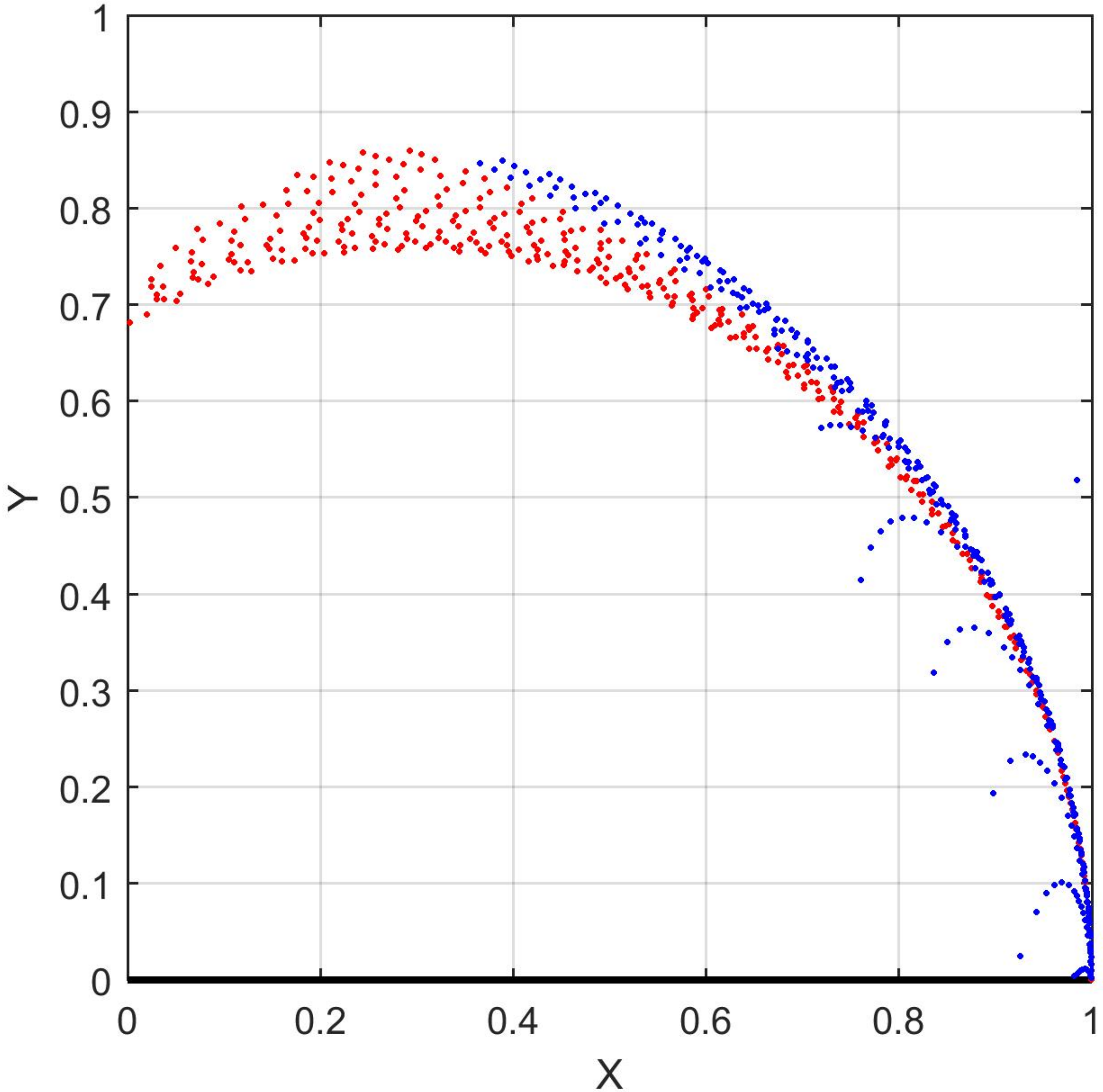
$$\int_{\sin^{-1}\left(\frac{1}{p\sqrt{2}}\right)}^{\cos^{-1}\left(\frac{M}{2p}\sqrt{\frac{1}{FEI}}\right)} \frac{d\phi}{\sqrt{\frac{F}{EI}}\sqrt{1-p^2\sin^2\phi}} = L$$

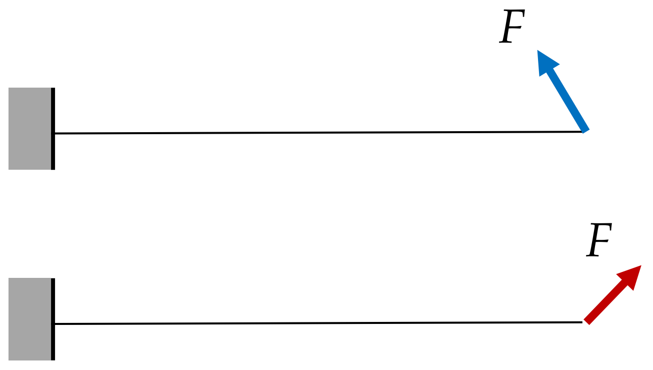
Is there a way out?

Locus of the tip under transverse loads

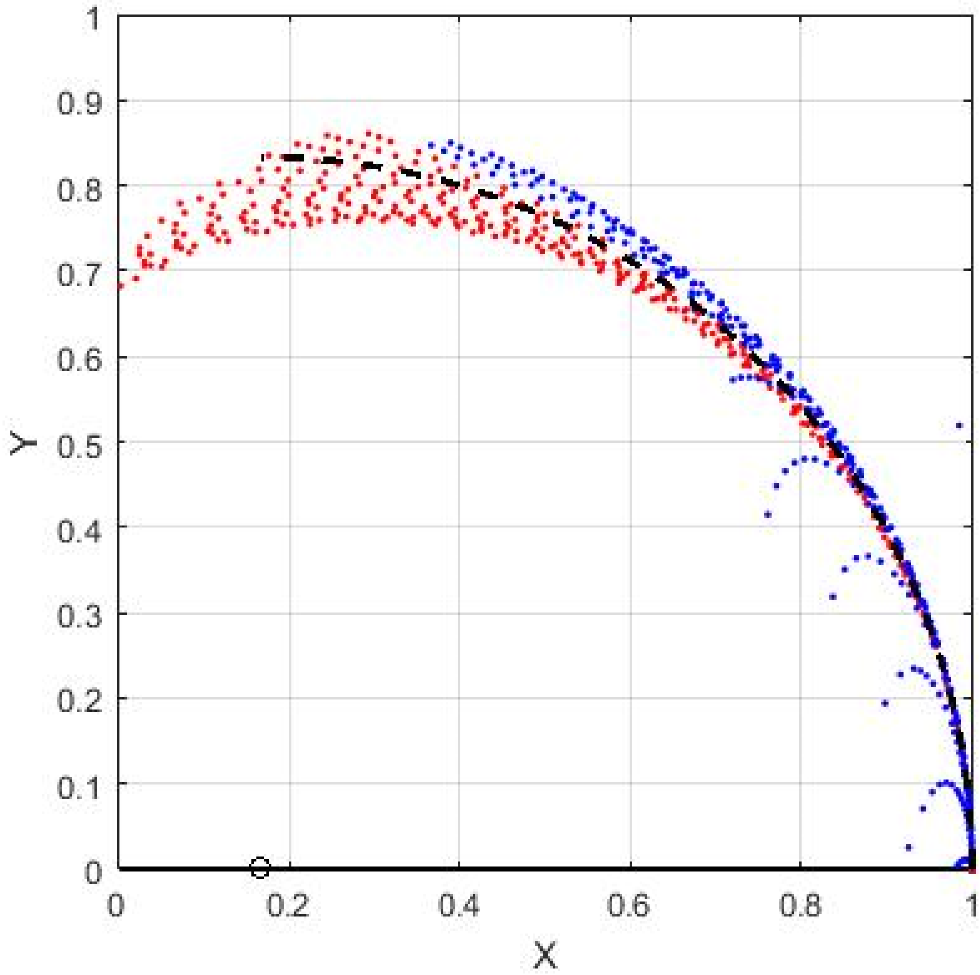


Locus of the tip under a variety of loads

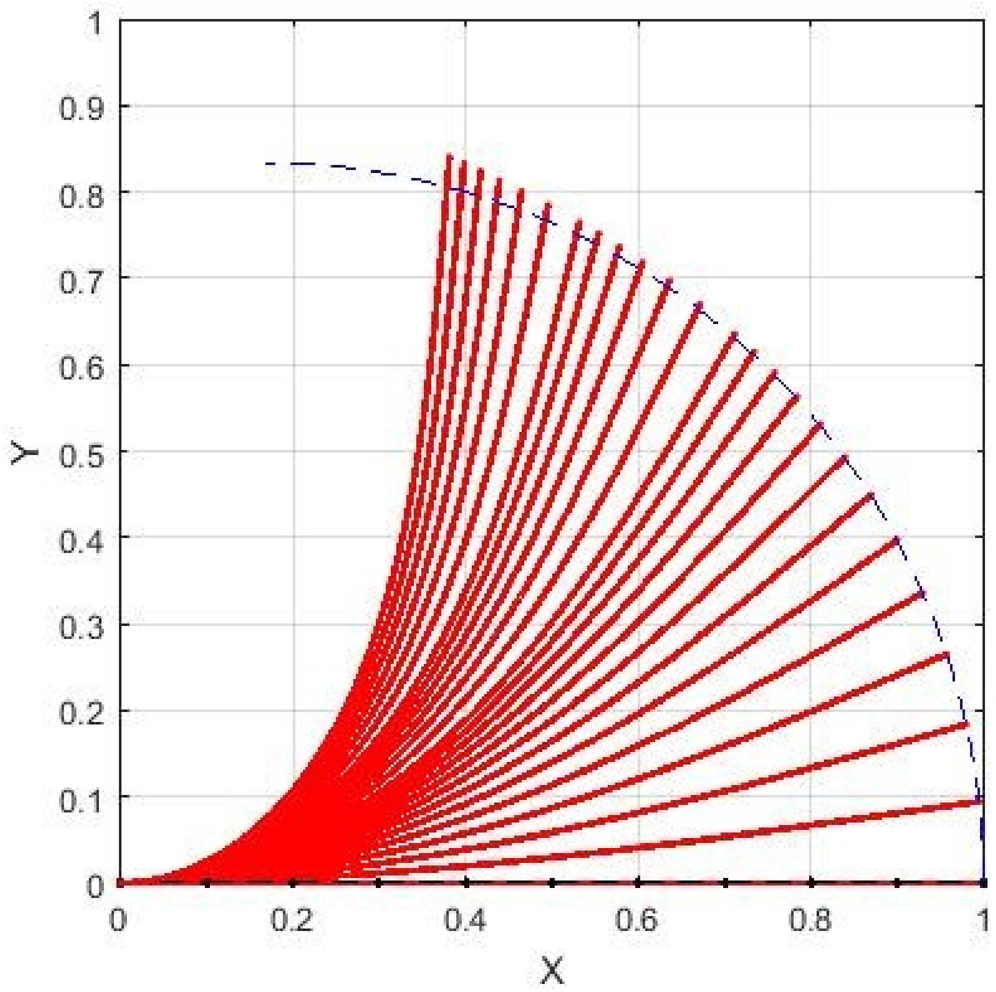




“Aha!” moment:
I see a circle there!



The circle more clearly visible with only transverse loads



Who had that "Aha!" moment?

68-MECH-36

Suresh

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Kinetostatic Synthesis of Flexible Link Mechanisms[1]

R. H. BURNS

Bausch & Lomb,
Rochester, N. Y.
Assoc. Mem. ASME.

F. R. E. CROSSLEY

Georgia Institute of Technology,
Atlanta, Ga. Mem. ASME.

A profound insight

Burns and Crossley, 1968

Burns, R. H. and Crossley, F. R. E., “Kinetostatic Synthesis of Flexible Link Mechanisms,” Trans. ASME, 68-MECH-36, 1968.

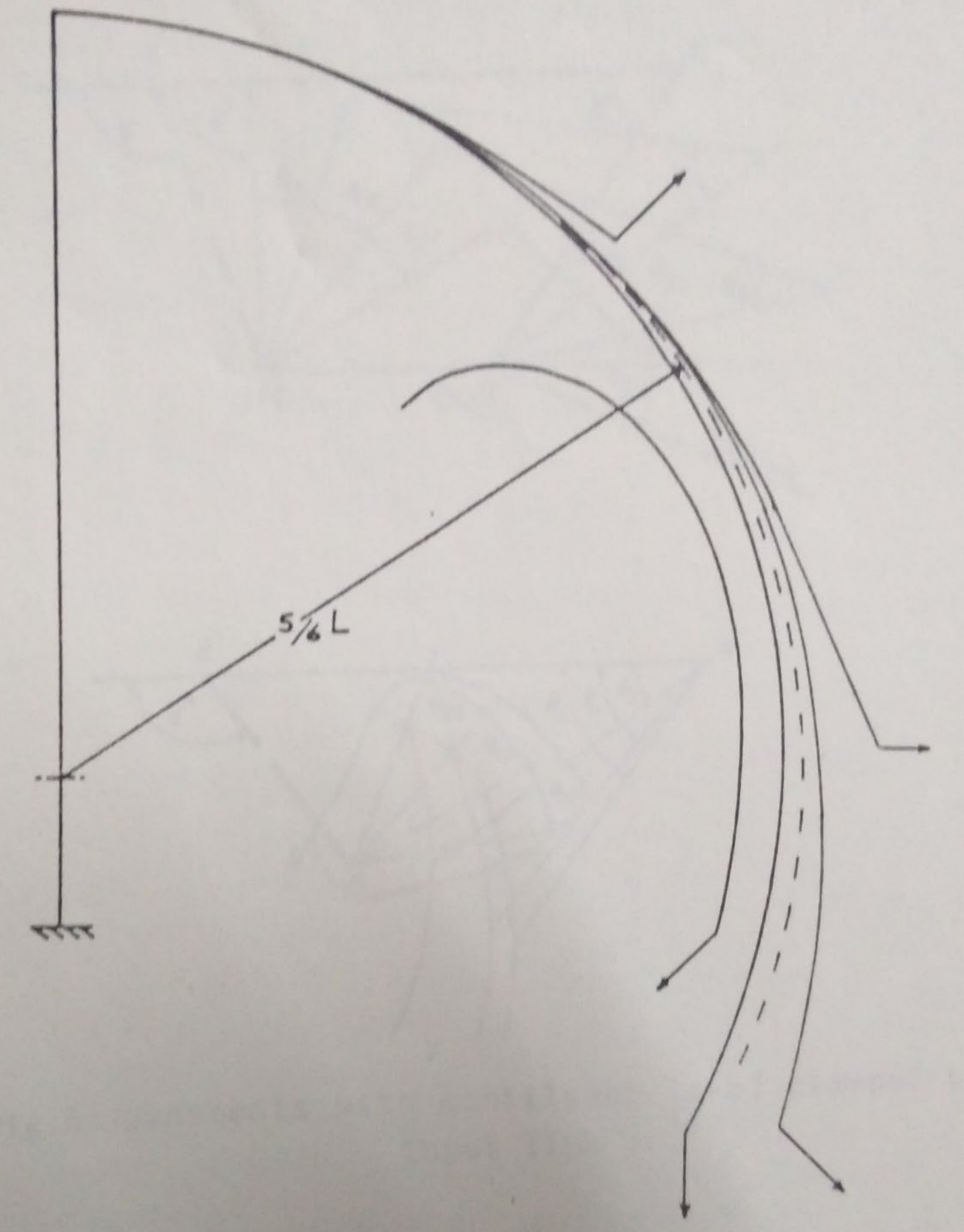
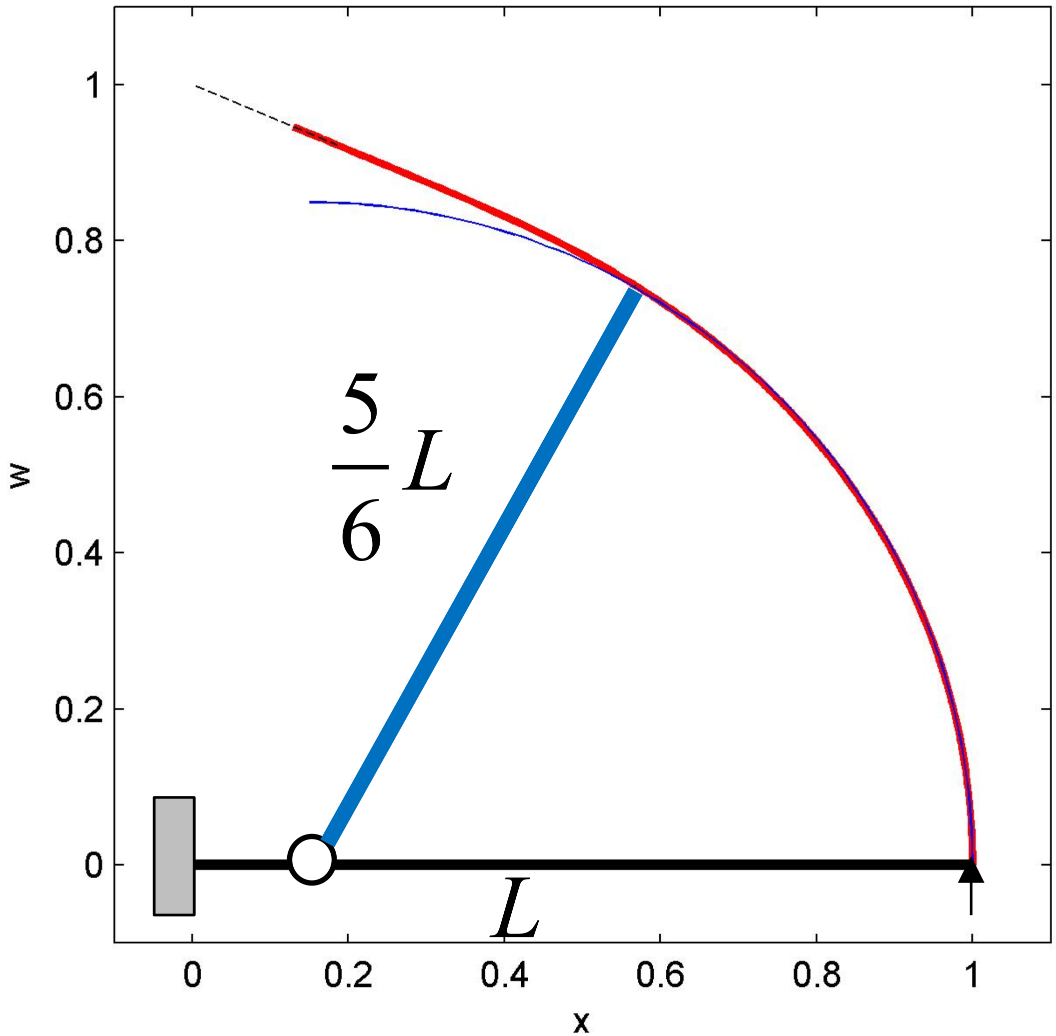


Fig.3 Deflection of a cantilever under varyin
forces of constant direction

An excerpt from the Burns-Crossley paper of 1968

There is no literature recording any previous work toward this problem of synthesis. Sieker (1,2),[2] however, has discussed the design of mechanisms which contain flat springs. Meyer zur Capellen (3), Parkus (4), and others have studied the lateral bending vibrations which occur in the connecting rod of a slider-crank or coupler of any four-bar; and Houben (5) has written on the elastic stability of such oscillations. The coupler of a four-bar mechanism in the form of a compression and tension spring has been considered by Dizioglu (6,7). Such a design must usually be taken as an elastic system with two degrees of freedom, constrained by the force of the spring.

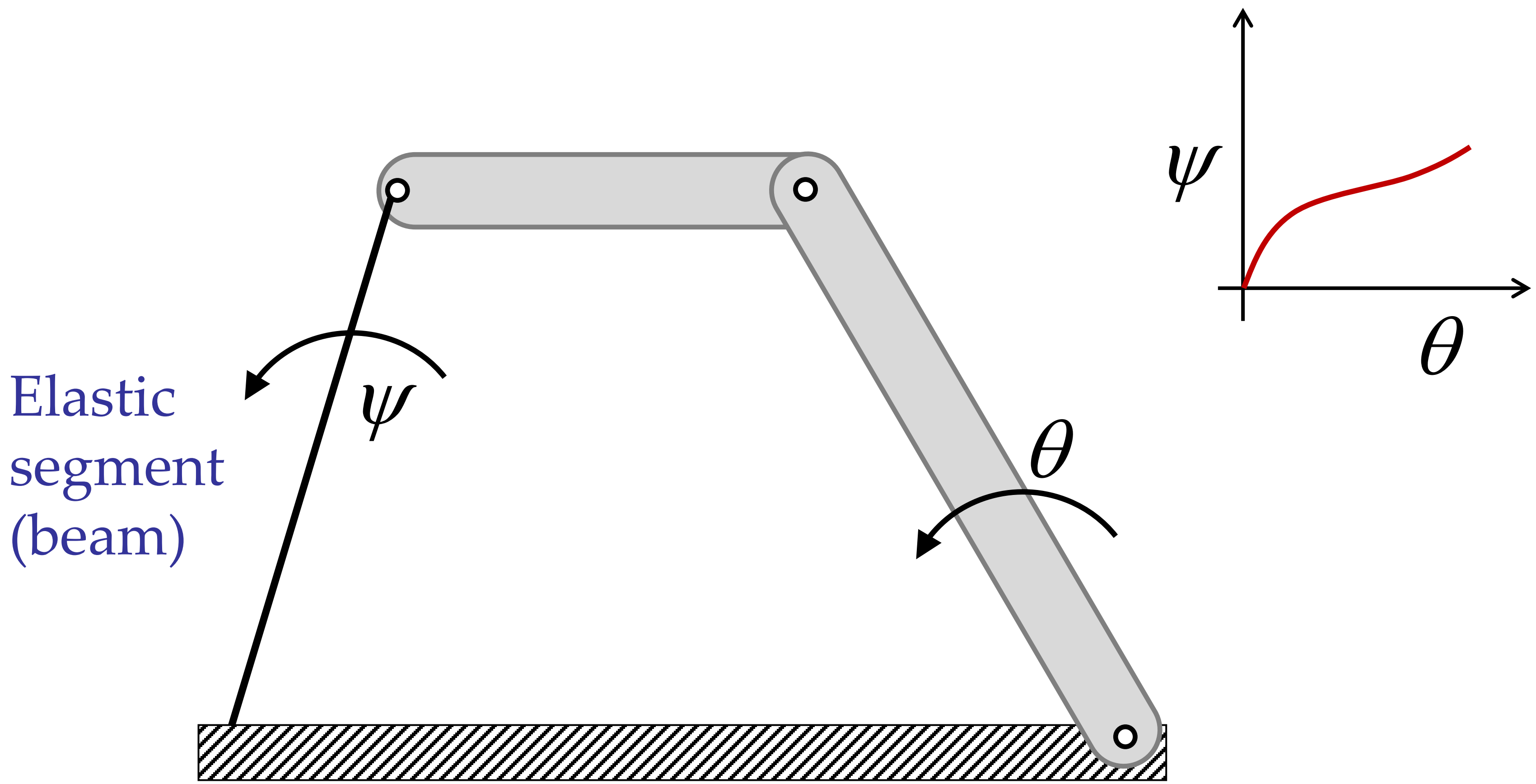
Kinematic approximation of the locus of the loaded tip



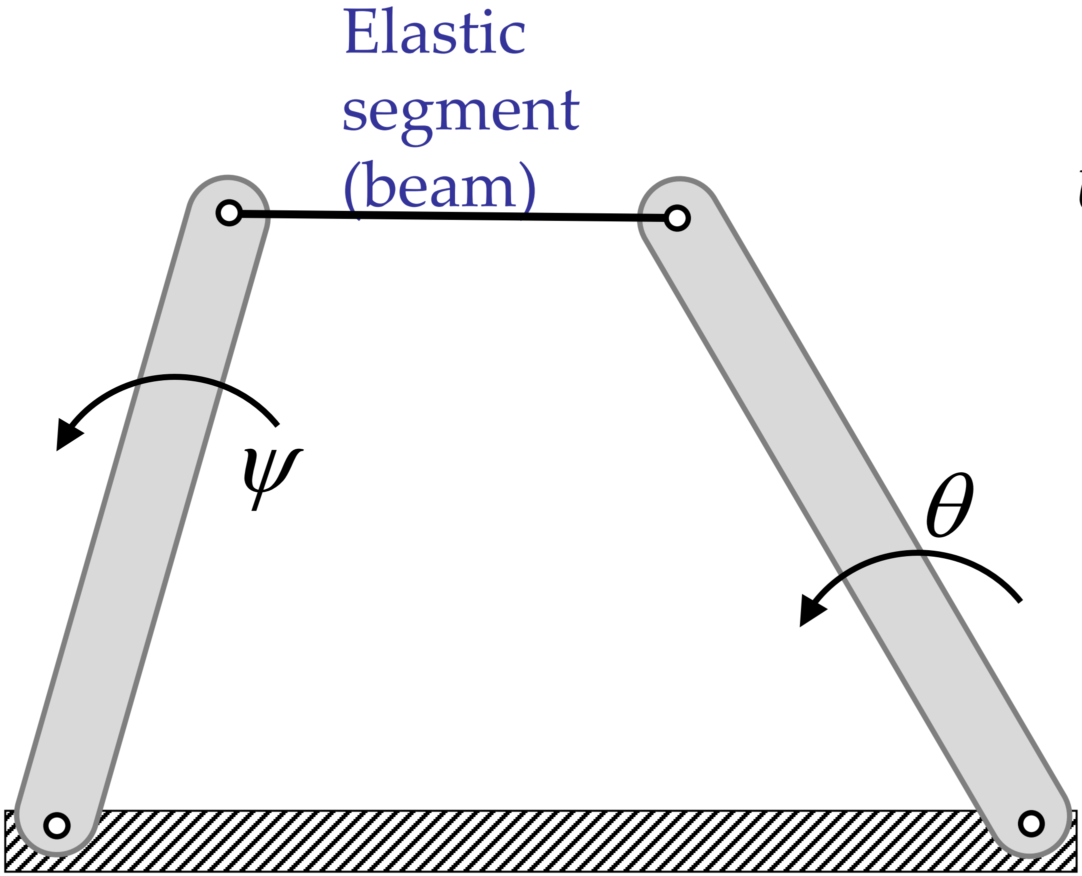
Burns and Crossley, 1968

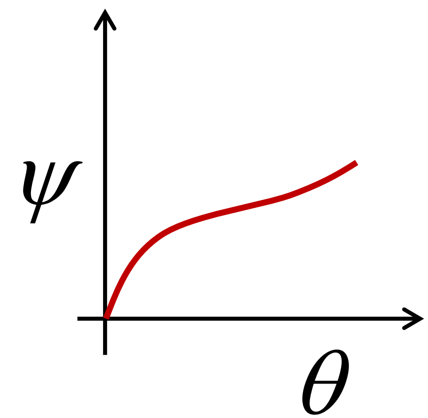
The locus can be approximated with a circular arc for a very large range of bending of a cantilever.

Now, can we solve this problem for "function generation"?

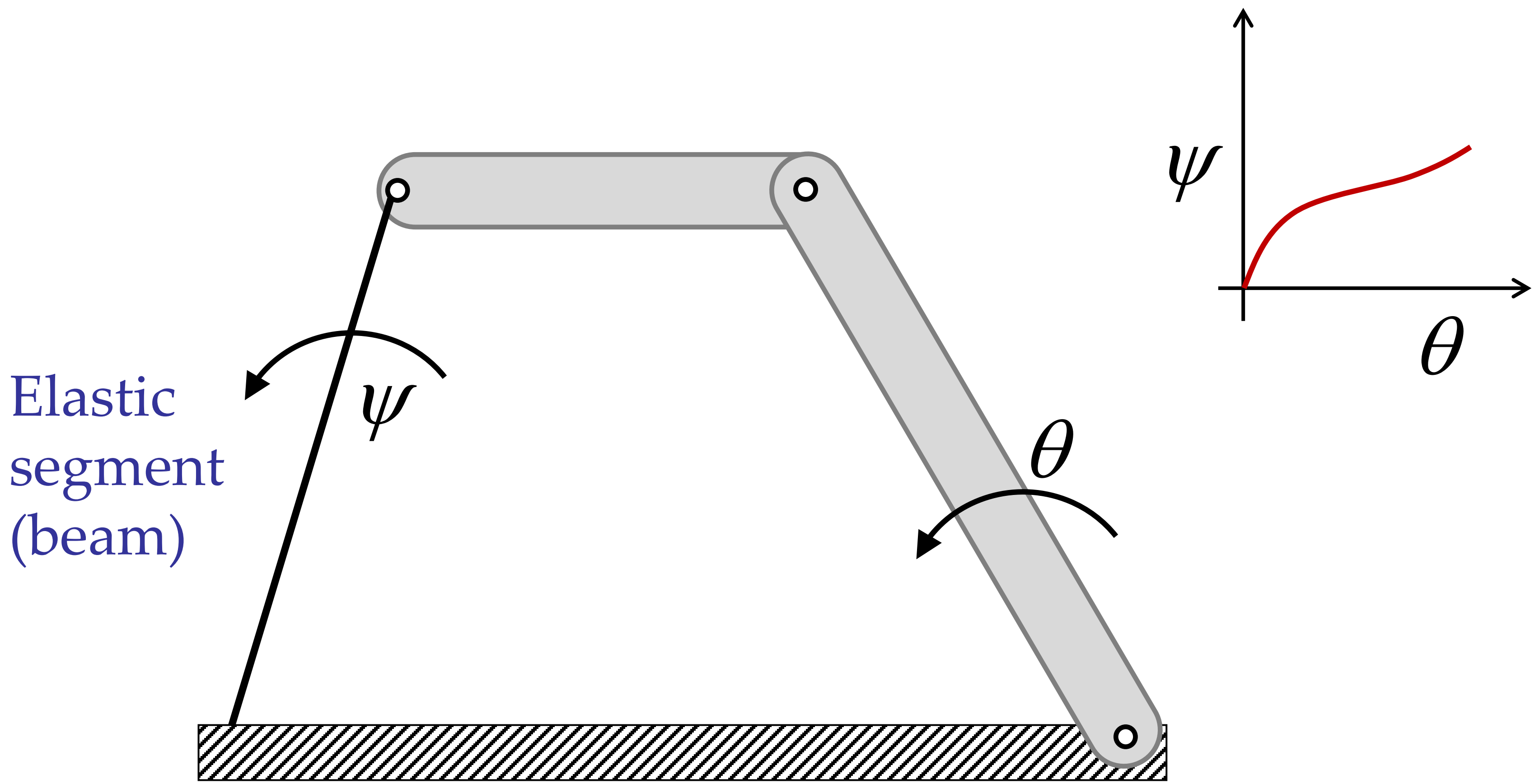


Or this solve this problem for "function generation"?

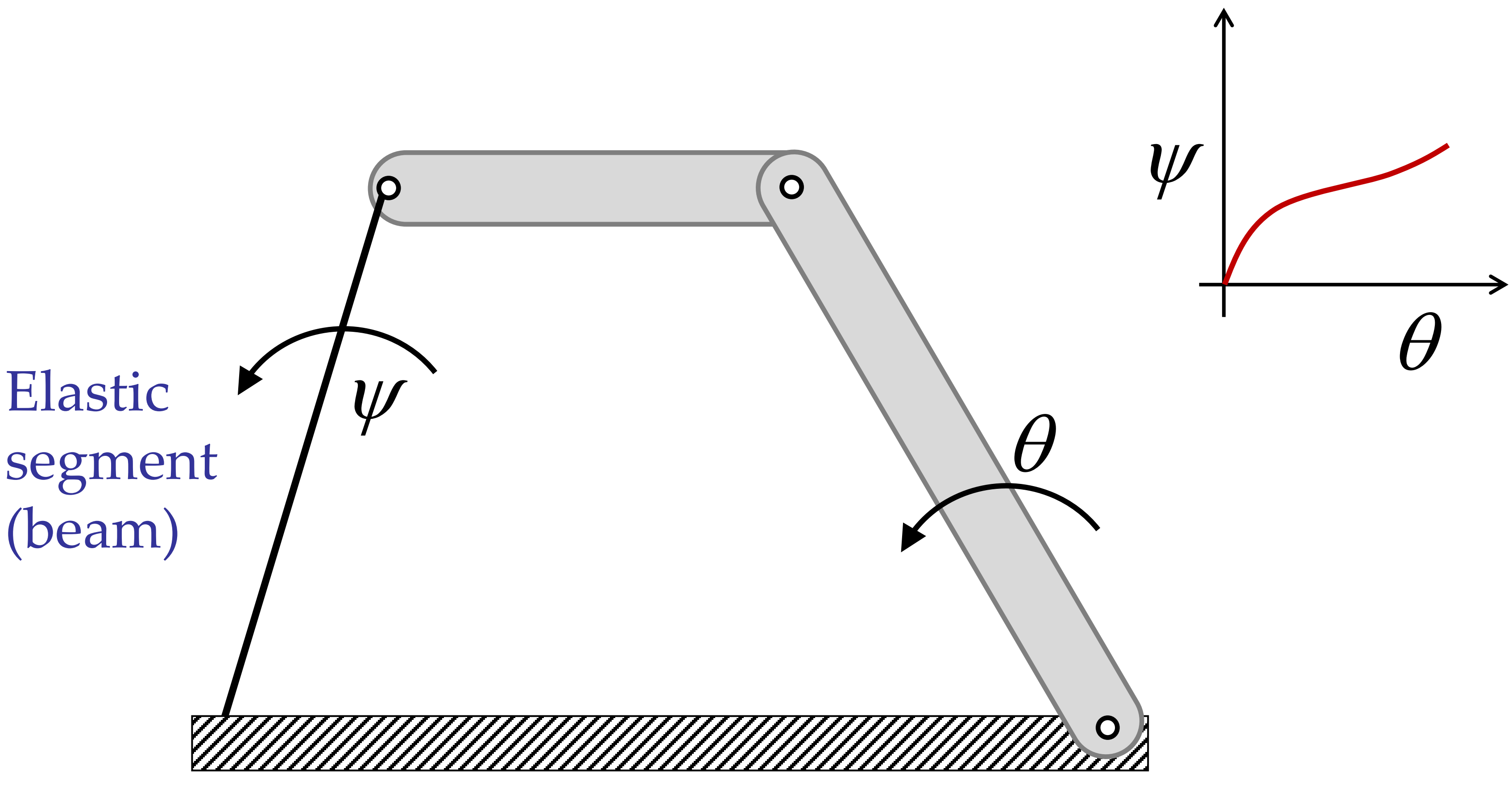




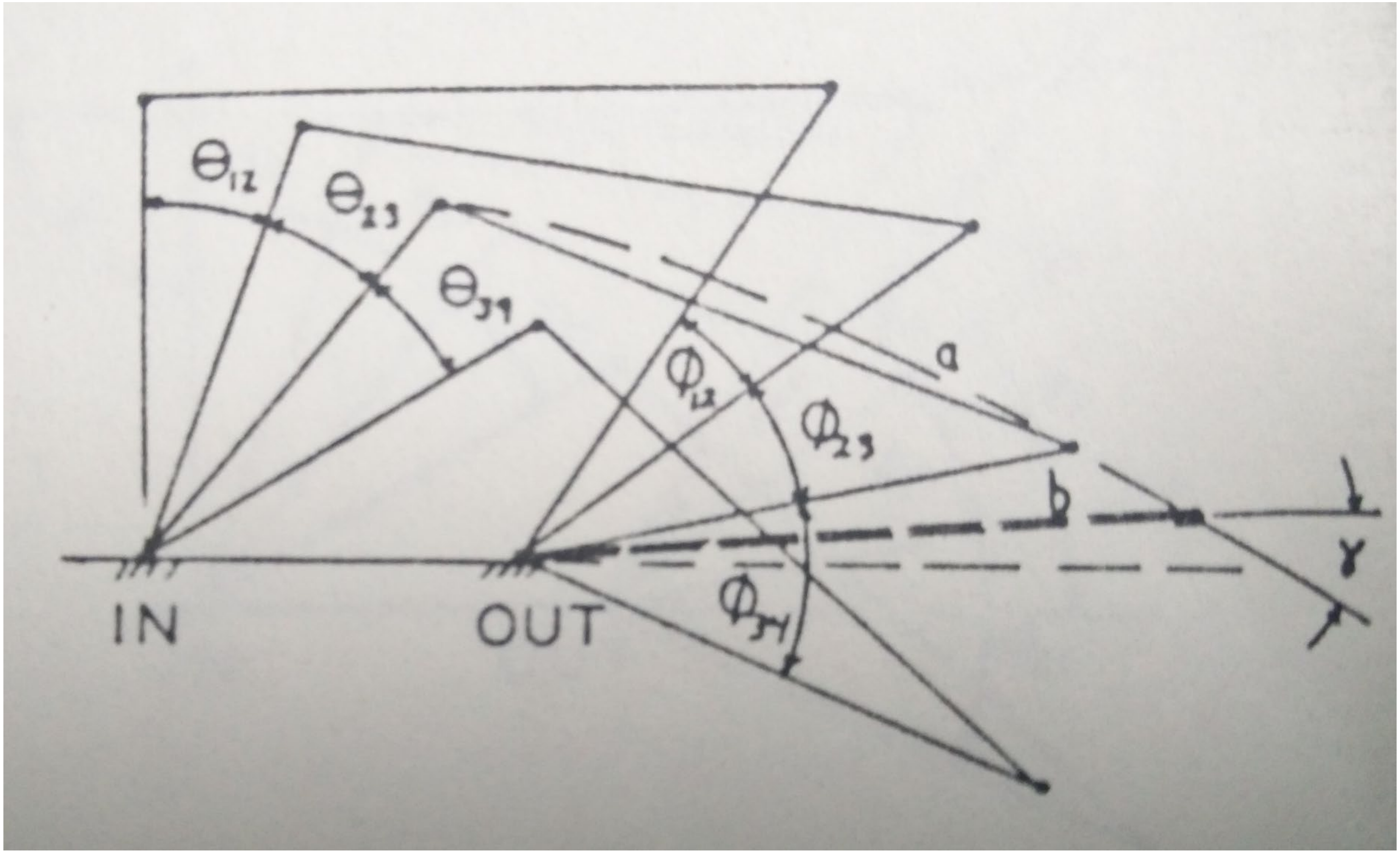
Now, can we solve this problem for "function generation"?



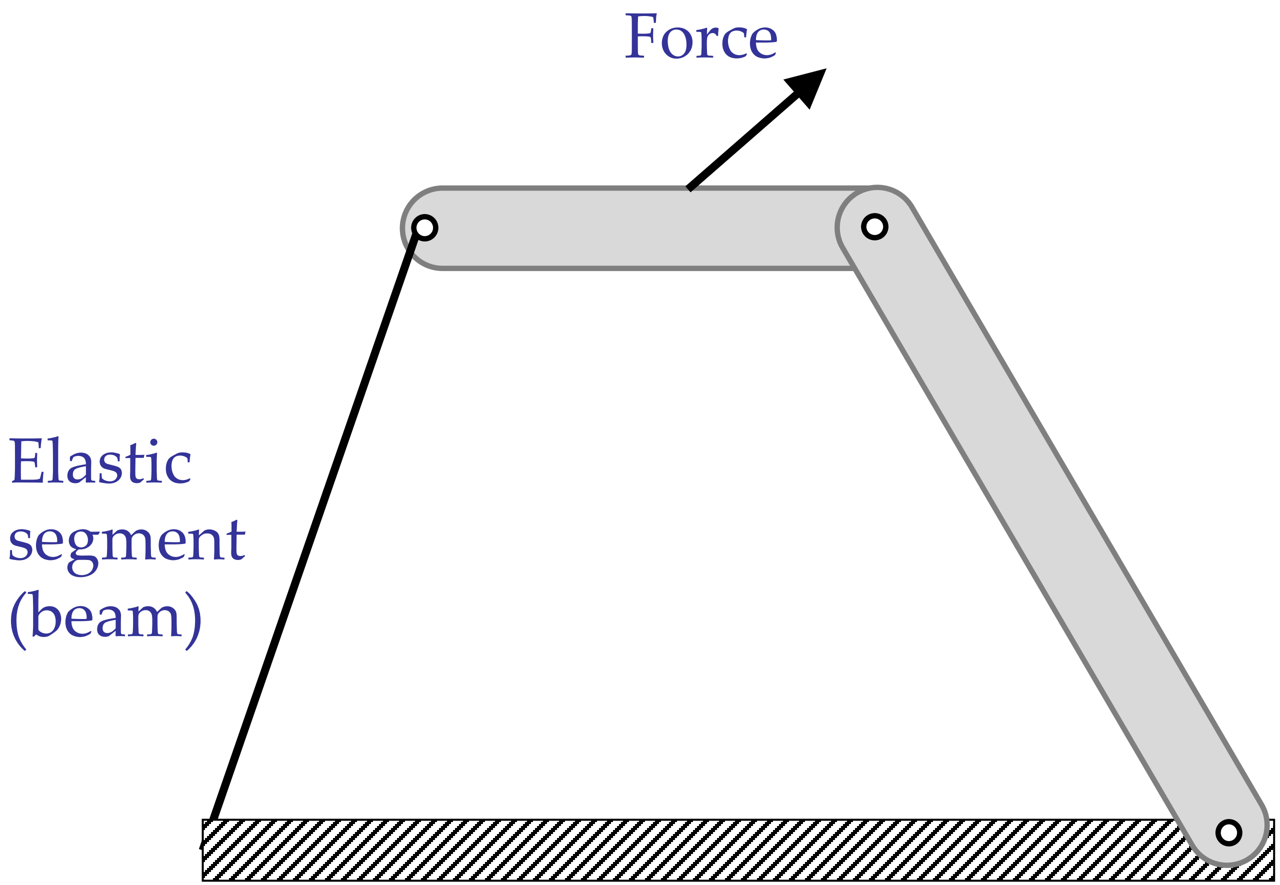
Now, can we solve this problem for "function generation"?



An overlay method for "elastic mechanisms" [Burns and Crossley]



Now, can we solve this problem?



Further reading

- Burns, R. H. and Crossley, F. R. E., "Kinetostatic Synthesis of Flexible Link Mechanisms," Trans. ASME, 68-MECH-36, 1968.
- Burns, R. H. and Crossley, F. R. E., "Structural Permuations of Flexible Link Mechanisms," Trans. ASME, 66-MECH-5, 1966.
- Burns, R. H., "The Kinetostatic Synthesis and Analysis of Flexible Link Mechanisms," Dr. Eng. Dissertation, Yale Univ., 1964.