ME 254

Building-block based synthesis: theory and implementation with stiffness and compliance ellipsoids

G. K. Ananthasuresh

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A key reference

ASME Journal of Mechanisms and Robotics, February 2011.

An Intrinsic Geometric Framework for the Building Block Synthesis of Single Point Compliant Mechanisms

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In this paper, we implement a characterization based on eigentwists and eigenwrenches for the synthesis of a compliant mechanism at a given point. For 2D mechanisms, this involves characterizing the compliance matrix at a unique point called the center of elasticity, where translational and rotational compliances are decoupled. Furthermore, the translational compliance may be represented graphically as an ellipse and the coupling between the translational and rotational components as vectors. These representations facilitate geometric insight into the operations of serial and parallel concatenations. Parametric trends are ascertained for the compliant dyad building block and are utilized in example problems involving serial concatenation of building blocks. The synthesis technique is also extended to combination of series and parallel concatenation to achieve any compliance requirements. [DOI: 10.1115/1.4002513]

Primary reference

STRUCTURE OF ROBOT COMPLIANCE

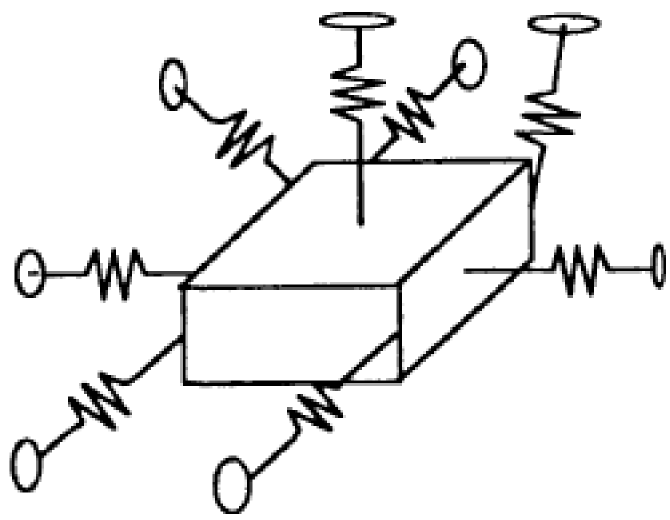
Timothy Patterson
Engineering Science and Mechanics

Harvey Lipkin
The George W. Woodruff School of Mechanical Engineering

Georgia Institute of Technology
Atlanta, Georgia

presented at

THE 1990 ASME DESIGN TECHNICAL CONFERENCES –
21st BIENNIAL MECHANISMS CONFERENCE
CHICAGO, ILLINOIS
SEPTEMBER 16–19, 1990



A comprehensive reference

A CONCEPTUAL APPROACH TO THE COMPUTATIONAL SYNTHESIS OF COMPLIANT MECHANISMS

by

Charles J. Kim

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Mechanical Engineering)
in The University of Michigan
2005

Another comprehensive reference

An Intrinsic and Geometric Framework for the Synthesis and Analysis of Distributed Compliant Mechanisms

by

Girish Krishnan

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Mechanical Engineering)
in The University of Michigan
2011

Some slides from…

Prof. Girish Krishnan, UIUC, USA
Prof. Charles Kim, Bucknell University, USA

The Building Block Method

Mathematical Representation

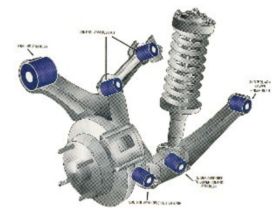
Characterize Building Blocks

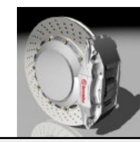
Evaluate Candidate Building Blocks

Decompose Problem Specification

Mechanism Assembly

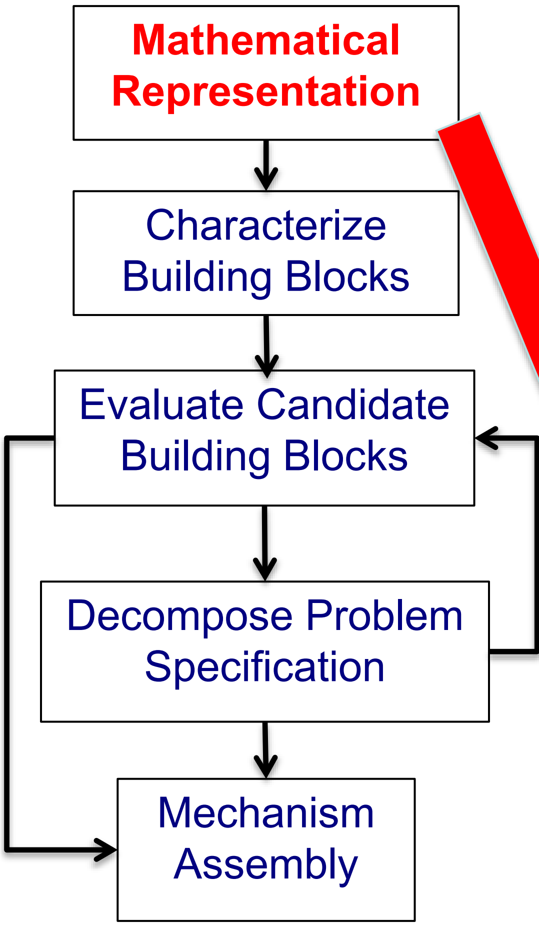
Engineering Systems: Functional characterization and decomposition into building blocks



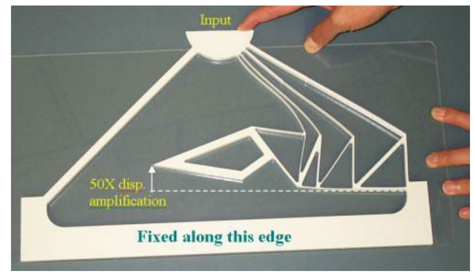


Wouldn’t it be insightful if such a decomposition strategy can be applied to compliant mechanisms?

The Building Block Method



For an elastic continuum, functional characterization of constituent members are not evident.



Saxena, Yin, and Ananthasuresh,2003

The deformation nature of each constituent member depends on the members connected to it.

Is there a fundamental building block for compliant mechanisms?

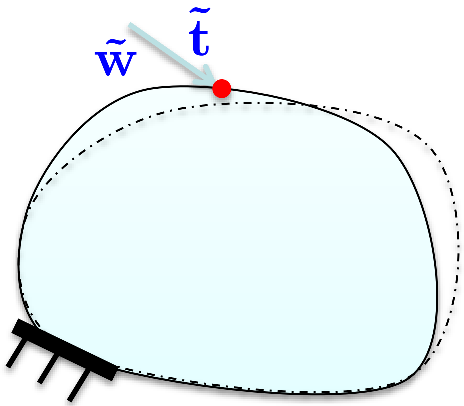
We need a new Mathematical Representation of Compliance that enables a building block approach

Representation of Compliance at a Single Port

And illustration of the building block method

An Intrinsic Geometric Representation

At the Input

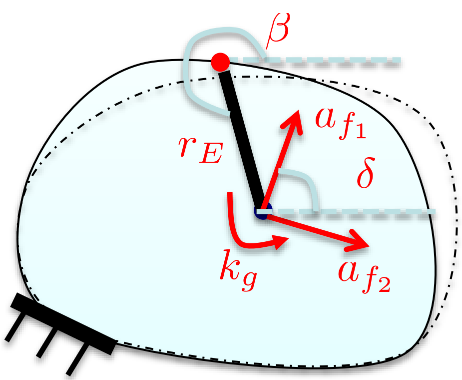


$\tilde{\mathbf{t}} = \mathbf{C}_{6\times 6}\tilde{\mathbf{w}}$

For Planar case

$\tilde{\mathbf{t}} = \mathbf{C}_{3\times 3}\tilde{\mathbf{w}}$

At the Center of Elasticity



$\mathbf{C}_{3\times 3}$

Eigen-Twist and Eigen-Wrench Parameters

Lipkin, Patterson 1992

Selectively normalizing translations…

Minimize $PE = \tilde{\mathbf{w}}^T \mathbf{C} \tilde{\mathbf{w}}$

Such that $\tilde{\mathbf{w}}^T \boldsymbol{\Gamma} \tilde{\mathbf{w}} = 1$

$=> \mathbf{C}\hat{\mathbf{w}}_{\mathbf{fi}} = a_{fi}\boldsymbol{\Gamma}\hat{\mathbf{w}}_{\mathbf{fi}} \;\; i = 1..3$

… and rotations

Minimize $PE = \tilde{\mathbf{t}}^T \mathbf{C}^{-1} \tilde{\mathbf{t}}$

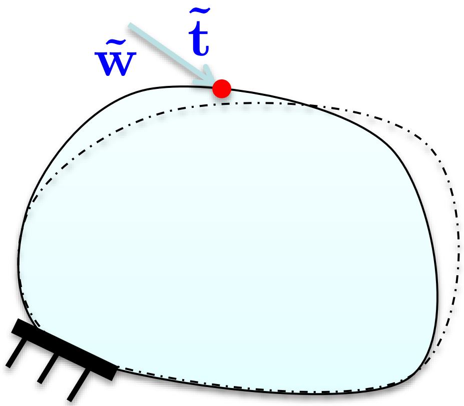
Such that $\tilde{\mathbf{t}}^T \xi \tilde{\mathbf{t}} = 1$

$=> \mathbf{C}^{-1}\hat{\mathbf{t}}_{\mathbf{gi}} = k_{gi}\xi\hat{\mathbf{t}}_{\mathbf{gi}} \;\; i = 1..3$

Six parameters: a_{f_1}, a_{f_2}, k_g
r_E, δ, β

An Intrinsic Geometric Representation

At the Input

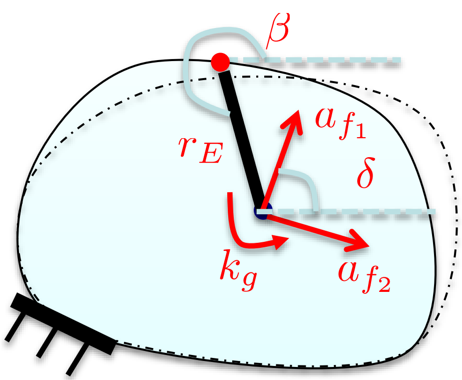


$$\tilde{\mathbf{t}} = \mathbf{C}_{6\times 6}\tilde{\mathbf{w}}$$

For Planar case

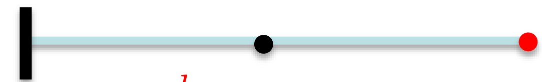
$$\tilde{\mathbf{t}} = \mathbf{C}_{3\times 3}\tilde{\mathbf{w}}$$

At the Center of Elasticity



$$\mathbf{C}_{3\times 3}$$

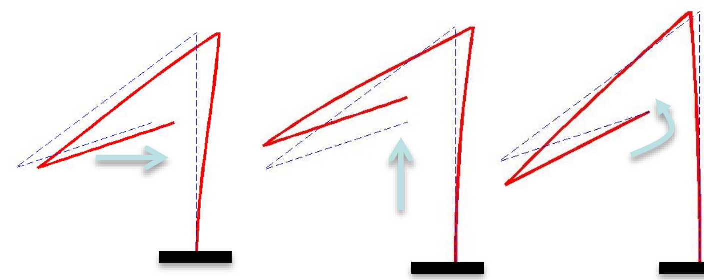
For A Beam



$$r_E = \frac{l}{2}, \delta = 0, \beta = 180^\circ$$

$$a_{f_1} = \frac{l^3}{EI}, a_{f_2} = \frac{l}{EA}, k_g = \frac{EI}{l}$$

For A Compliant Dyad



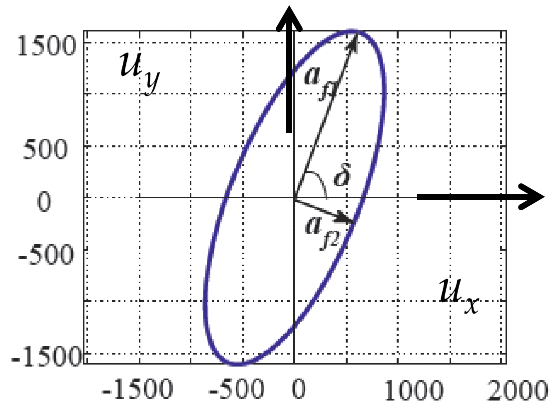
Translational and rotational compliance are decoupled at the Center of Elasticity

A Geometric Representation

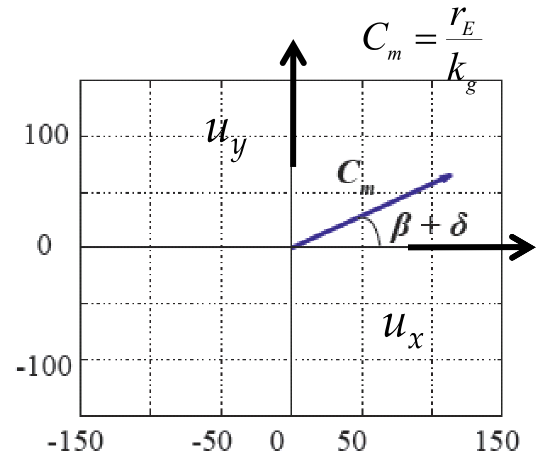
$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Series Combination: Compliance ellipses add, coupling vectors add with modifications

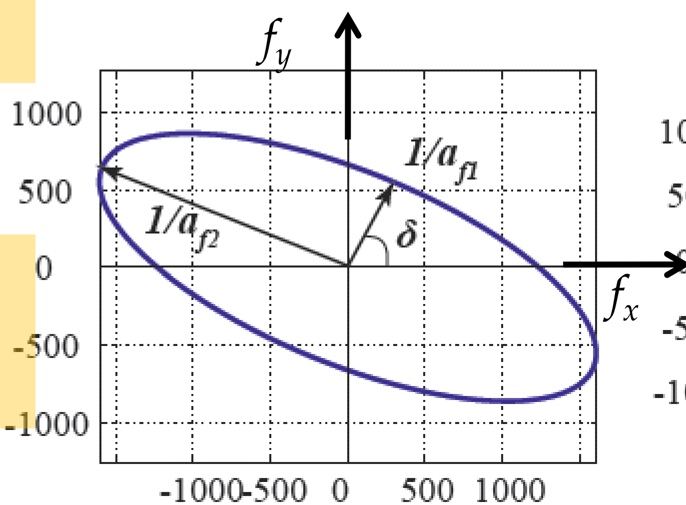
Parallel Combination: Stiffness ellipses add, coupling vectors add



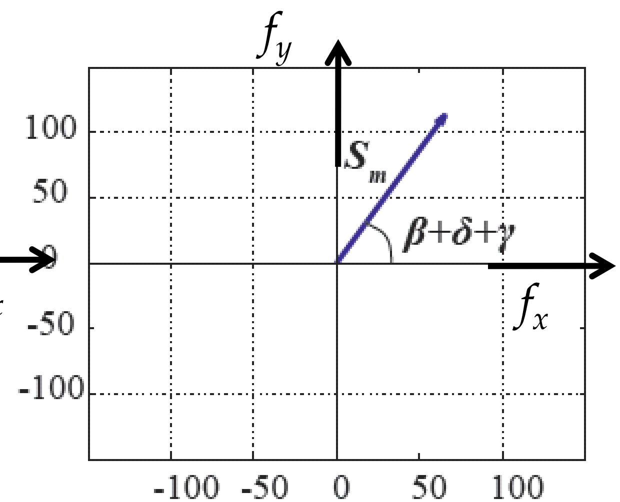
Compliance ellipse



Coupling vector

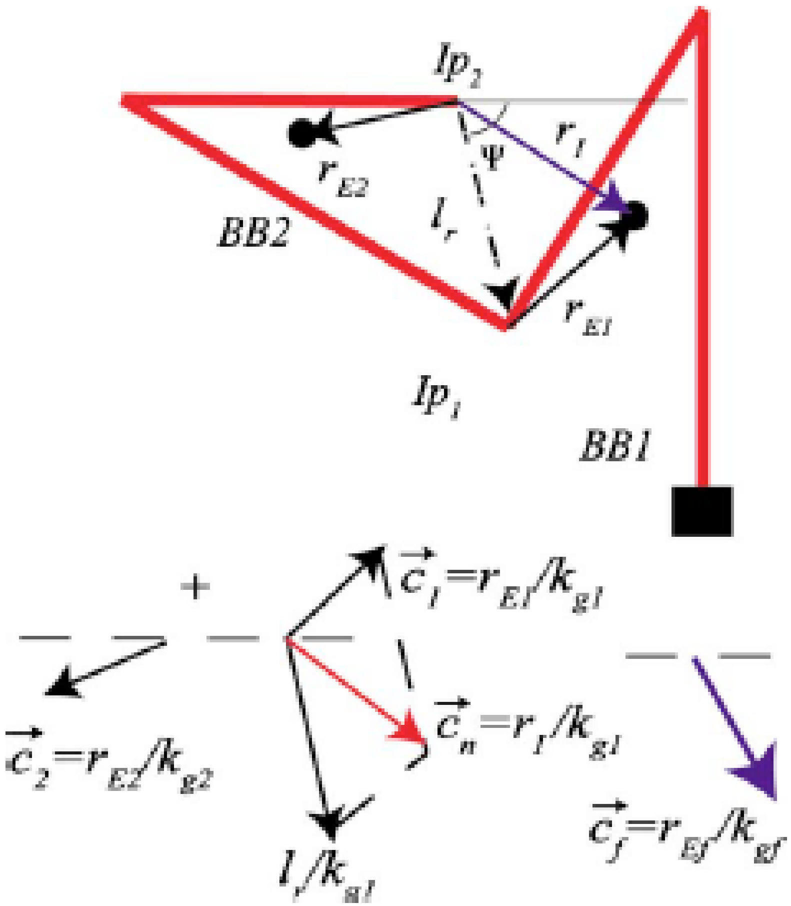


Stiffness ellipse



Stiffness coupling vector

Concatenation… computationally.



$$\mathbf{C}_{\text{resultant}} = \mathbf{C}_{\text{BB2}} + \mathbf{T}^T \mathbf{C}_{\text{BB1}} \mathbf{T}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_r \sin\psi & l_r \cos\psi & 1 \end{bmatrix}$$

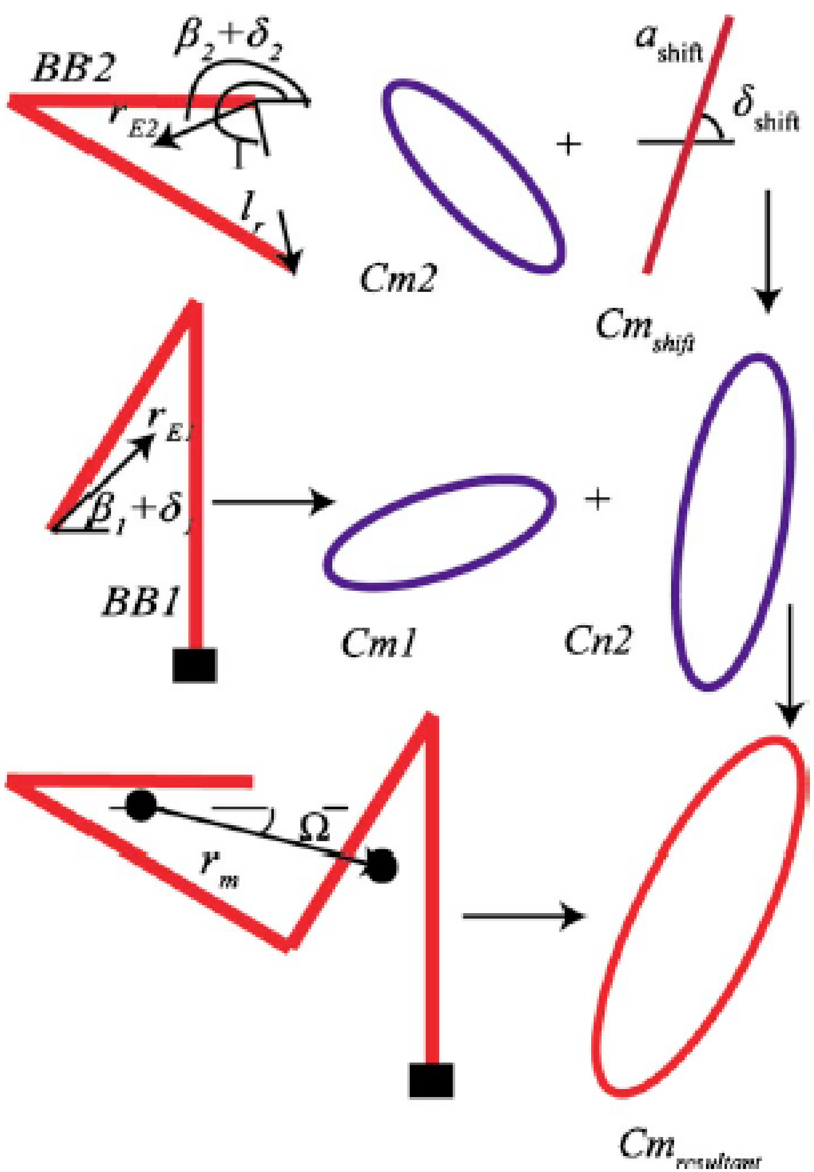
$$\frac{1}{k_{g_f}} = \frac{1}{k_{g_1}} + \frac{1}{k_{g_2}}$$

$$c_f = \left(c_1 + \frac{l_r}{k_{g_1}} \right) + c_2$$

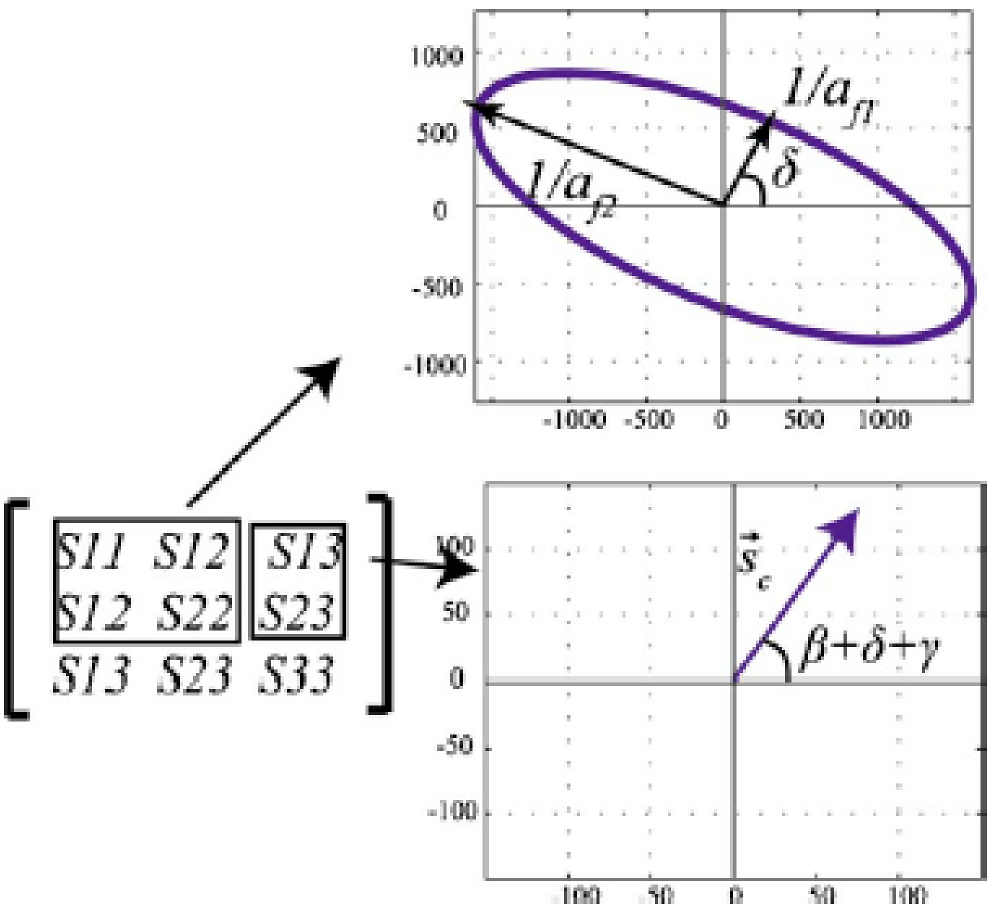
$$c_f = \frac{r_1}{k_{g_1}} + \frac{r_{E_2}}{k_{g_2}} = c_n + c_2$$

$$r_{Ef} = \frac{|c_f|}{k_{g_f}}$$

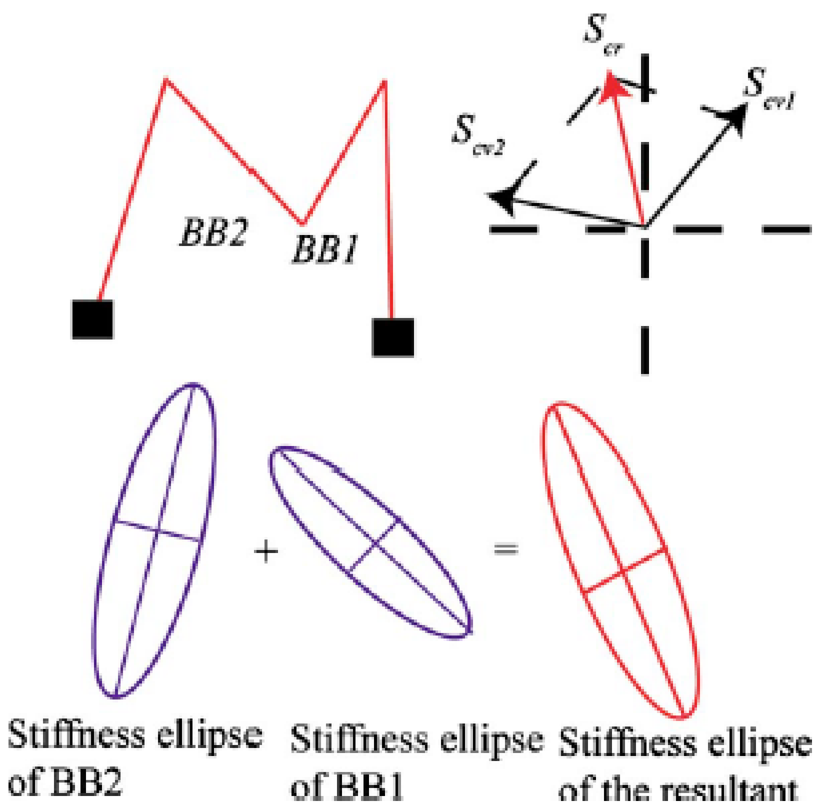
Concatenation... visually



Stiffness ellipsoids



Parallel concatenation

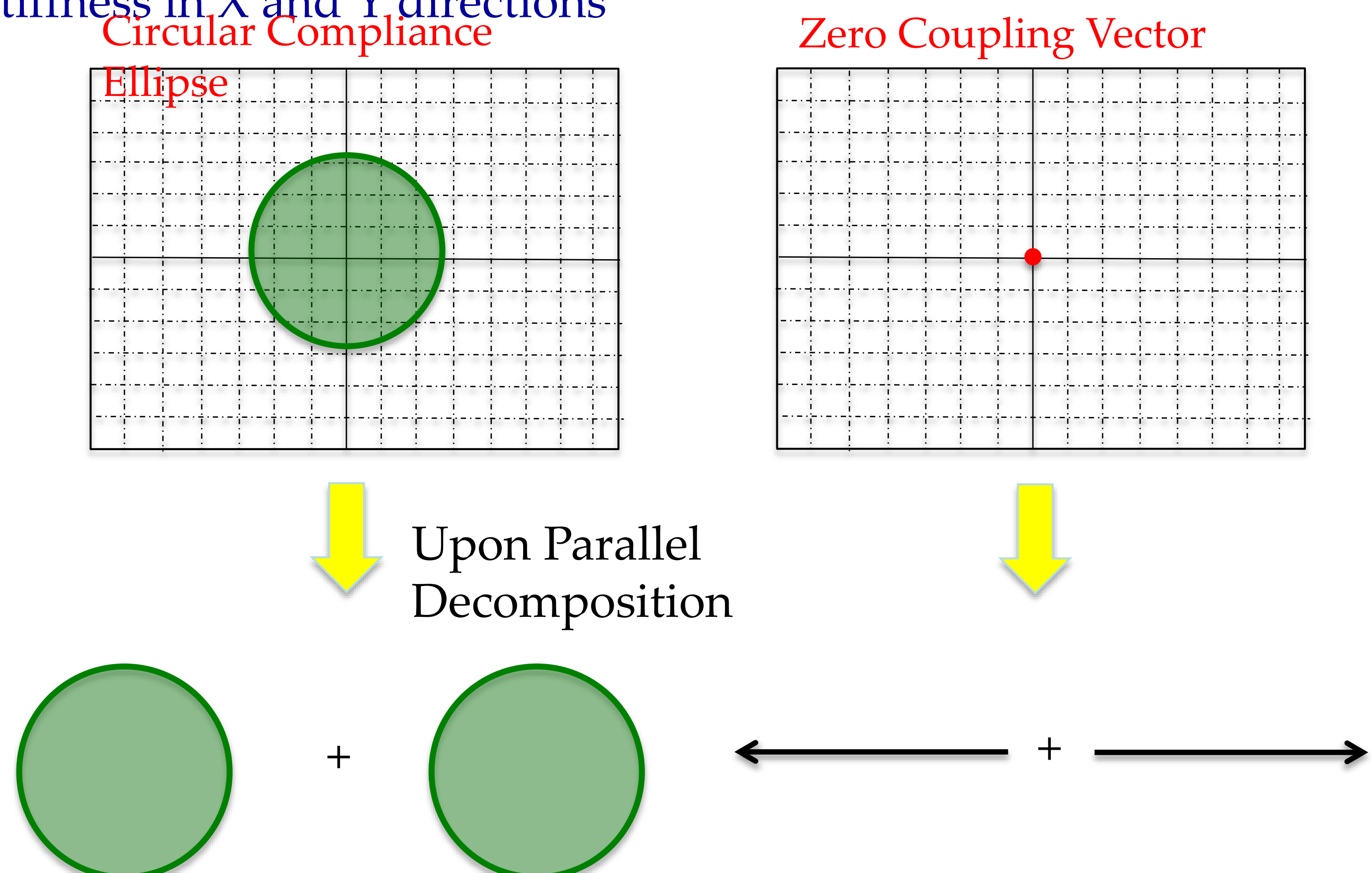


SYNTHESIS WITH BUILDING BLOCKS

Problem Restatement

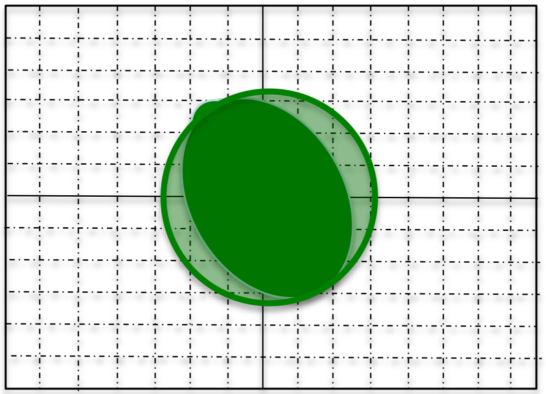
<u>Vision Based Force Design Goal:</u>

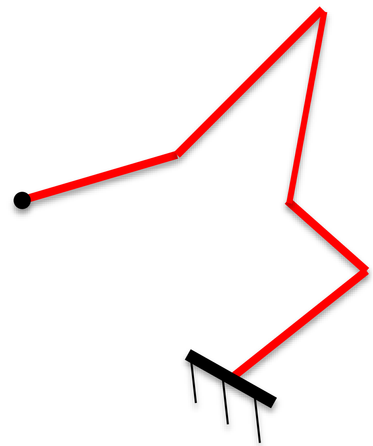
1. Design a compliant suspension with decoupled translations and rotations in the XY-plane
2. Equal stiffness in X and Y directions

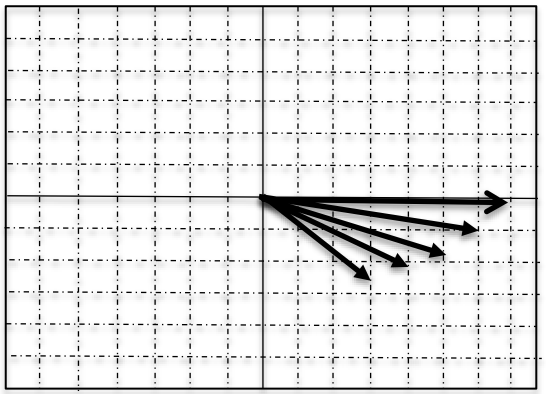


Problem Decomposition

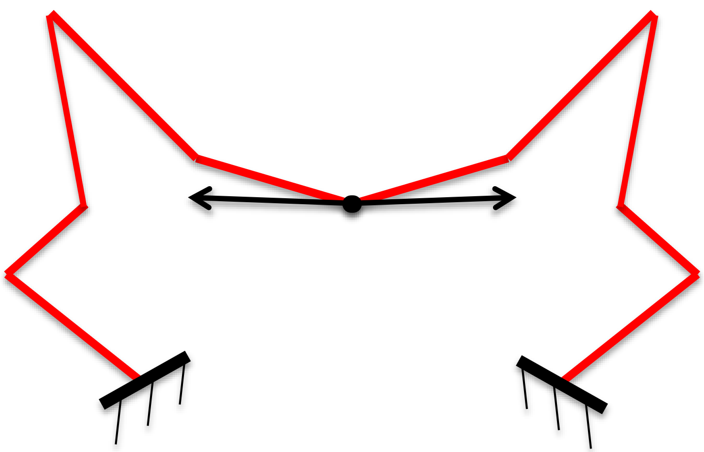
Symmetric half design by series decomposition



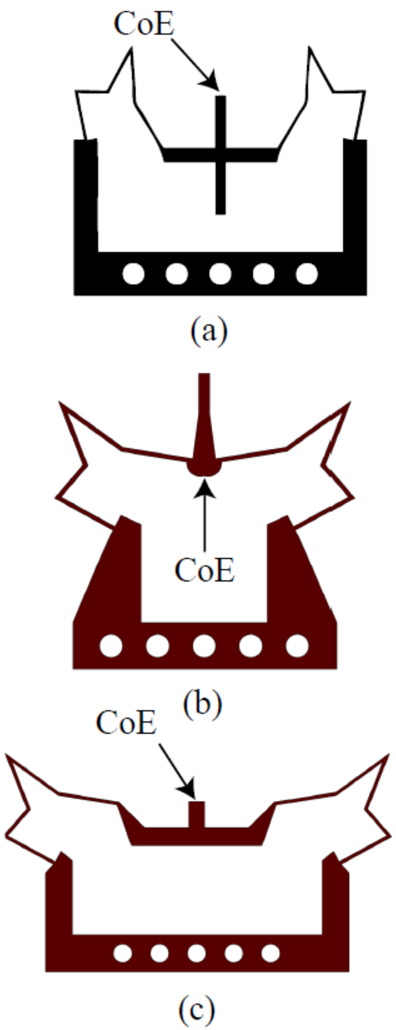


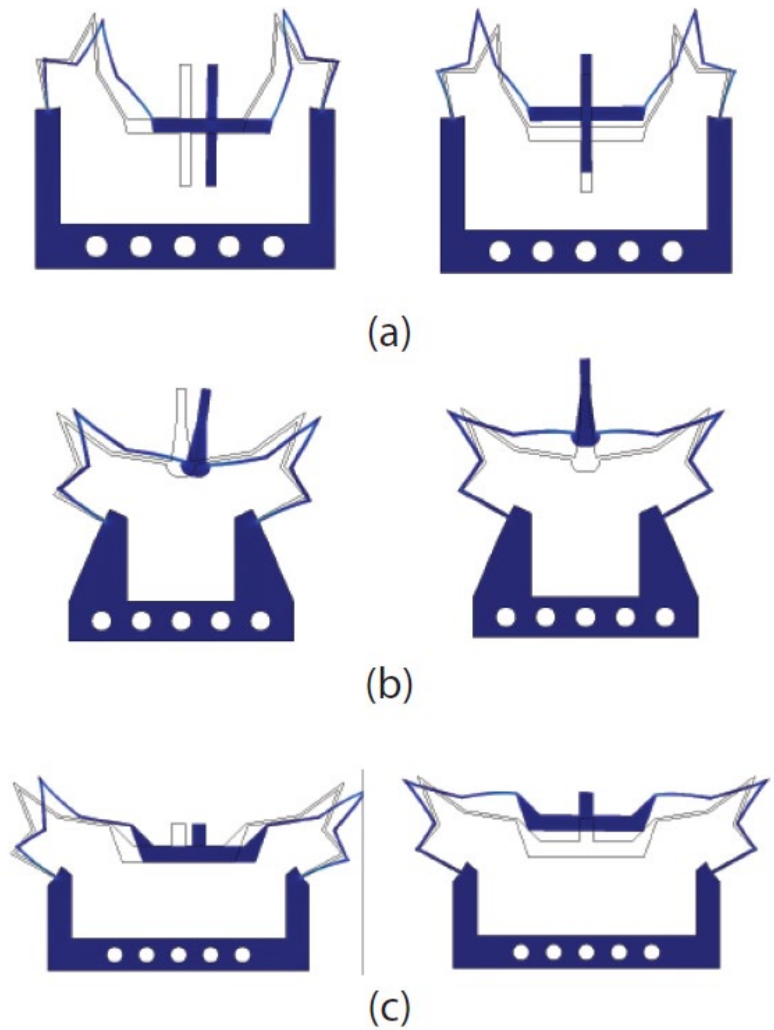


Final Mechanism Assembly



Force Sensor Solution





Main points

- A mathematical representation of lumped stiffness of a compliant dyad.
- Computational concatenation of compliant dyads.
- Synthesis (intuition-based)

Further reading

ASME Journal of Mechanisms and Robotics, February 2011.

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