ME 254

Empirical formulae for elastic pairs (flexure joints)

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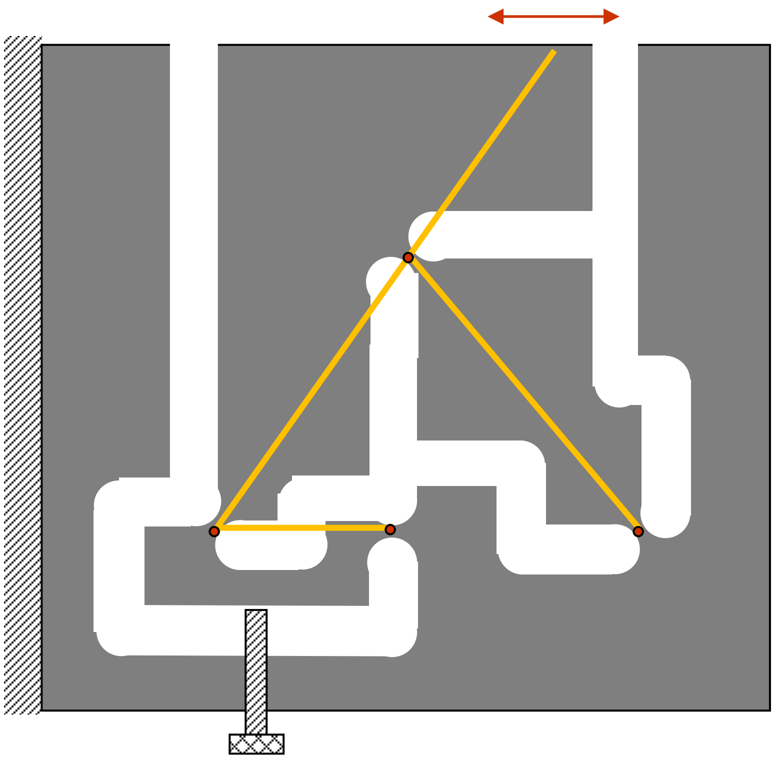
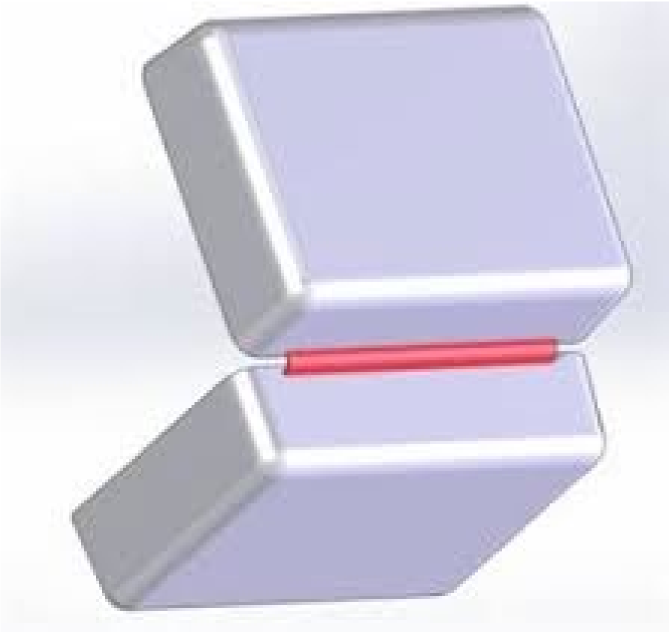
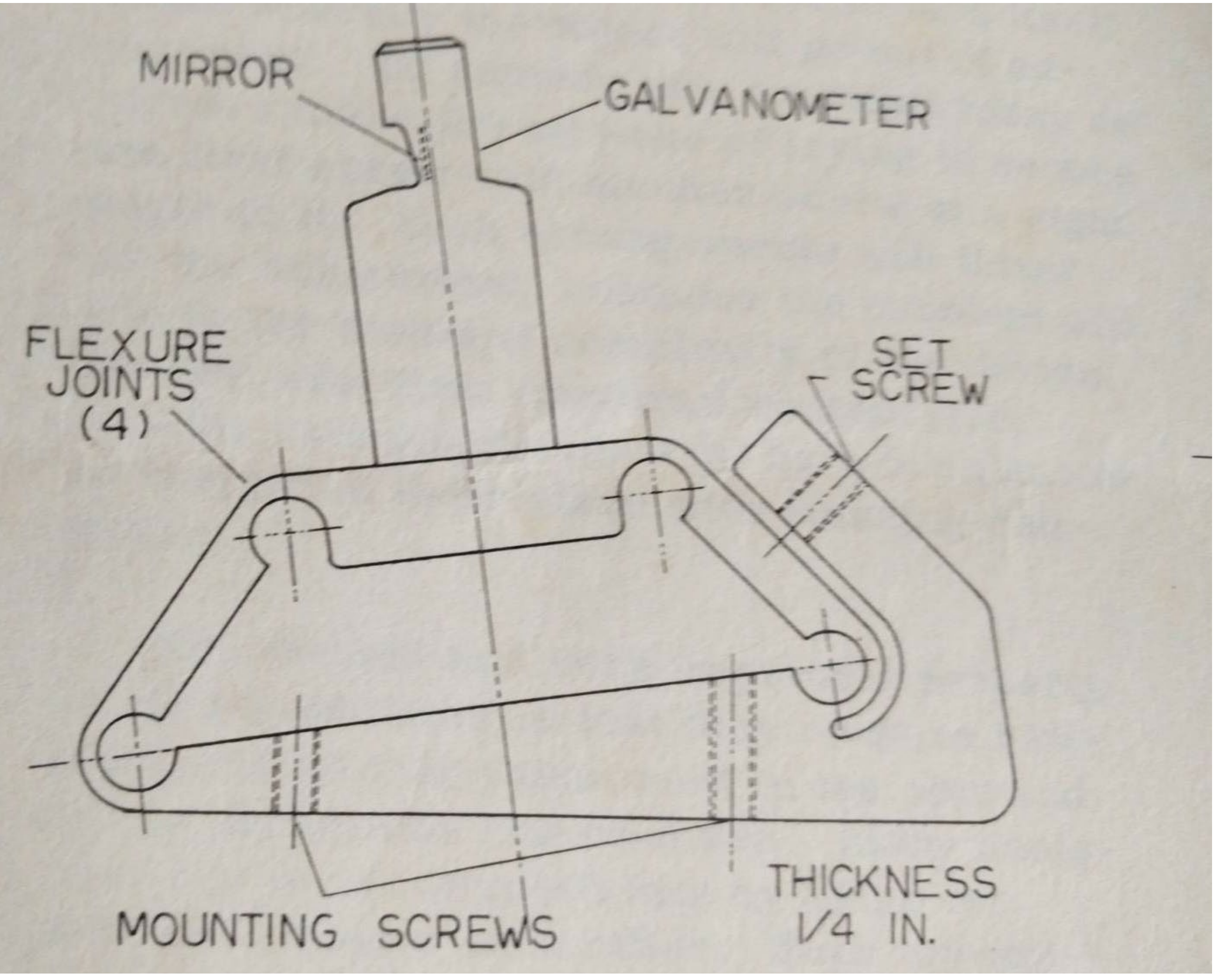
Discrete compliance



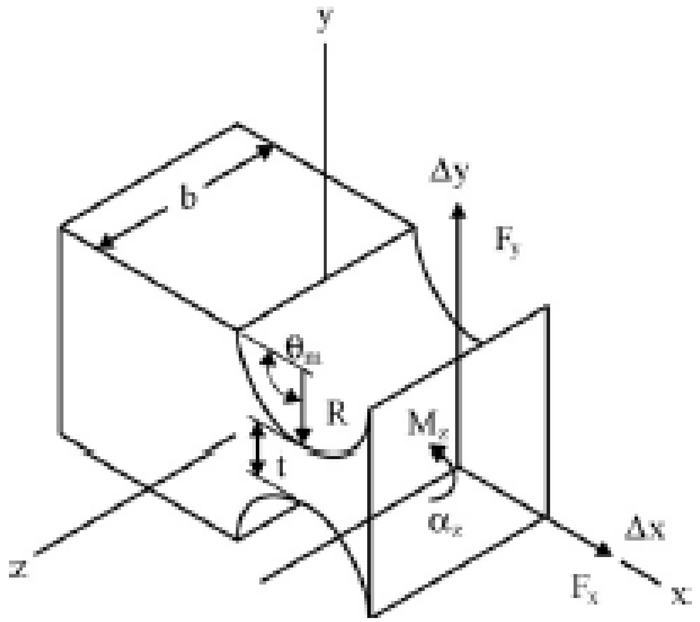
Living hinge



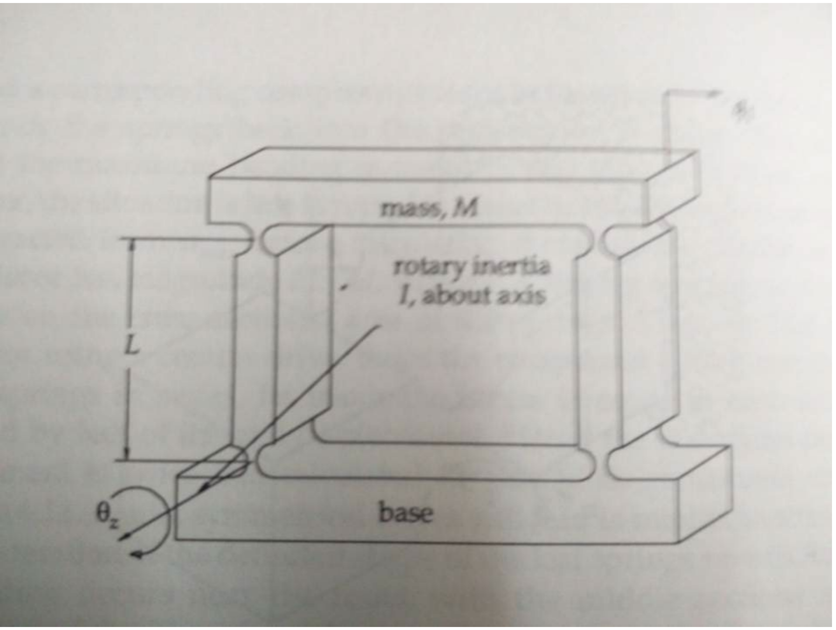
Towfigh's adjustment mechanism



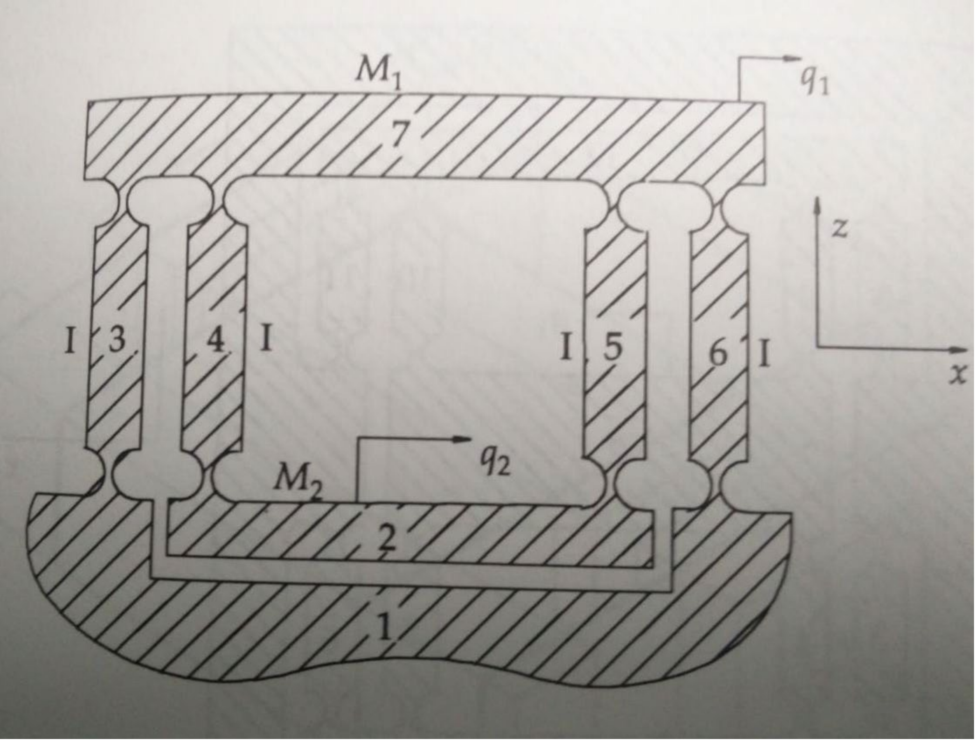
Circular-notch elastic pair



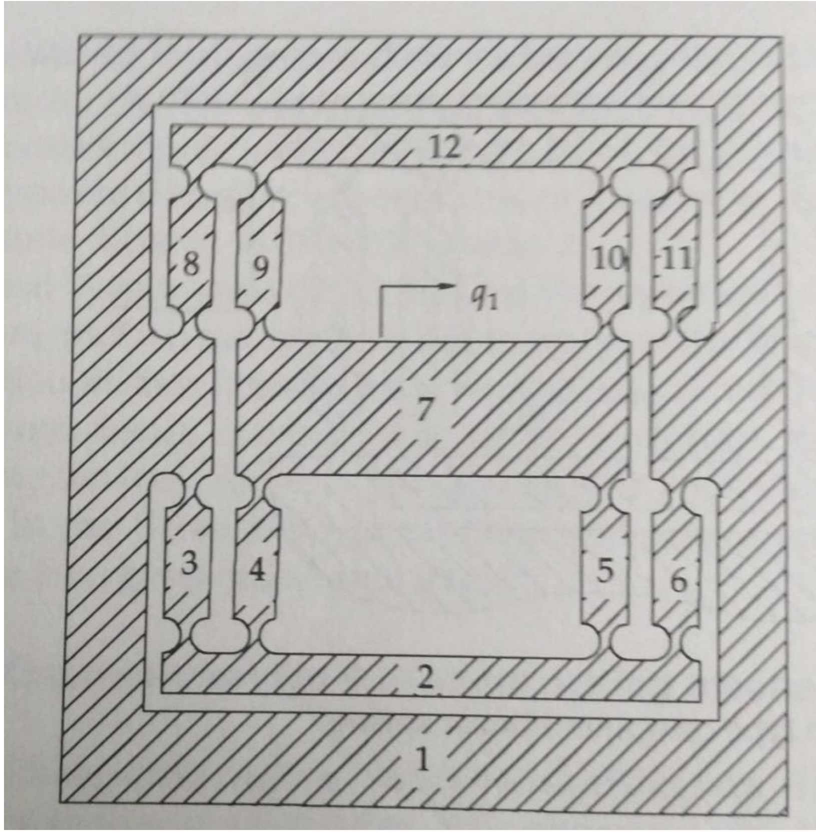
Parallel-motion flexure mechanism



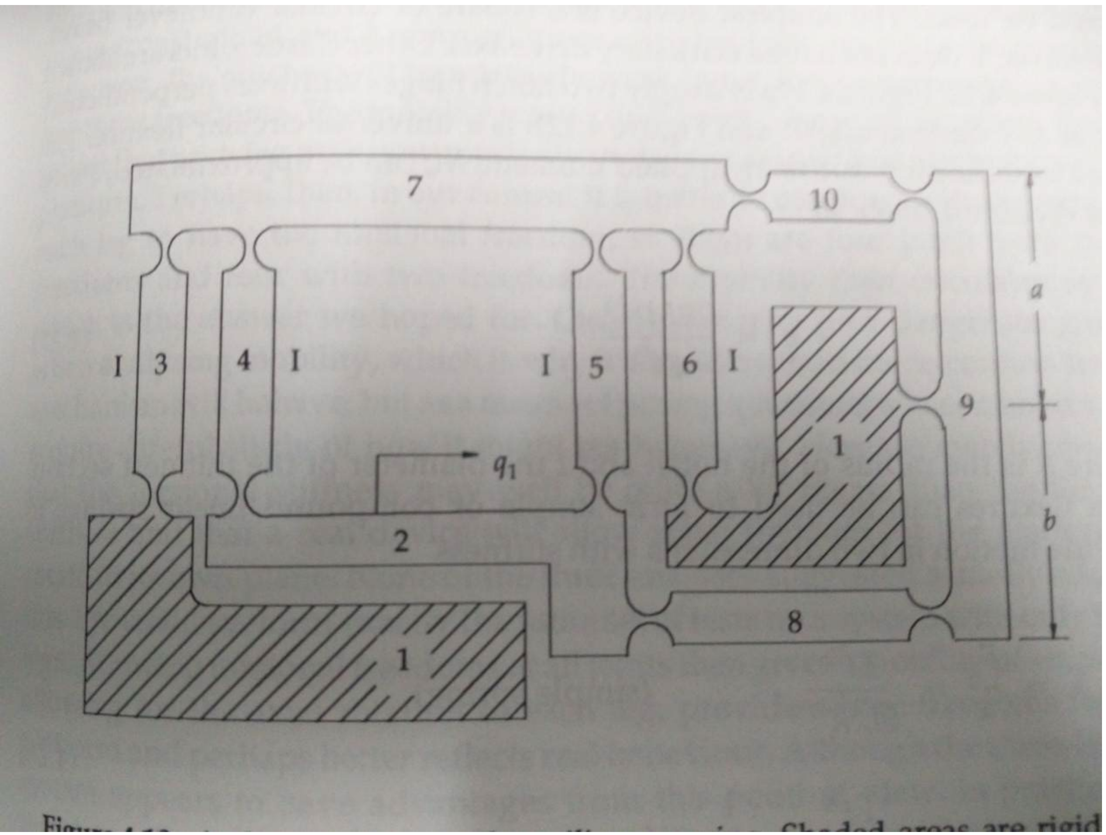
Folded flexure mechanism



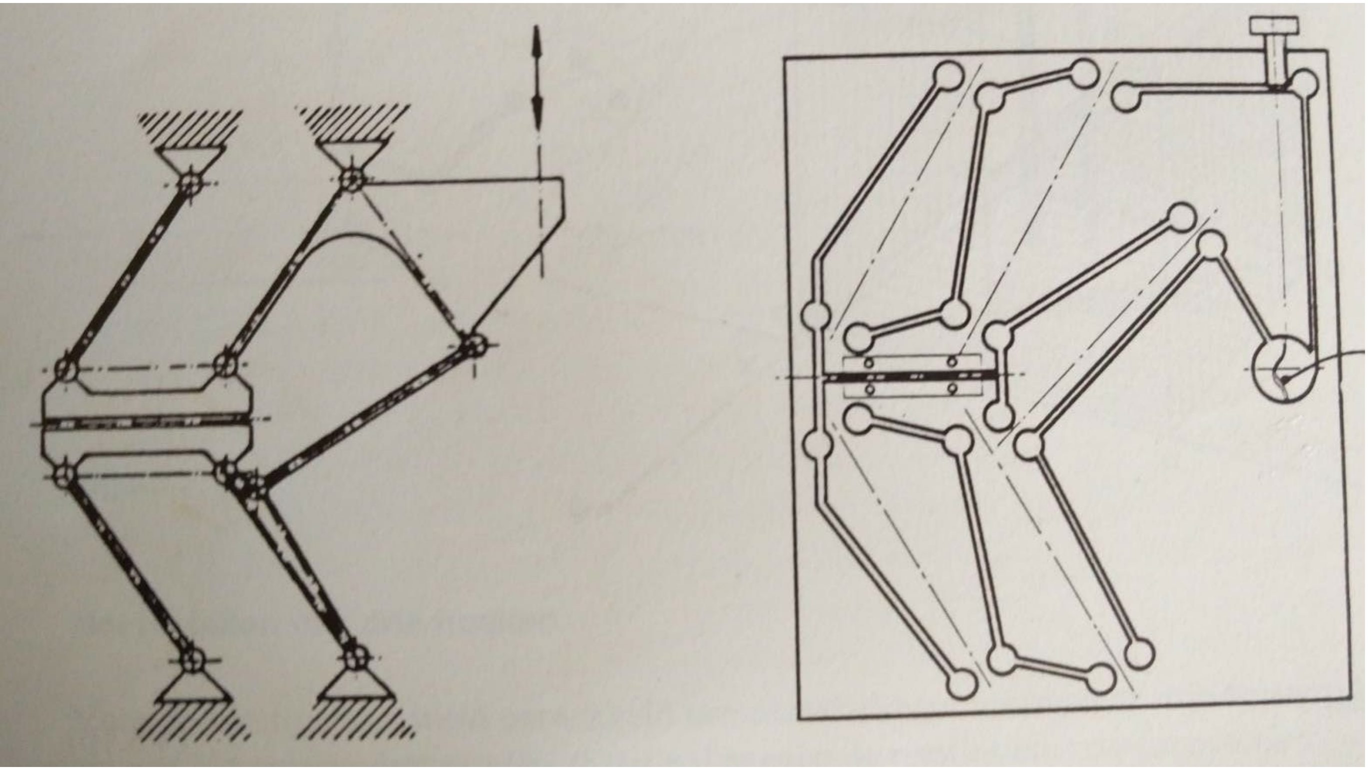
Complete folded flexure mechanism



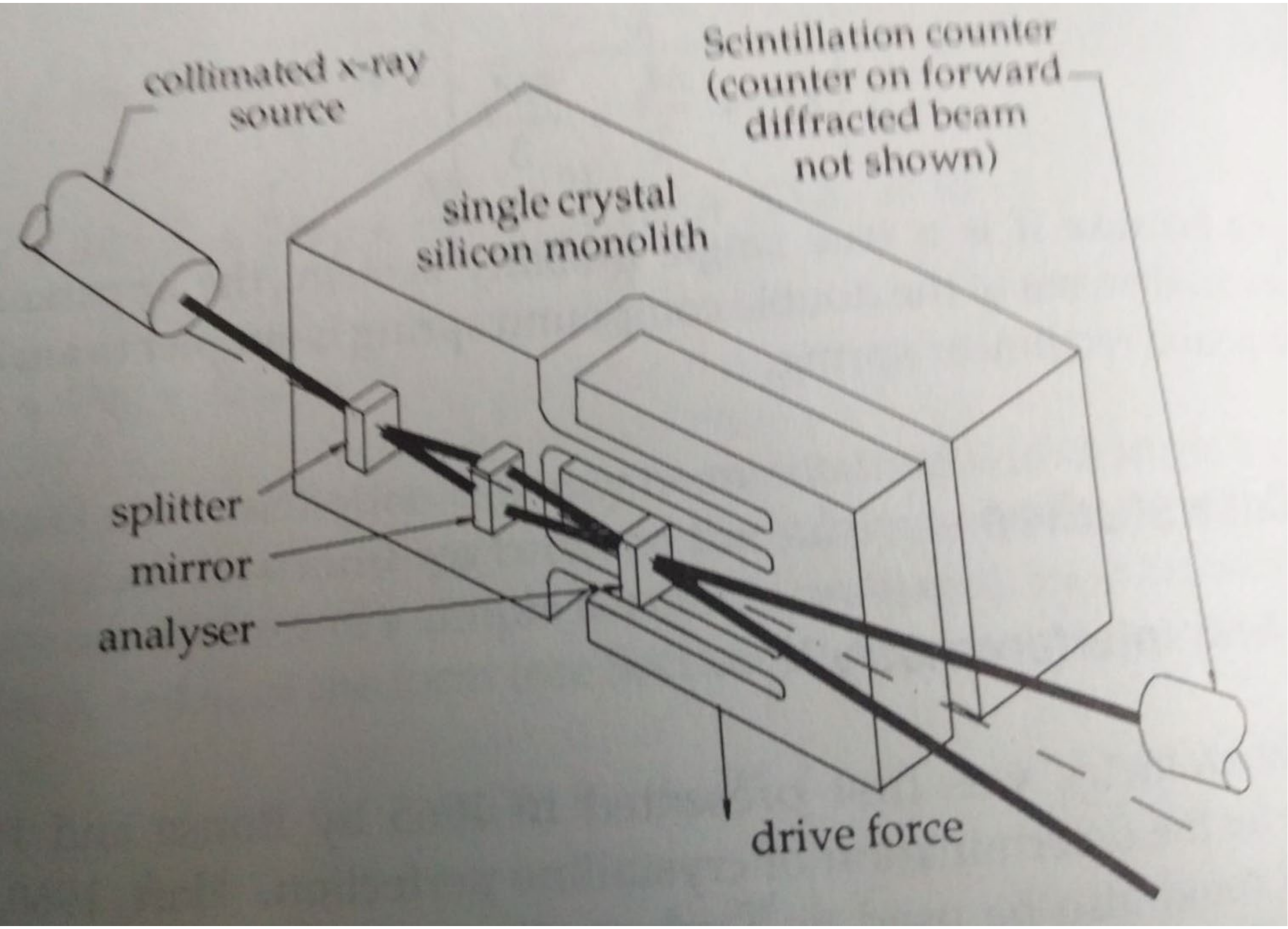
Folded flexure with a lever



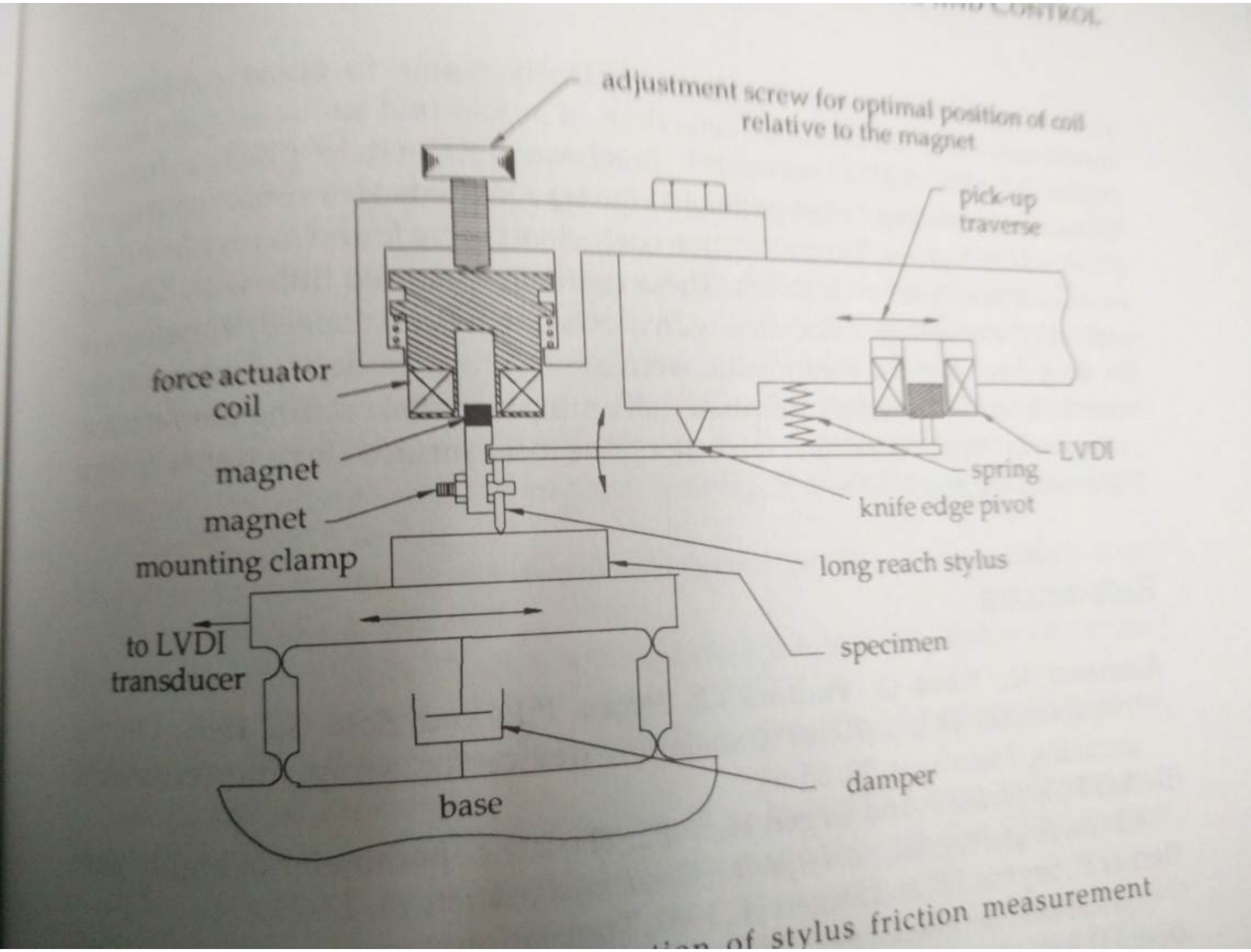
Rigid-body mechanism → compliant mechanism



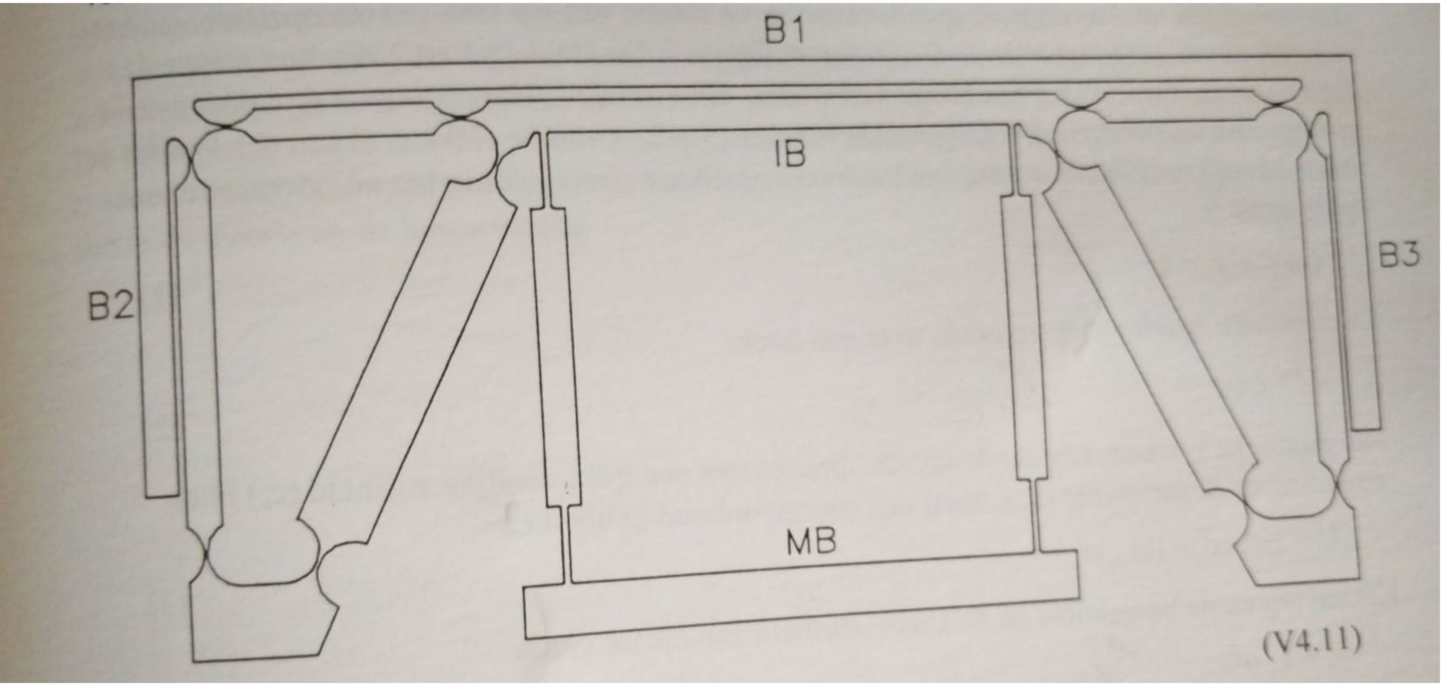
A precision instrument



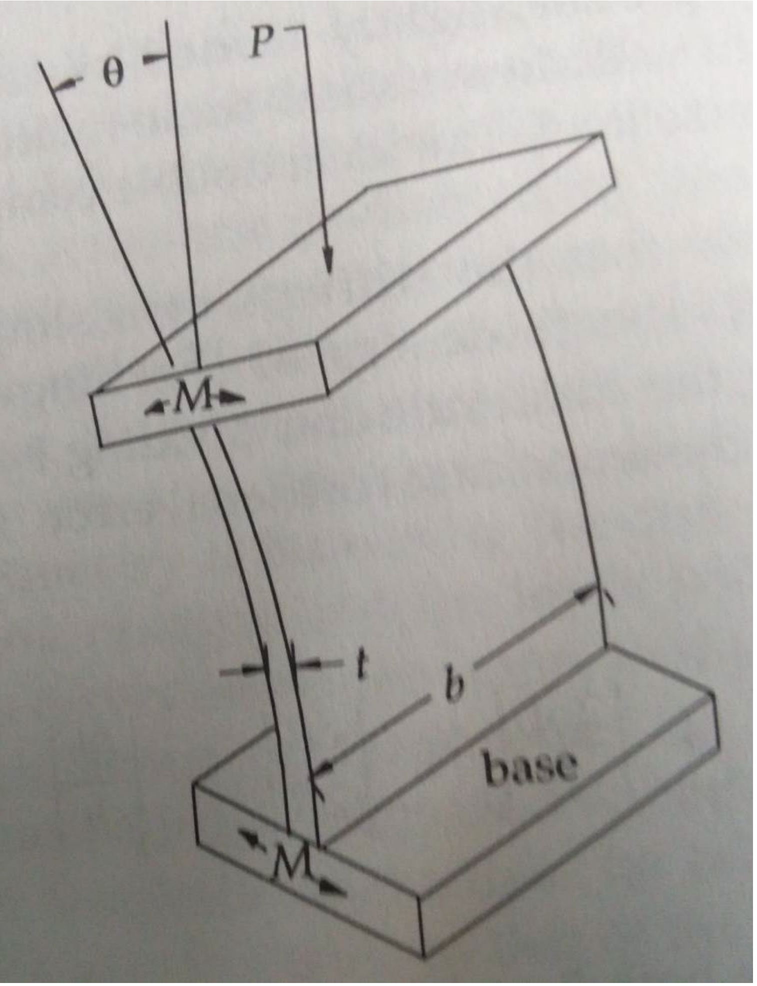
A precision mechanism



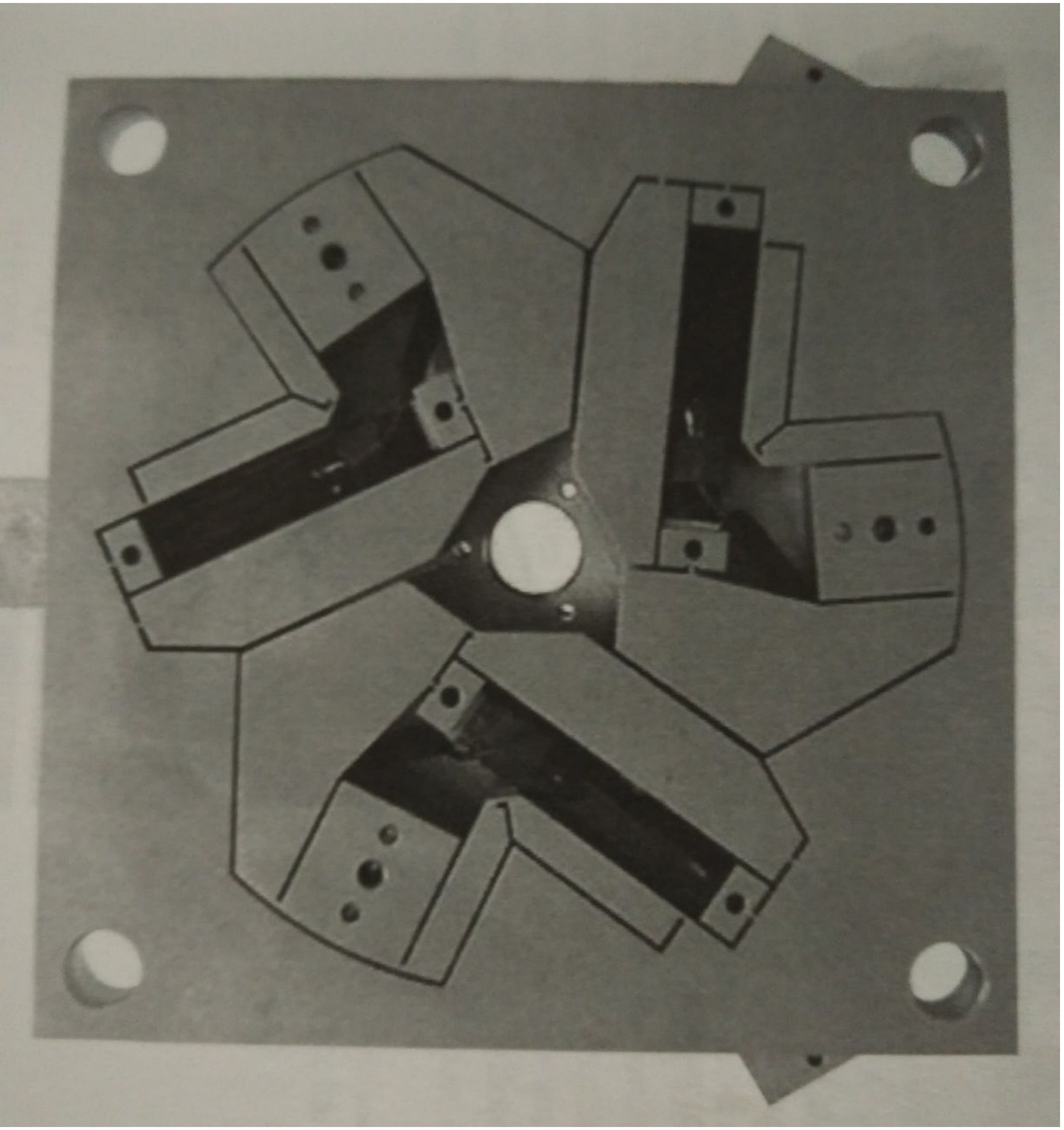
A precision flexure mechanism



A small-length rotational elastic pair



A precision motion stage



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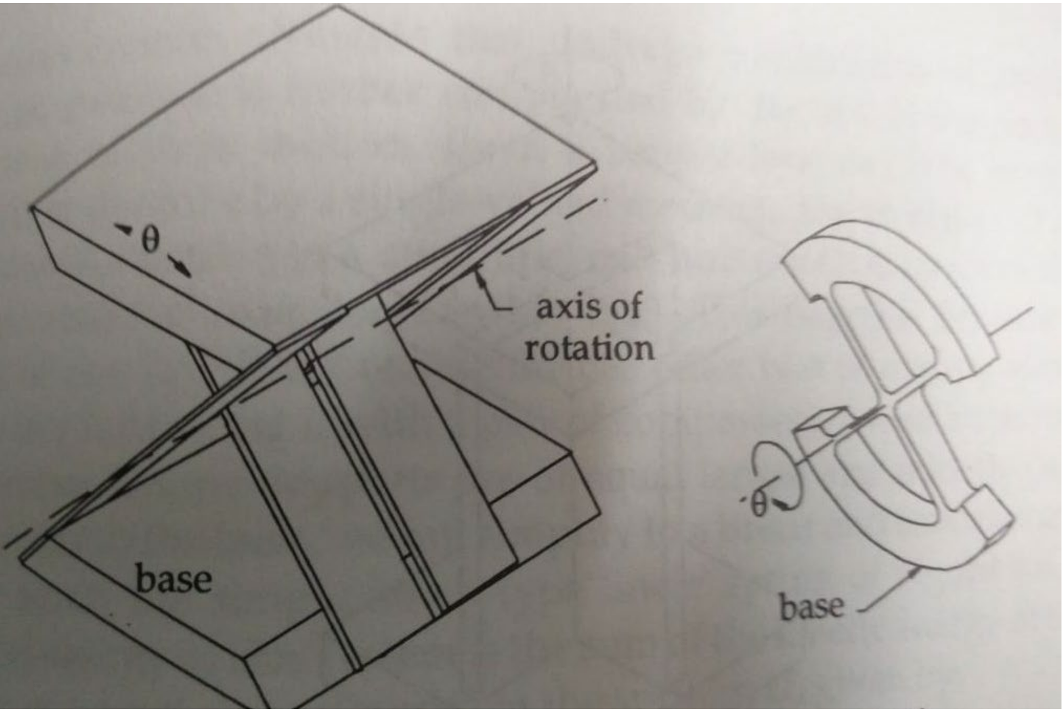
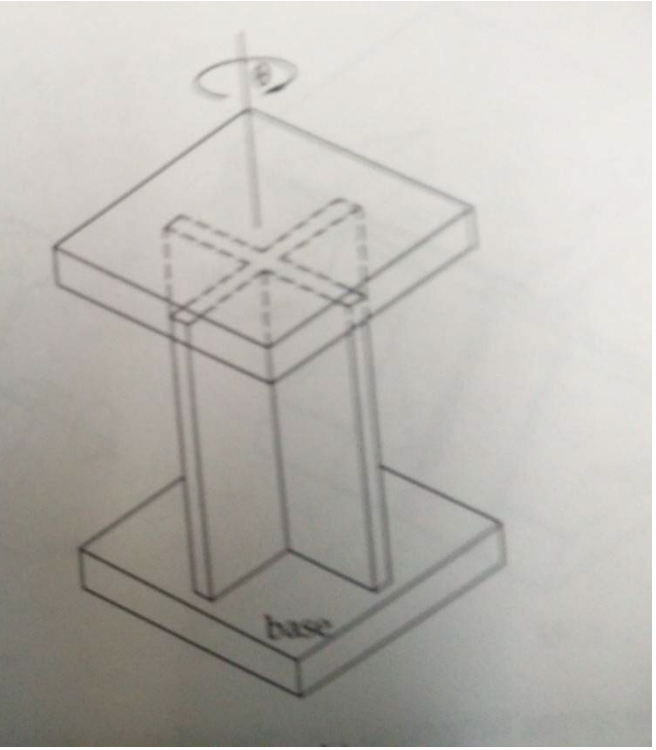
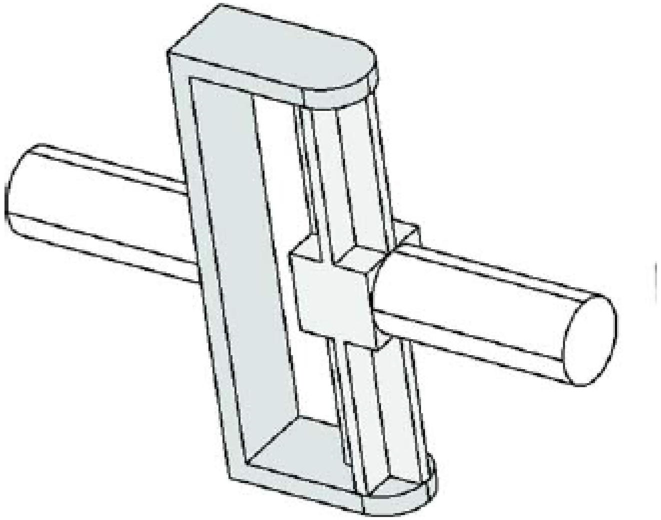
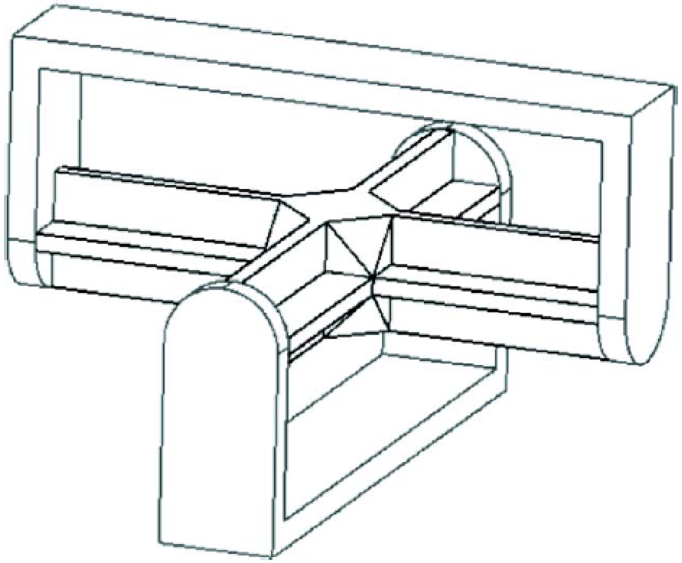
Cross-strip flexure



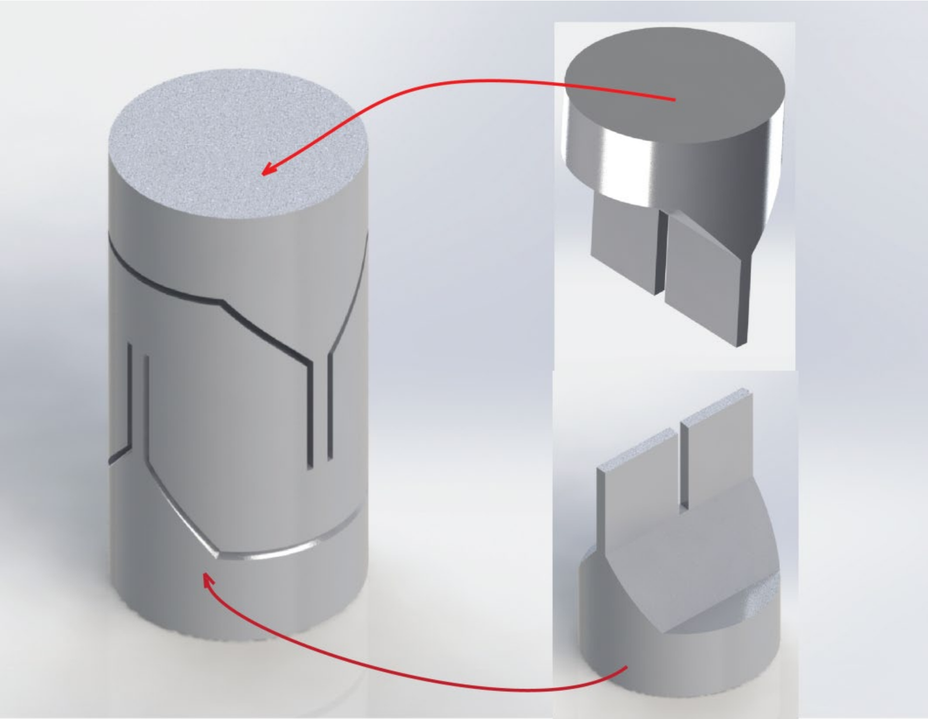
Cruciform flexure

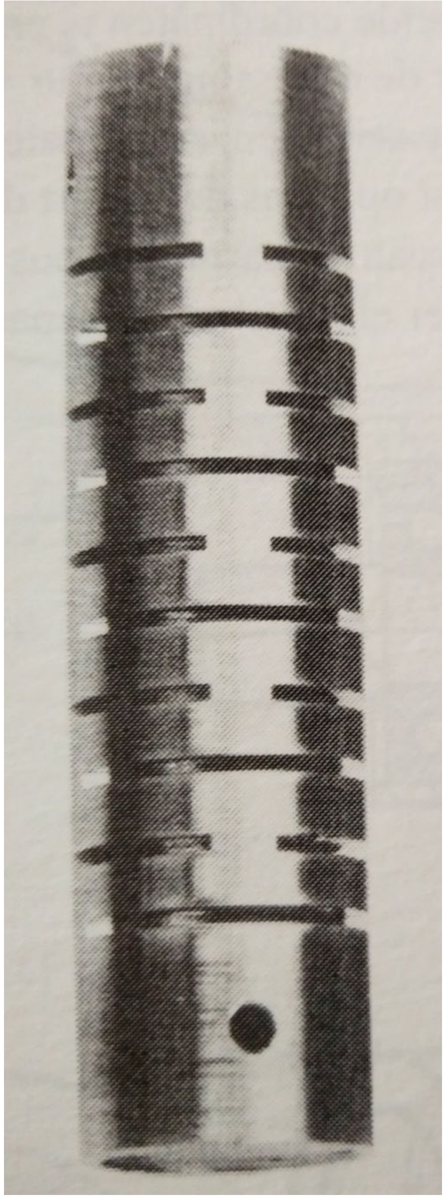




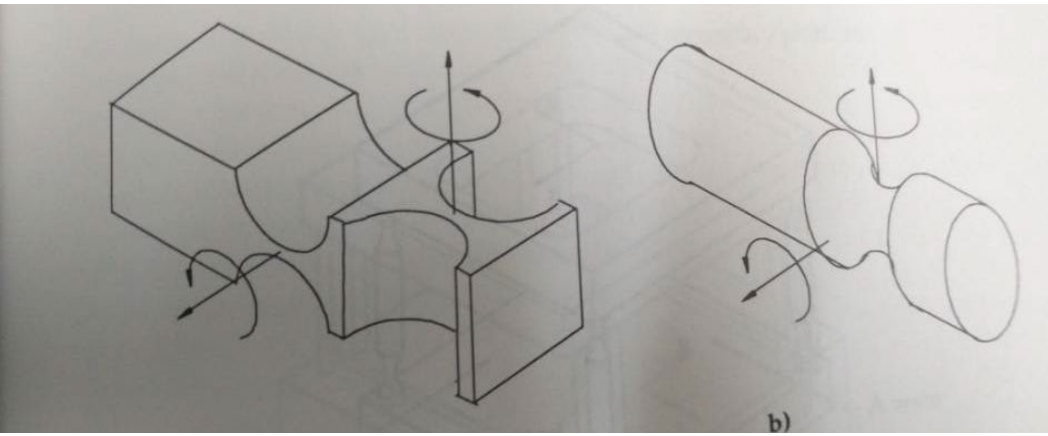


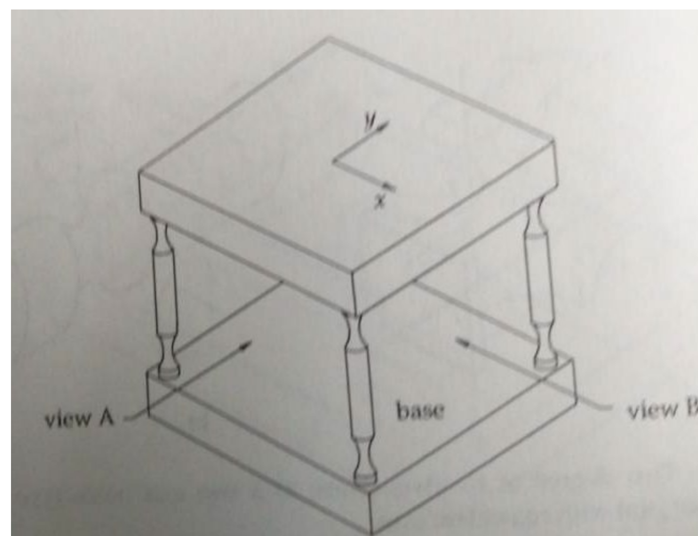
3D flexure





3D flexures and their use





Paros-Weisbord (simplified)

$$\frac{\alpha_z}{M_z} = \frac{9\pi R^{1/2}}{2Ebt^{5/2}}$$

$$\frac{\Delta y}{F_y} = \frac{9\pi}{2Eb}\left(\frac{R}{t}\right)^{5/2}$$

$$\frac{\Delta x}{F_x} = \frac{1}{Eb}[\pi(R/t)^{1/2} - 2.57]$$

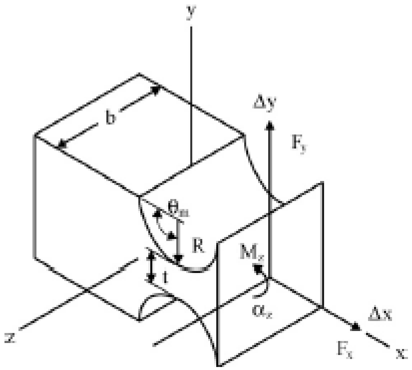
Lobontiu; Wu and Zhou

$$\frac{\alpha_z}{M_z} = \frac{24R}{Ebt^3(2R+t)(4R+t)^3}\left[t(4R+t)(6R^2+4Rt+t^2)\right.$$
$$\left.+6R(2R+t)^2\sqrt{t(4R+t)}\arctan\left(\sqrt{1+\frac{4R}{t}}\right)\right]$$

$$s = R/t$$

$$\frac{\alpha_z}{M_z} = \frac{12}{EbR^2}\left[\frac{2s^3(6s^2+4s+1)}{(2s+1)(4s+1)^2}\right.$$
$$\left.+\frac{12s^4(2s+1)}{(4s+1)^{5/2}}\arctan\sqrt{4s+1}\right]$$

Circular-notch elastic pair



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Review

Review of circular flexure hinge design equations and derivation of empirical formulations

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School of Mechanical Engineering, The University of Adelaide, SA 5005, Australia

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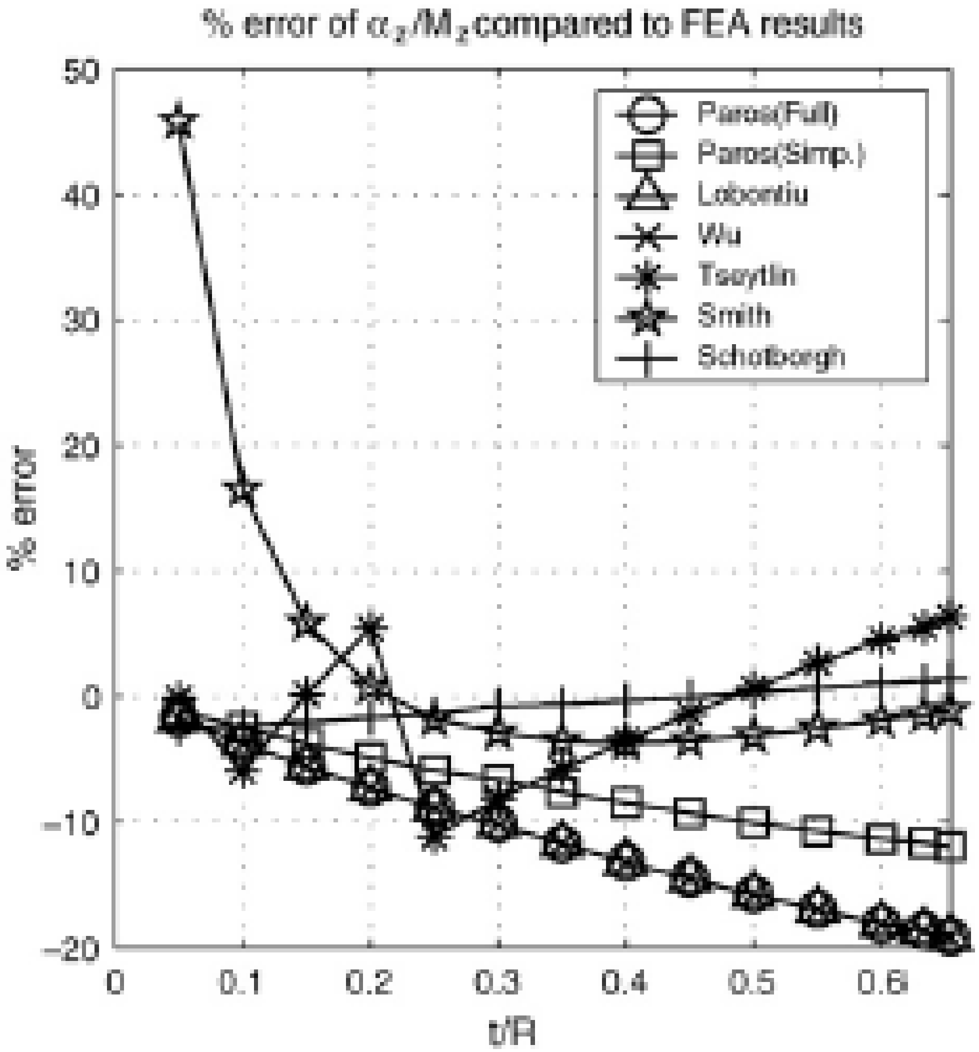
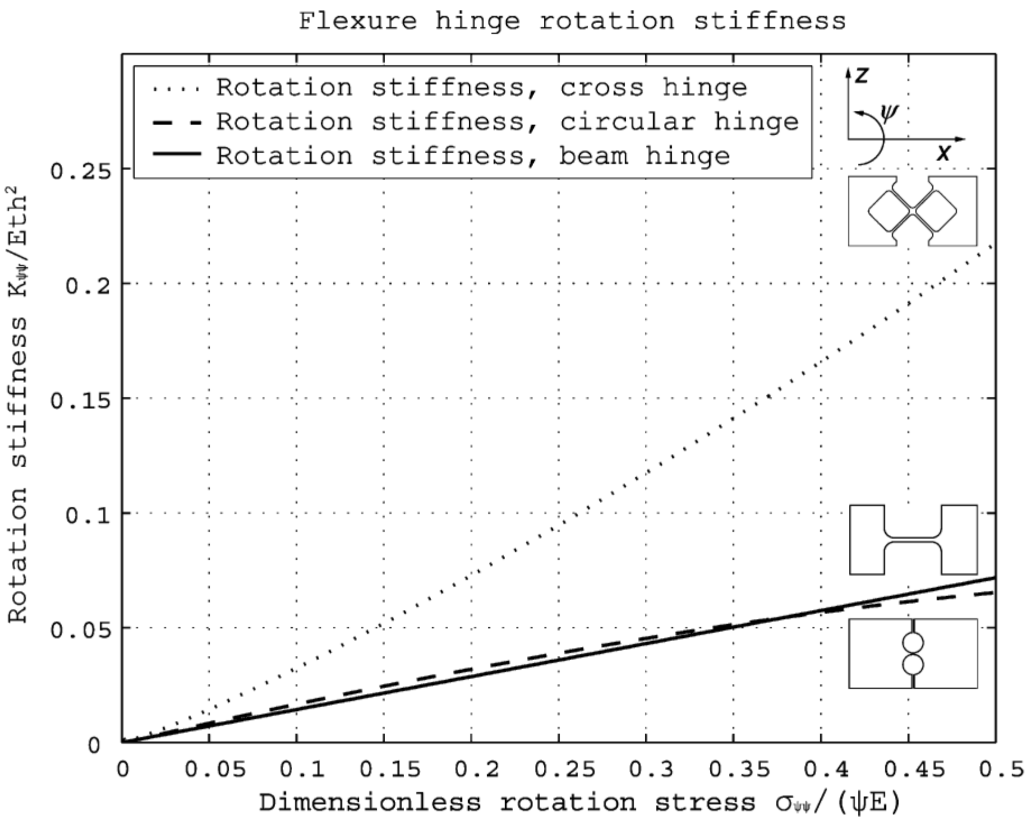
Comparative review





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Dimensionless design graphs for flexure elements and a comparison between three flexure elements

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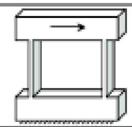
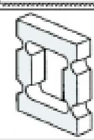
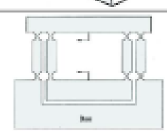
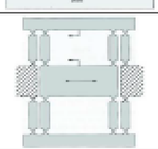
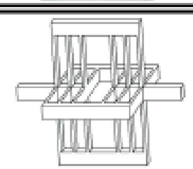
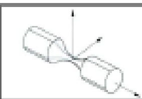
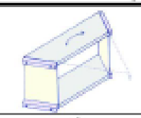
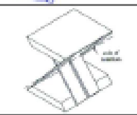
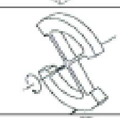
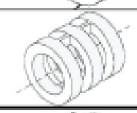
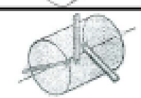
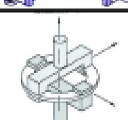
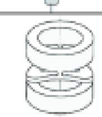
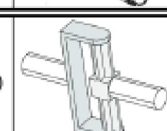
Received 16 May 2003; received in revised form 31 March 2004; accepted 21 April 2004
Available online 19 June 2004

Ananthasu

Comparison

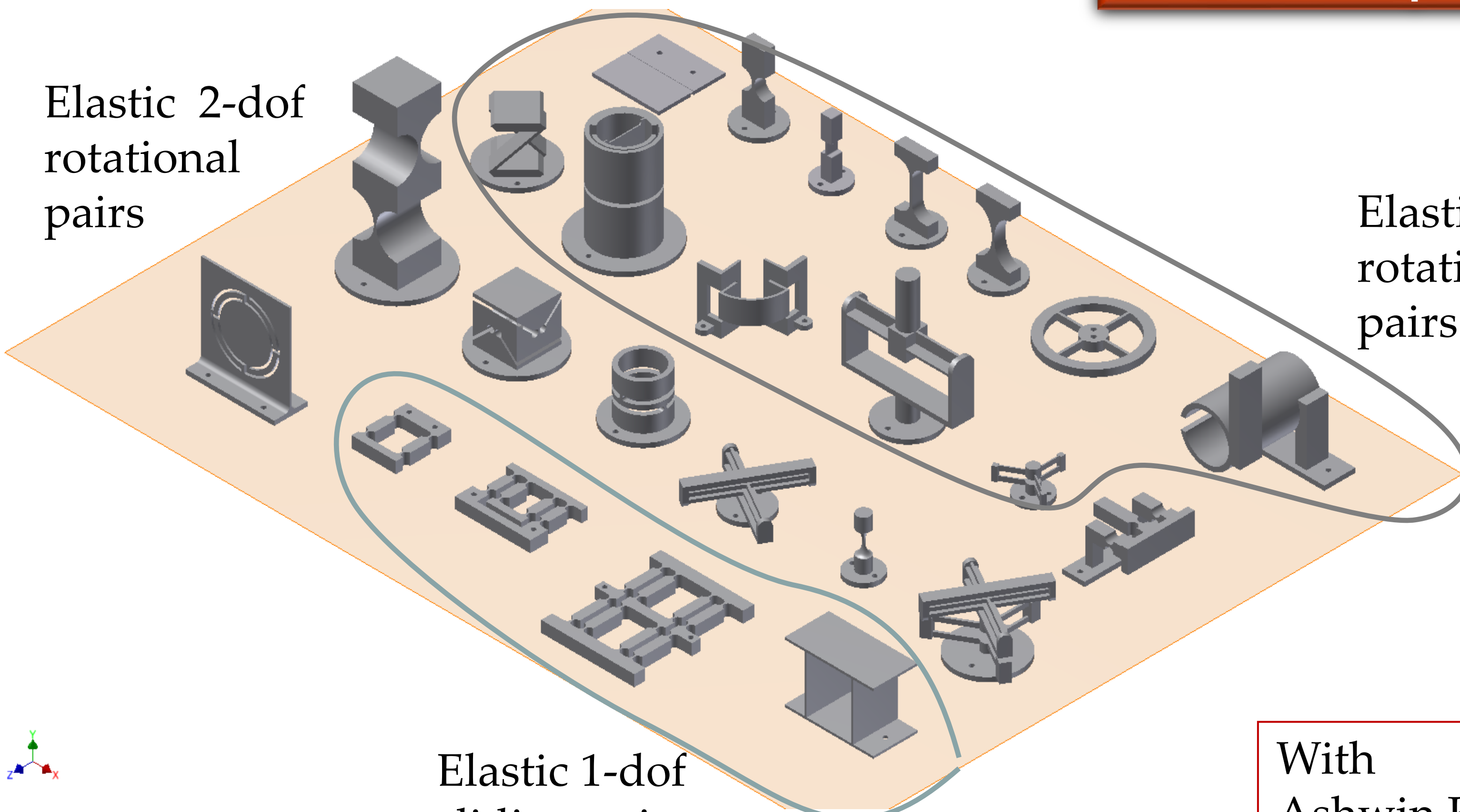
Table 1 Benchmarked flexible translational joints (–: poor, 0: normal; +: good)

		Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
(a)		0	–	0	0	+
(b)		–	–	–	0	+
(c)		–	0	–	0	+
(d)		–	+	–	0	+
(e)		+	+	+	+	+

		Range of Motion	Axis Drift	Stress Concentration	Off-Axis Stiffness	Compactness
(a)		–	–	–	–	+
(b)		0	–	+	–	0
(c)		+	–	+	–	–
(d)		–	–	0	–	+
(e)		–	0	–	0	0
(f)		–	+	0	–	–
(g)		+	+	+	–	–
(h)		–	+	–	–	–
(i)		–	0	–	–	0
(j)		+	0	+	+	0
(k)		+	+	+	+	0

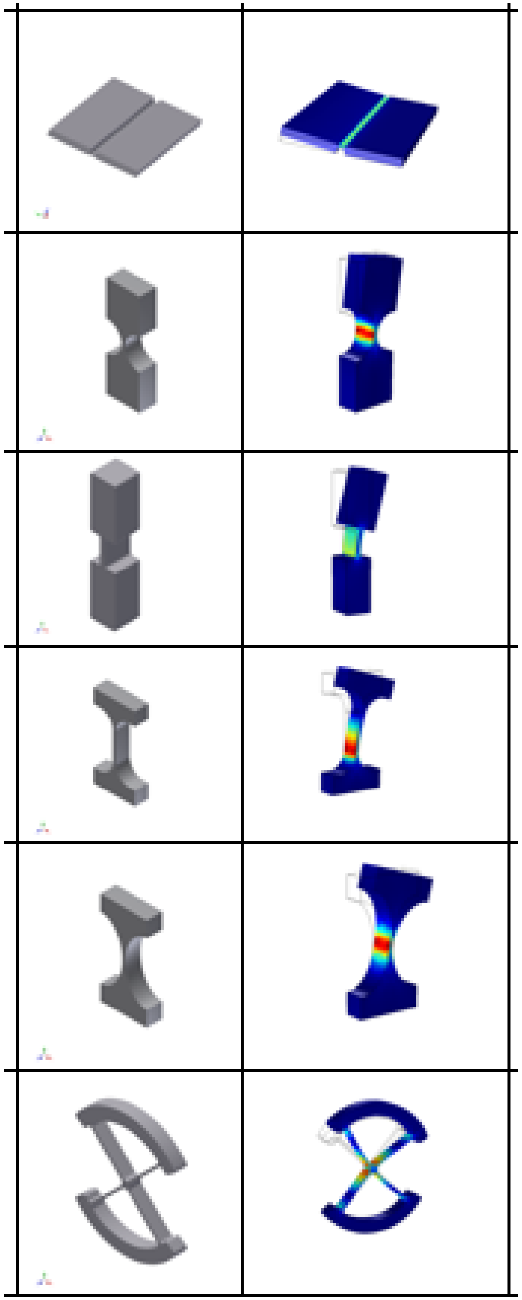
Elastic pairs

No specific shape



With
Ashwin Rao
Santosh Bhargav

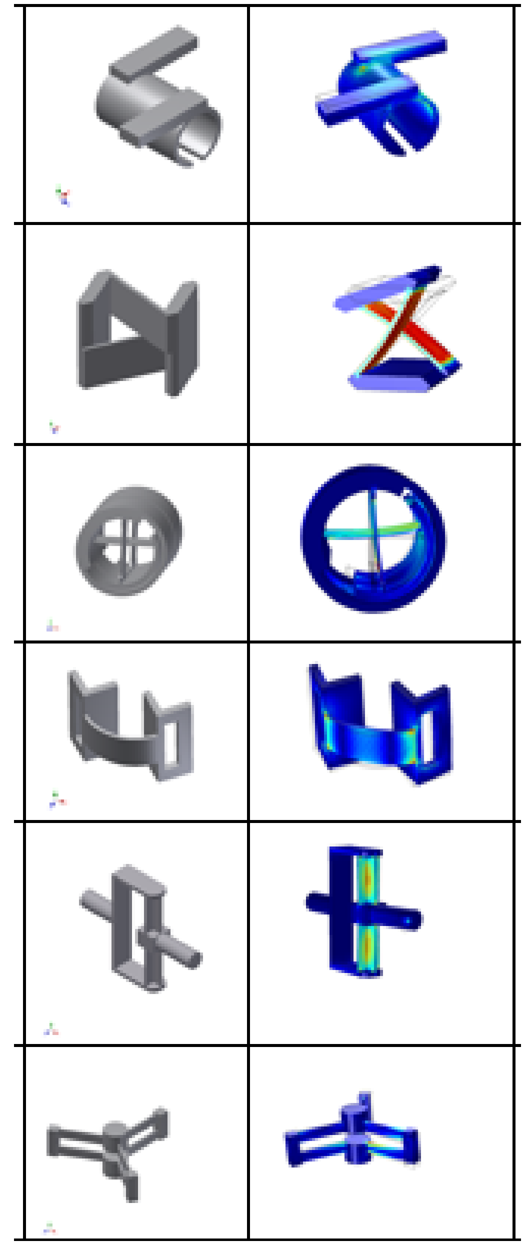
Bendix joint

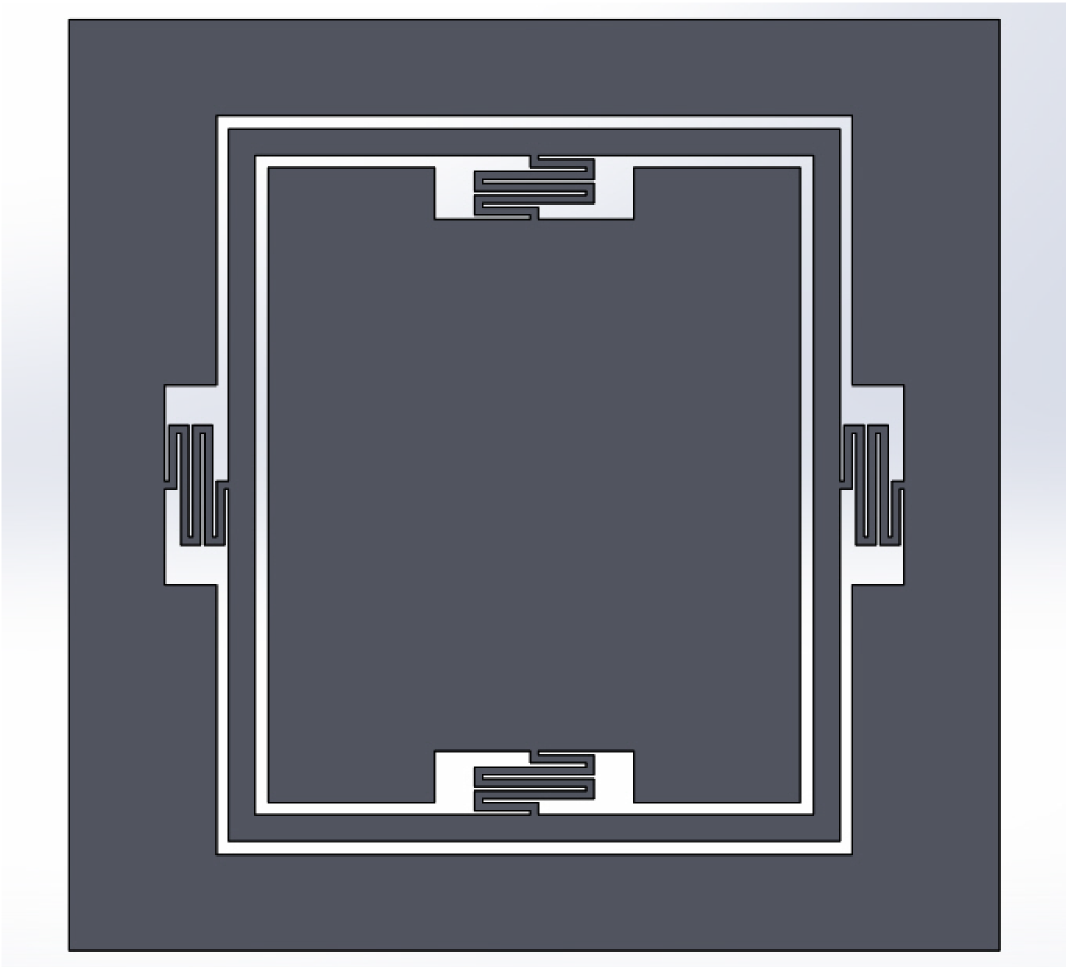


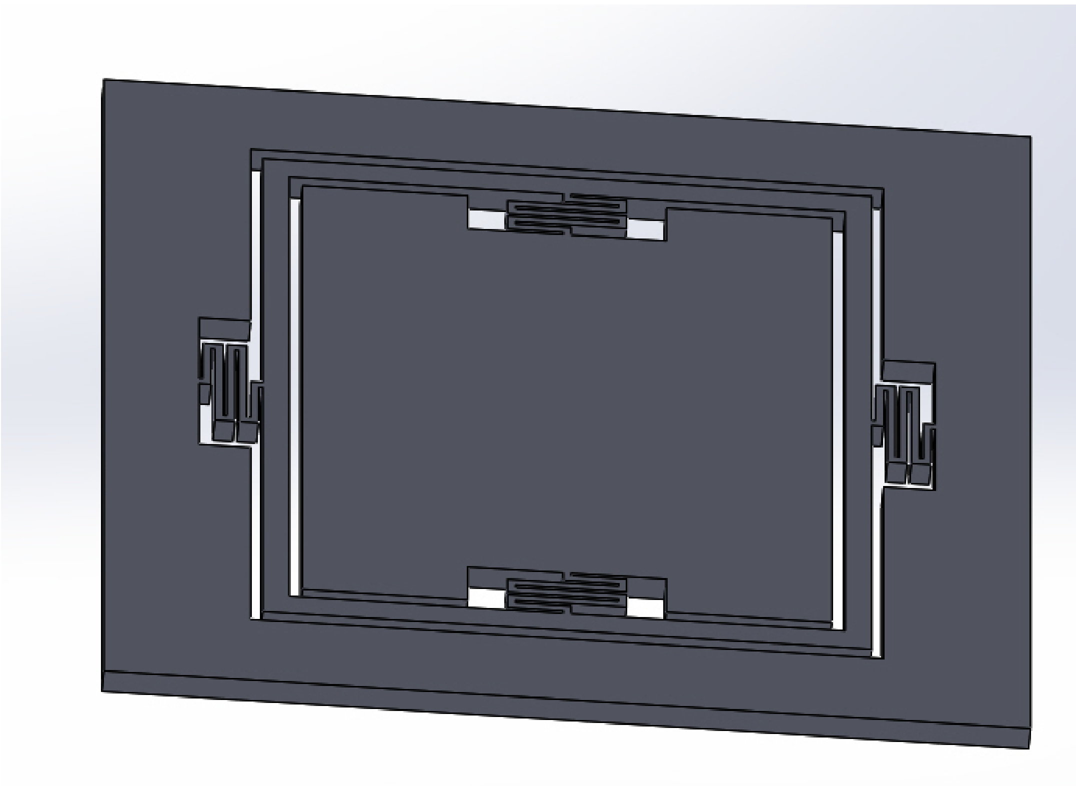
1-dof rotational elastic pairs

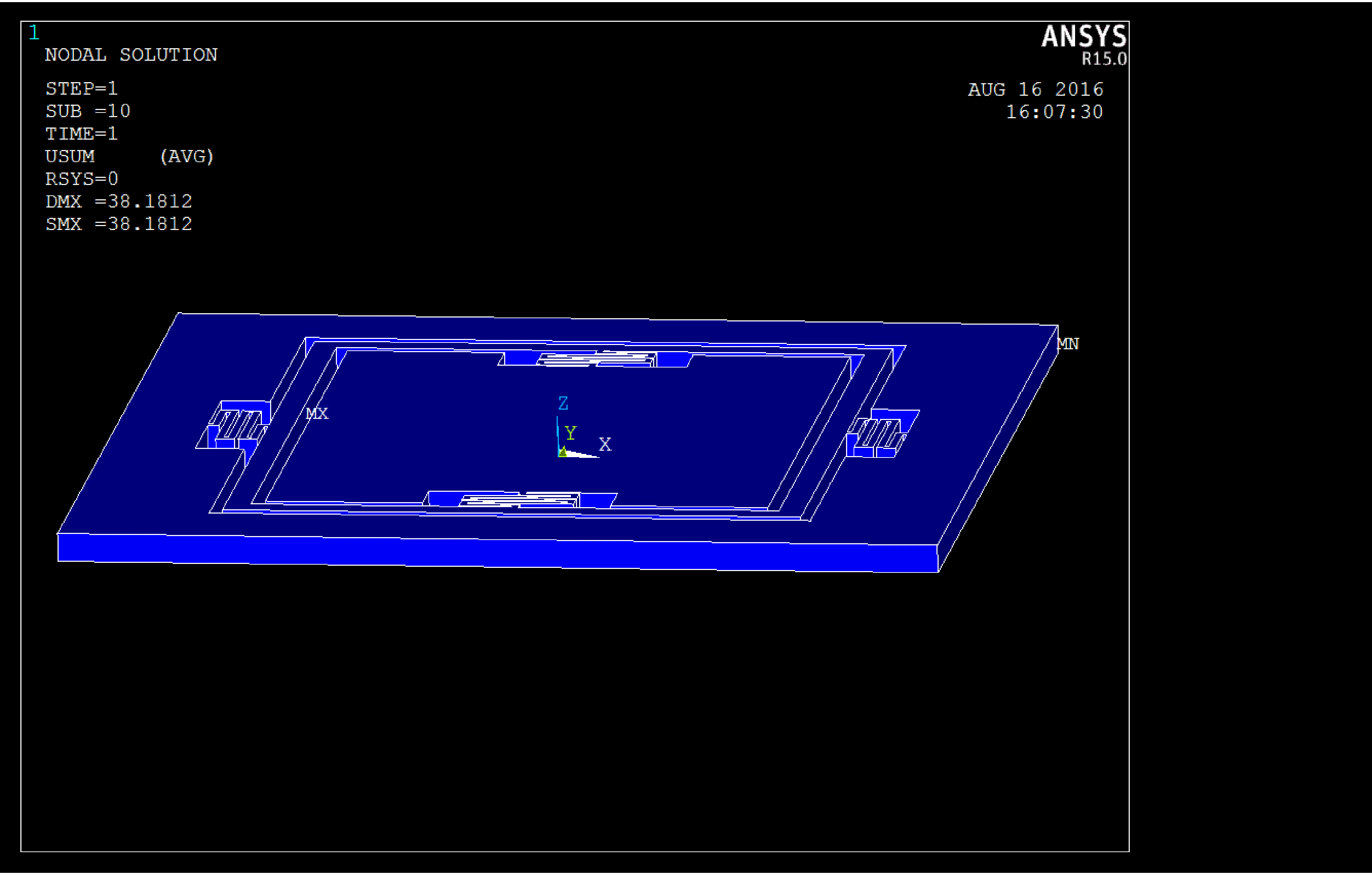
More than a dozen shapes for 1-dof elastic rotational pairs!

Bendix elastic rotational pair





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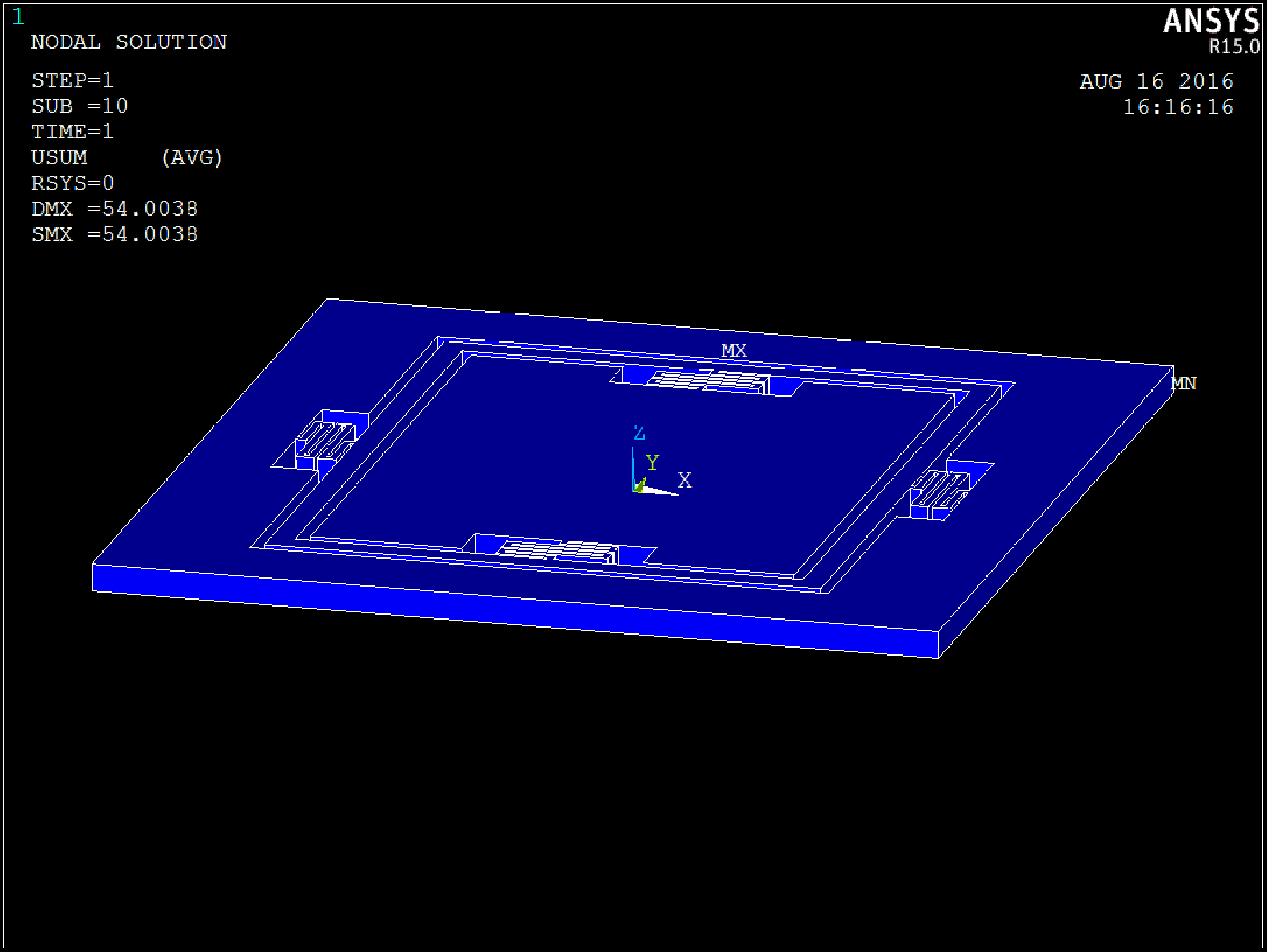
MN
Z
Y
X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AUG 16 2016
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MX
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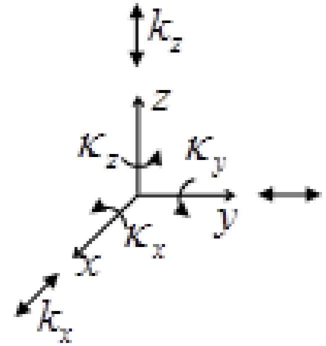
X

Multi-axis stiffness of an elastic pair



$$\mathbf{Ku}=\mathbf{f} \Rightarrow \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} & k_{x\theta} & k_{x\phi} & k_{x\psi} \\ & k_{yy} & k_{yz} & k_{y\theta} & k_{y\phi} & k_{y\psi} \\ & & k_{zz} & k_{z\theta} & k_{z\phi} & k_{z\psi} \\ & & & k_{\theta\theta} & k_{\theta\phi} & k_{\theta\psi} \\ & \text{Symmetric} & & & k_{\phi\phi} & k_{\phi\psi} \\ & & & & & k_{\psi\psi} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \\ \theta \\ \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$$

Elastic deformation analysis, analytical or numerical, via the compliance matrix can be used to compute **K**.

Computing the multi-axis compliance matrix

$$\mathbf{K}^{-1} = \mathbf{C} \Rightarrow \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} & c_{x\theta} & c_{x\phi} & c_{x\psi} \\ & c_{yy} & c_{yz} & c_{y\theta} & c_{y\phi} & c_{y\psi} \\ & & c_{zz} & c_{z\theta} & c_{z\phi} & c_{z\psi} \\ & & & c_{\theta\theta} & c_{\theta\phi} & c_{\theta\psi} \\ & \text{Symmetric} & & & c_{\phi\phi} & c_{\phi\psi} \\ & & & & & c_{\psi\psi} \end{bmatrix} \begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} u_x \\ u_y \\ u_z \\ \theta \\ \phi \\ \psi \end{Bmatrix}$$

Up to six analysis runs…
Three finite element analysis runs in 2D.
Six finite element analysis runs in 3D.

Further reading

- How to Design Flexure Hinges—Paros J. M. and L. Weisbord, *Machine Design*, Nov. 25, 1965
- Foundations of Ultraprecision Mechanism Design—S. T. Smith and D. G. Chetwynd
- Compliant Mechanisms: Design of Flexure Hinges—Nicolae Lobontiu, CRC Press
- Flexures: Elements of Elastic Mechanisms—St. Smith, CRC Press