ME 254, Lectures 4 and 5

Compliant mechanisms —Mobility Analysis

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Degrees of freedom (DoF)

- Minimum number of scalar inputs needed to define the complete configuration of the mechanism.
- Also equal to the minimum number of inputs needed for deterministic motion.

Kutzbach-Grübler's formula

3D

$$DoF = 6(n-1) - 5f_1 - 4f_2 - 3f_3 - 2f_4 - f_5$$

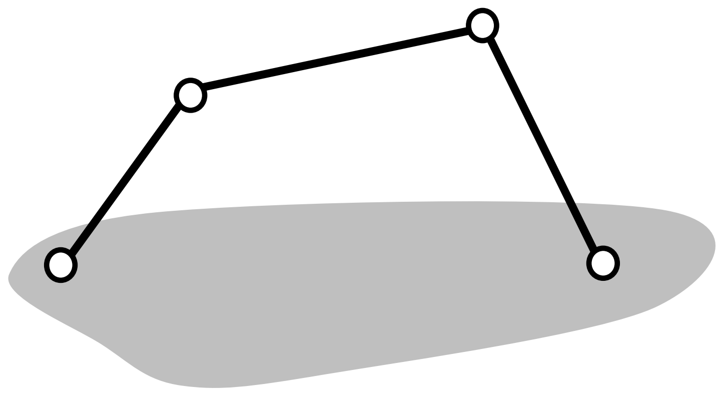
$n =$ number of rigid bodies (called links)

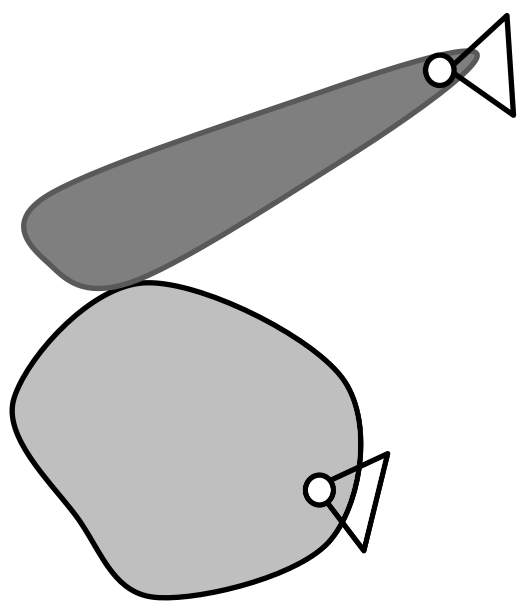
$f_i =$ joints that allow i relative motions

2D

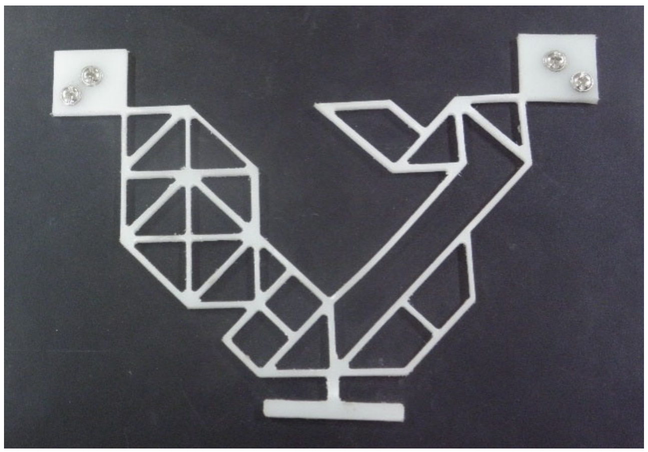
$$DoF = 3(n-1) - 2f_1 - f_2$$

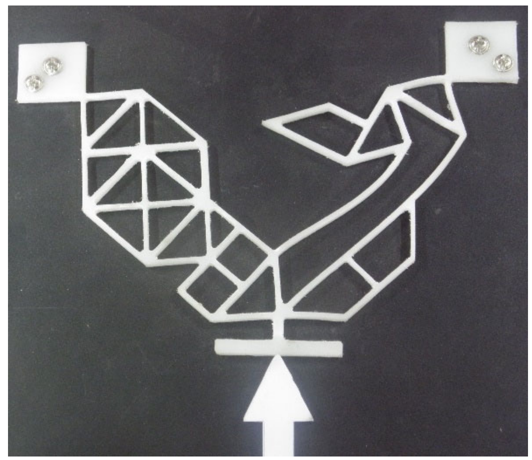
Familiar examples

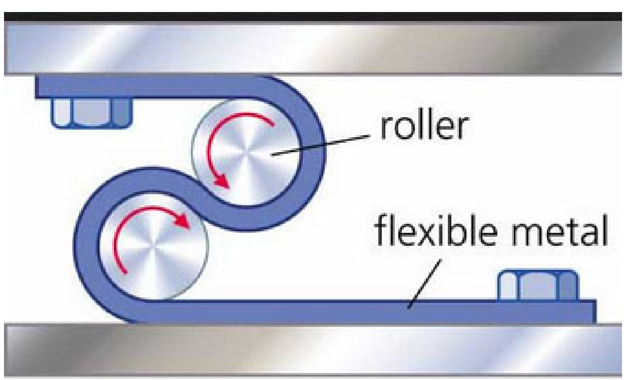




How many DoF do these have?







Let's derive the Grübler's formula.

DoF formula extended to compliant mechanisms

Midha, Murphy, and Howell (1995)
Ananthasuresh and Howell (1996)

3D

$$DoF = 6(n_{seg} - 1) - \sum_{j=1}^{5} (6 - j) n_{Kj} - \sum_{j=1}^{5} (6 - j) n_{Cj} - 6 n_{fix} + \sum_{j=1}^{6} j \, n_{scj}$$

2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2} (3 - j) n_{Kj} - \sum_{j=1}^{2} (3 - j) n_{Cj} - 3 n_{fix} + \sum_{j=1}^{3} j \, n_{scj}$$

n_{seg} = number of segments (rigid or compliant)

n_{Kj} = number of kinematic pairs allowing j relative dof

n_{Cj} = number of elastic pairs allowing j relative dof

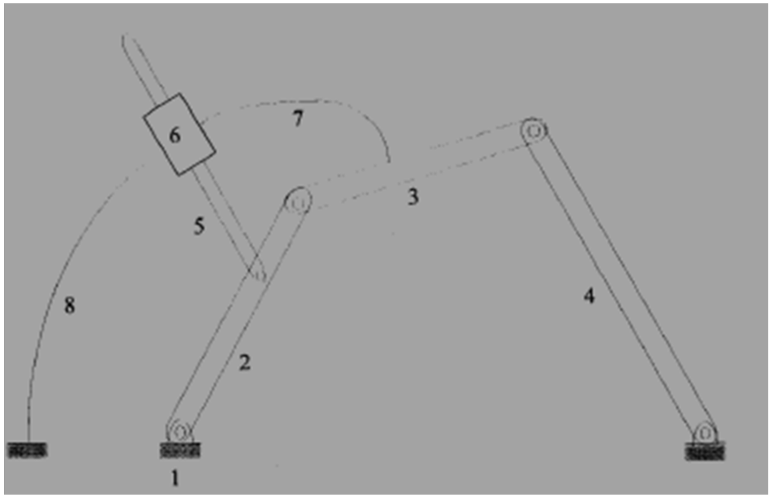
n_{fix} = number of fixed connections

n_{scj} = number of segments with segment compliance of j

Let us understand the formula.

2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2} (3 - j) n_{Kj} - \sum_{j=1}^{2} (3 - j) n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\, n_{scj}$$



Let us understand the formulae.

2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2}(3-j)\,n_{Kj} - \sum_{j=1}^{2}(3-j)\,n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\,n_{scj}$$

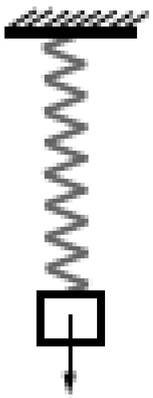
3D

$$DoF = 6(n_{seg} - 1) - \sum_{j=1}^{5}(6-j)\,n_{Kj} - \sum_{j=1}^{5}(6-j)\,n_{Cj} - 6n_{fix} + \sum_{j=1}^{6} j\,n_{scj}$$

Example 1

2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2}(3-j)\,n_{Kj} - \sum_{j=1}^{2}(3-j)\,n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\,n_{scj}$$

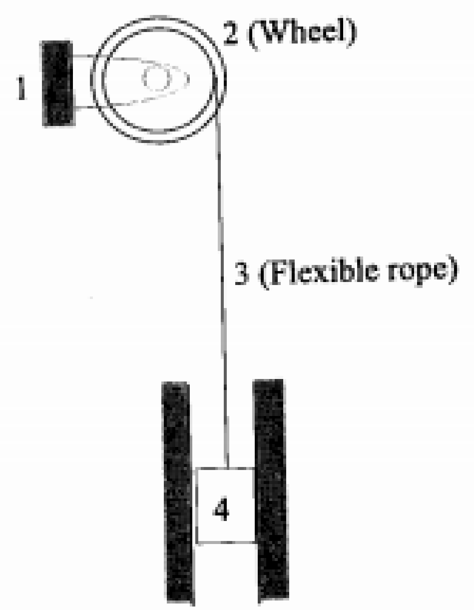


Understanding "segment compliance"

2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2}(3-j)n_{Kj} - \sum_{j=1}^{2}(3-j)n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\, n_{scj}$$

A simple example



Case	*nseg*	*nfix*	*nK1*	*nsc3*	*dof*
Winch	4	2	2	1	2

Example 2

2D

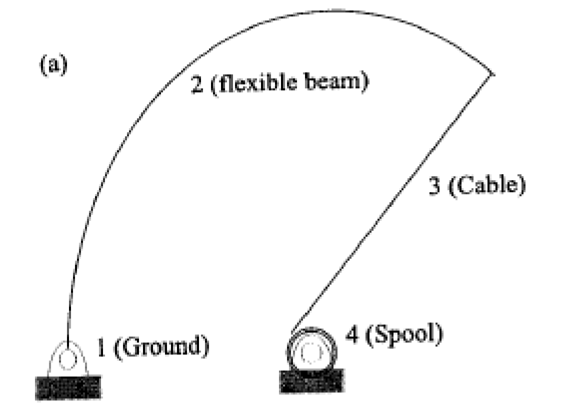
$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2}(3-j)\,n_{Kj} - \sum_{j=1}^{2}(3-j)\,n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\,n_{scj}$$

Virtual rigid segments (VRSs) to take care of applied forces/moments

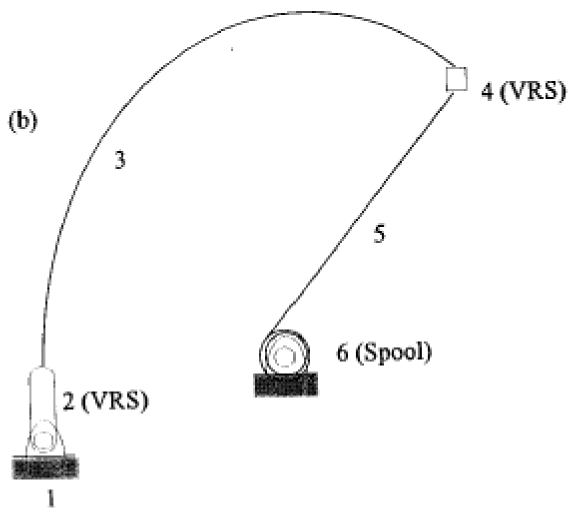
2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2}(3-j)\,n_{Kj} - \sum_{j=1}^{2}(3-j)\,n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\,n_{scj}$$

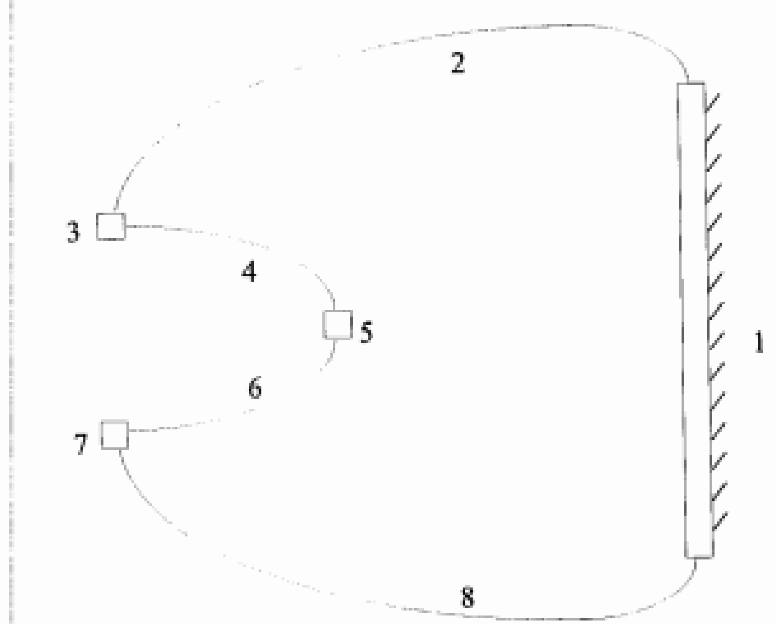
Elastic arm example



Case	nseg	nfix	nK1	nsc3	dof
a	4	2	2	2	5
b	6	4	2	2	5



A compliant gripper

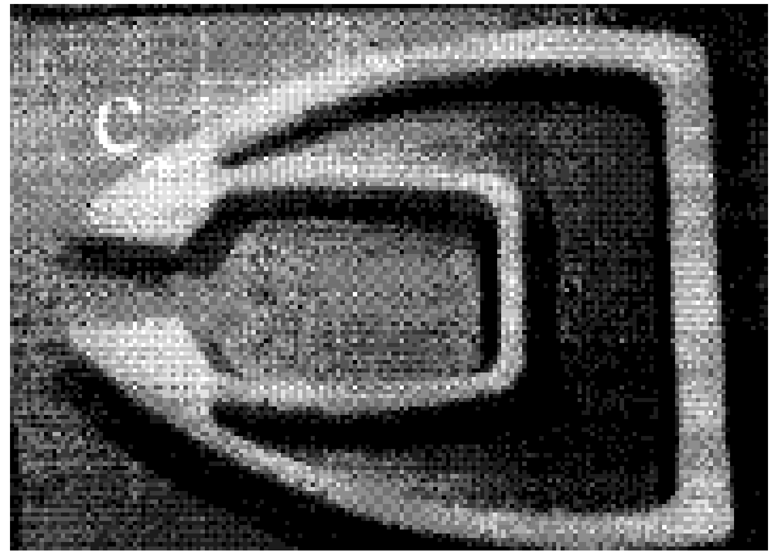


Ground segment: 1

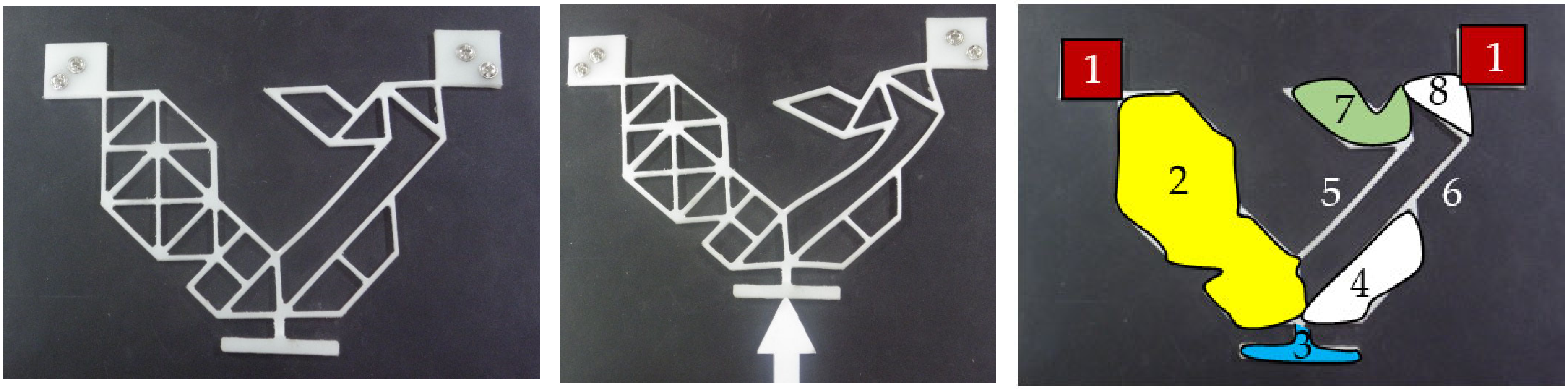
VRS's: 3, 5, 7

Compliant segments: 2, 4, 6, 8

Case	*nseg*	*nfix*	*nsc1*	*nsc3*	*dof*
a	8	8	0	4	9
b	8	8	4	0	1



Let us count…



$n_{seg} = 8$

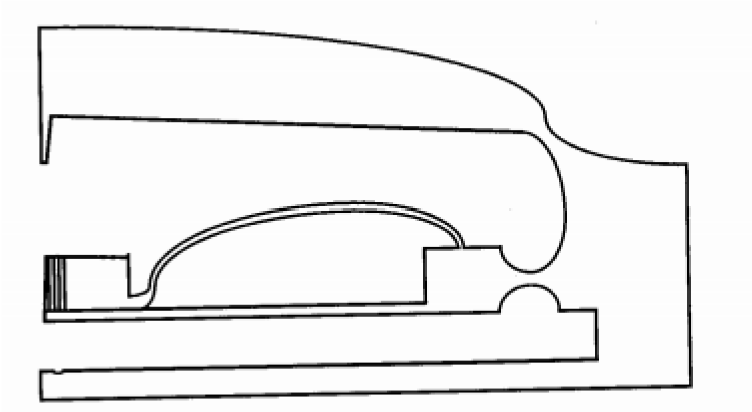
$n_{C1} = 5$ (Between segments 1-2, 2-3, 3-4, 1-8, and 7-8)

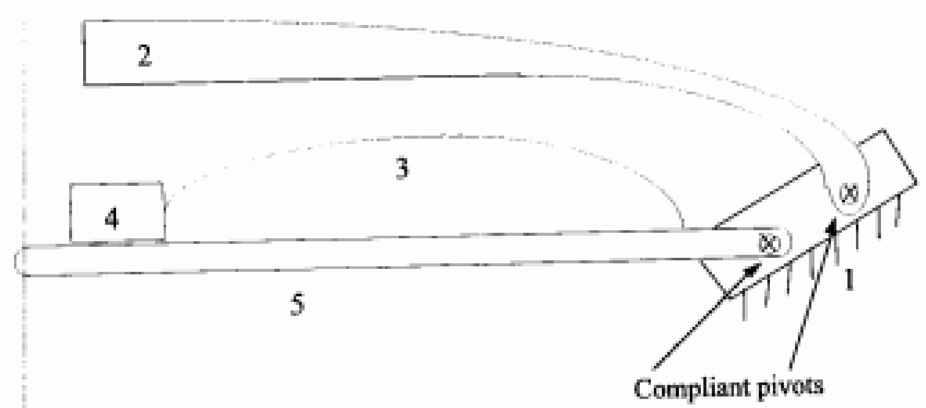
$n_{fix} = 4$ (Between 2-5, 4-6, 5-7, and 6-8)

$n_{sc3} = 2$ (Bodies 5 and 6; in case of doubt, assume segment compliance of 3 in 2D)

$f = 3(n_{seg} - 1) - 2n_{C1} - 3n_{fix} + 3n_{sc3}$ = 3(8-1) – 2(5) – 3(4) + 3(2) = 21-10-12+6 = 5

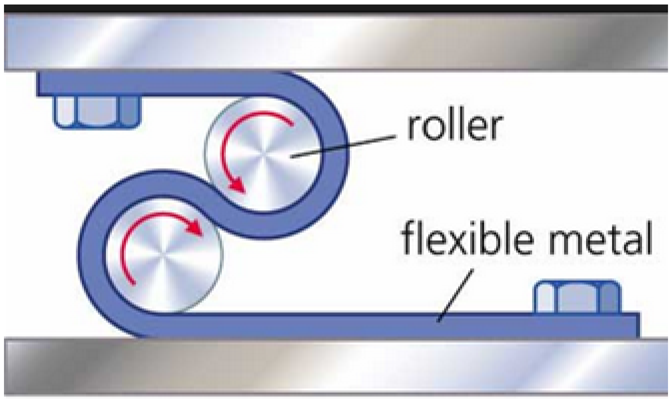
Compliant stapler





Case	*nseg*	*nfix*	*nK1*	*nC1*	*nsc3*	*dof*
Stapler	5	2	1	2	1	3

DoF of the rolamite elastic pair



This rolamite elastic pair has one degree of freedom. Let us see if our counting also gives the same.

n_{seg} = 4 (two rollers, one flexible segment, and the fixed frame above and below)

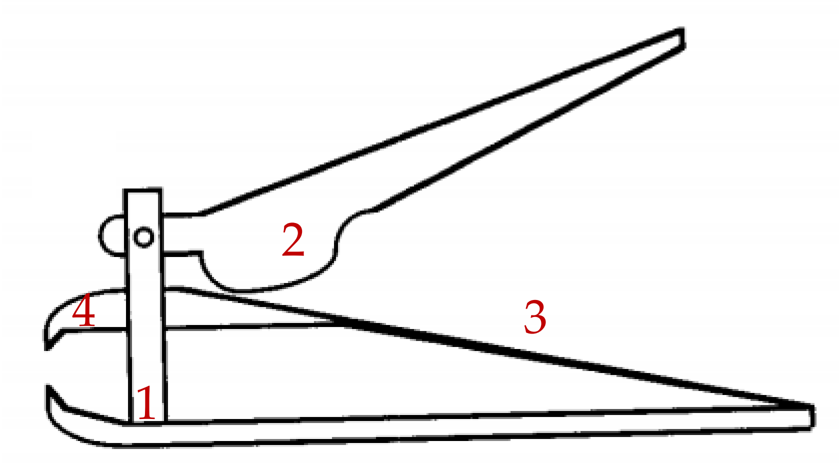
n_{K1} = 2 (two rollers with the flexible segment)

$n_{sc3} = 1$

n_{fix} = 2 (the flexible segment with the fixed frame at two places)

$f = 3(4\text{-}1) - 2(2) - 3(2) + 3(1) = 9 - 4 - 6 + 3 = 2$

DoF of a nail-clipper



$$dof = 3(n_{seg} - 1) - \sum_{j=1}^{2}(3-j)\,n_{Kj} - \sum_{j=1}^{2}(3-j)\,n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\,n_{scj}$$

$$\begin{aligned} dof &= 3(4-1) - 2n_{K1} - n_{K2} - 3n_{fix} + n_{sc1} \\ &= (3\times 3) - (2\times 1) - 1 - (3\times 2) + (1\times 1) \\ &= 9 - 2 - 1 - 6 + 1 = 1 \end{aligned}$$

Main points

2D

$$DoF = 3(n_{seg} - 1) - \sum_{j=1}^{2} (3 - j) n_{Kj} - \sum_{j=1}^{2} (3 - j) n_{Cj} - 3n_{fix} + \sum_{j=1}^{3} j\, n_{scj}$$

3D

$$DoF = 6(n_{seg} - 1) - \sum_{j=1}^{5} (6 - j) n_{Kj} - \sum_{j=1}^{5} (6 - j) n_{Cj} - 6n_{fix} + \sum_{j=1}^{6} j\, n_{scj}$$

- Extended Grubler's formula gives maximum DoF for compliant mechanisms.
- Segment compliance
  - Needs interpretation
  - Assume 3 (in 2D) and 6 (3D) when in doubt
- Introduce a Virtual Rigid Segment (VRS)
  - where there is a force on an elastic segment
  - where two elastic segments meet

Further reading

Proceedings of
The 1996 ASME Design Engineering Technical Conferences and
Computers in Engineering Conference
August 18-22, 1996, Irvine, California

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CASE STUDIES AND A NOTE ON THE DEGREES-OF-FREEDOM IN COMPLIANT MECHANISMS

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An example that does not work

Kaleidocycle mechanism with elastic pairs

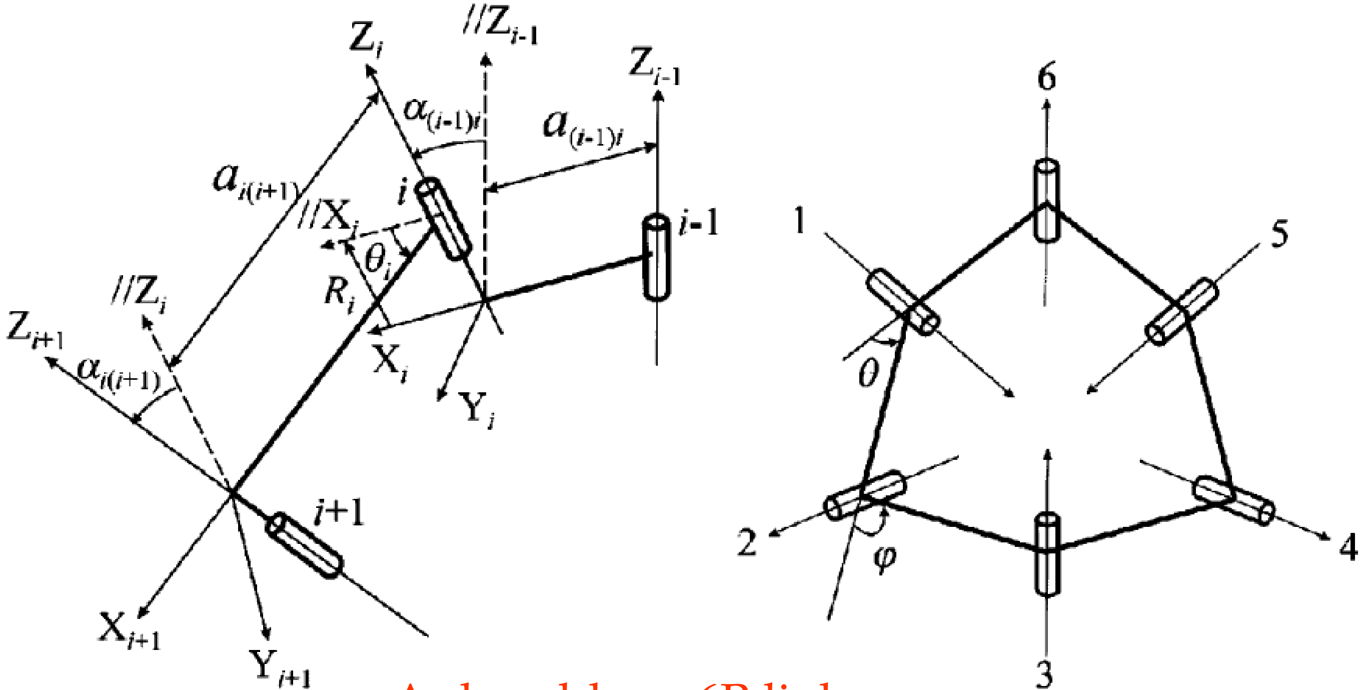
Six-tetrahedrons joined together with six flexural hinges


Bricard linkage: formula fails!

$$DoF = 6(n-1) - 5f_1 - 4f_2 - 3f_3 - 2f_4 - f_5$$

$$= 6(6-1) - (5 \times 6) - (4 \times 0) - (3 \times 0) - (2 \times 0) - (1 \times 0)$$

$$= 30 - 30 = 0$$



A closed-loop 6R linkage