ME 254: Compliant Mechanisms		
Assigned: Jan. 31, 2023	Quiz 1	Due: Jan. 31, 2023

This is an in-class quiz from 9:30 AM to 9:50 AM. Answer any two of the three questions.

Question 1 (1 point)

Ossur Pro-flex ankle-foot prosthesis is shown in Fig. 1. Draw a kinematic sketch that best represents its form and function. First think of it as a 2D (planar) mechanism and then think of it as a 3D (spatial) mechanism.

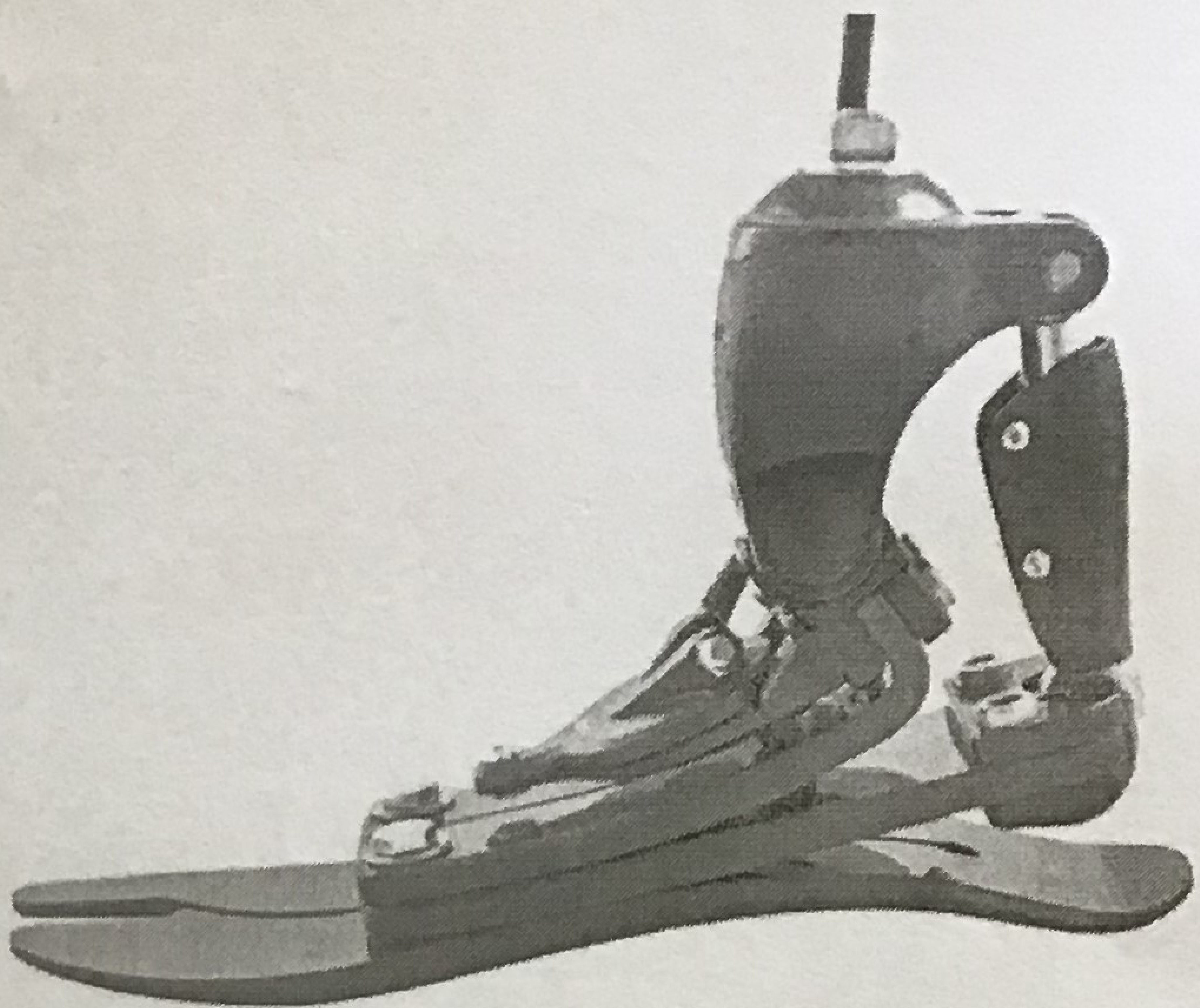
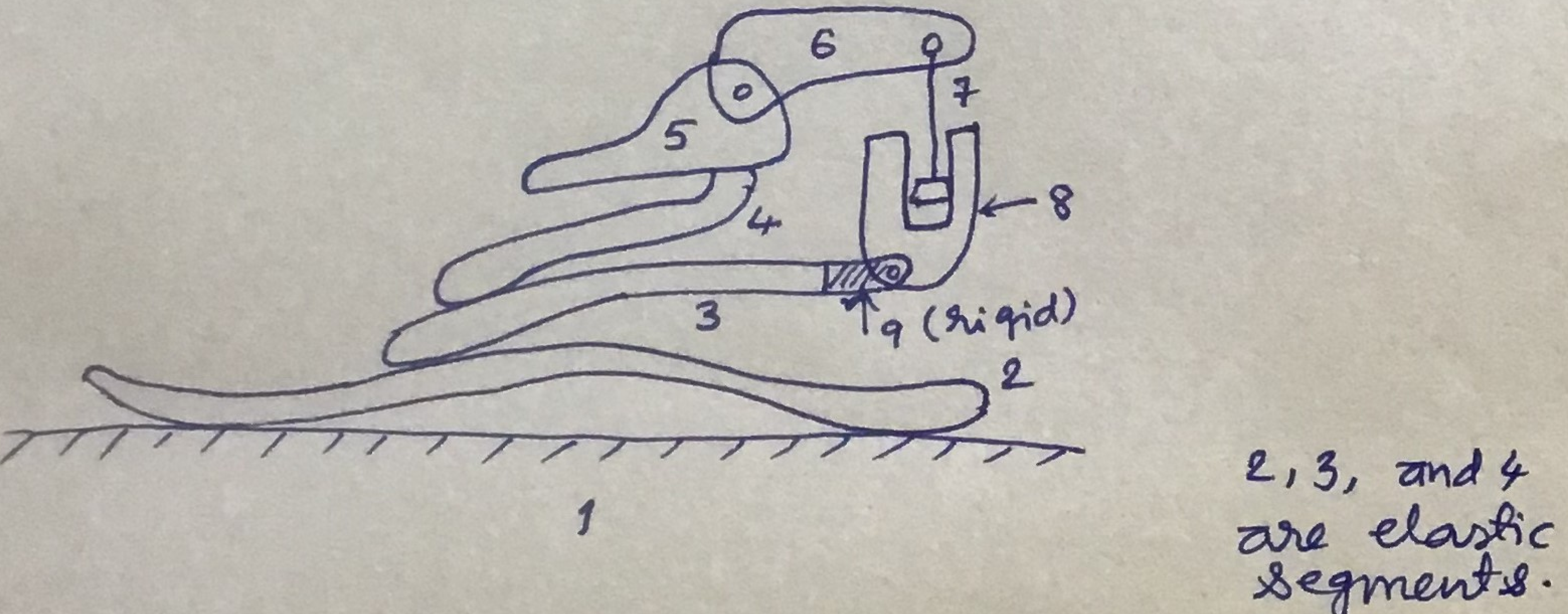


Fig. 1: Ossur Pro-flex ankle-foot prosthesis (Image from https://www.indiamart.com/proddetail/ossur-pro-flex-ankle-20238604448.html)

(a) 2D (planar) interpretation

2D Kinematic sketch (number all segments to help you interpret DoF as actuations)



Compute the degrees of freedom using the extended Grübler's formula. For your convenience, the table is given to indicate the number of segments, joints, etc. Fill this table as you compute the DoF.

n_{seg}	n_{fix}	n_{k1}	n_{k2}	n_{c1}	n_{c2}	n_{sc1}	n_{sc2}	n_{sc3}
9	5	4	0	0	0	3	0	0

DoF = $3(9-1) - 3(5) - 2(4) + 1(3) = 24 - 15 - 8 + 3 = 4$

Interpret the DoF as actuations by referring to segment numbers. Use virtual rigid segments as may be needed.

Three DoF (actuations) to segment 9 and an actuation between segments 7 and 8.

(b) 3D (spatial) interpretation

3D kinematic sketch (number all segments to help you interpret DoF as actuations) or mention the changes to the 2D sketch by referring to its segments and joints.

Same sketch as before with the following modifications:

- Joint between segments 9 and 8 becomes a spherical joint. (A)
- Segments 4 and 5 have an additional hinge for sideways rotation. (B)
- Segment 4 is duplicated to place on either side of segment 3. This will increase segment count by 1 and n_{fix} by 2. (C)

n_{seg}	n_{fix}	n_{k1}	n_{k2}	n_{k3}	n_{k4}	n_{k5}	n_{c1}	n_{c2}	n_{c3}	n_{c4}	n_{c5}	n_{sc1}	n_{sc2}	n_{sc3}	n_{sc4}	n_{sc5}	n_{sc6}
10	7	~~3~~ 4	0	1	0	0	0	0	0	0	0	1	0	3	0	0	0
(C)	(C)	-1 (B), (A) +1		(A)								Segment 2		Segments 3, 10, and 4			

DoF = $6(10-1) - 6(7) - 4(3) - 3(1) + 1(1) + 3(3)$
$= 54 - 42 - 14 - 3 + 1 + 9 = 7$

Interpret the DoF as actuations by referring to segment numbers (including VRSs, if needed.

Segment 9 has 6 DoF and can be given 6 actuations. Translation between 7 and 8 has one actuation.

Question 2 (1 point)

Compute the nullspace vector(s) of the following matrices. Do not use Matlab or any other program. Do it by hand yourself and show your steps and reasoning (e.g., rank?)

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 6 \end{bmatrix}$$

$\rightarrow 2x + 3y + z = 0 \Rightarrow z = -2x - 3y$

$\rightarrow x + 3y + 6(-2x - 3y) = 0$

$\Rightarrow (x - 12x) + (3y - 18y) = 0$

$\Rightarrow -11x = 15y \Rightarrow y = -\frac{11}{15}x$

Nullspace vector: $\begin{Bmatrix} 1 \\ -11/15 \\ -2 - 3(-\frac{11}{15}) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0.733 \\ 0.2 \end{Bmatrix}$

Rank is 2 because there is only one of x, y, and z can be chosen freely. Rank-deficiency is 1.

$$\begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 1 & 6 \end{bmatrix}$$

3X2

$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} 2x + y = 0 \\ 3x + 3y = 0 \\ y + 6y = 0 \end{Bmatrix}$

$x = 0; y = 0$ is the only solution.

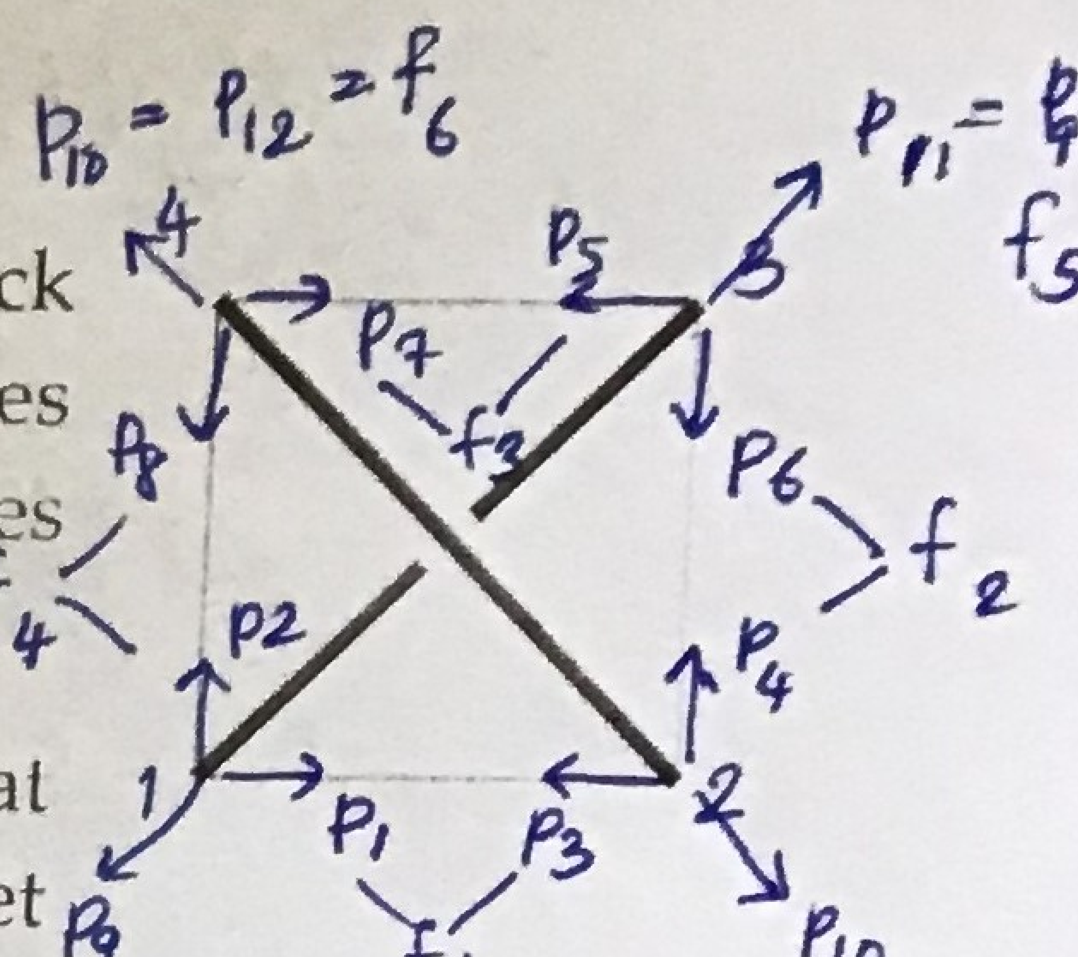
Two linearly independent equations. $\therefore$ Rank=2

No rank deficiency.

$\Rightarrow$ No nullspace vectors.

Question 3 (1 point)

A 2D tensegrity system is shown in the figure on the right. Thick lines represent compression elements (struts) and thin lines represent tension elements (wires). The struts are in parallel planes and do not intersect. So, one pf them is shown broken, for clarity.



The tension elements form a square of 1 unit side. If the diagonal at 45° expands by 0.01 units left to itself, the other members do not let it expand that much. So, all elements will have some internal force. If the internal force in the expanded diagonal strut that is at 45° is p, we want to know the internal forces in the other members. This can be found as the nullspace vector of a matrix. Construct such a matrix. You need not find the nullspace.

Hint: balance forces at the four vertices and write the force-equilibrium matrix and transpose it.

$$
\begin{array}{c}
1\\2\\3\\4\\5\\6\\7\\ \\
\end{array}
\underbrace{\begin{bmatrix}
1 & 0 & 0 & 0 & -1/\sqrt{2} & 0 \\
0 & 0 & 0 & 1 & -1/\sqrt{2} & 0 \\
-1 & 0 & 0 & 0 & 0 & 1/\sqrt{2} \\
0 & 1 & 0 & 0 & 0 & -1/\sqrt{2} \\
0 & 0 & -1 & 0 & 1/\sqrt{2} & 0 \\
0 & -1 & 0 & 0 & 1/\sqrt{2} & 0 \\
0 & 0 & 1 & 0 & 0 & -1/\sqrt{2} \\
0 & 0 & 0 & -1 & 0 & 1/\sqrt{2}
\end{bmatrix}}_{8\times 6 \;=\; \bar{\bar{H}}}
\underbrace{\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix}}_{6\times 1}
=
\underbrace{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}}_{8\times 1}
$$

$$\bar{\bar{C}} = \bar{\bar{H}}^T$$

$\uparrow$ Null space vector of this matrix will give what we need here.