Assignment - III

13751

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1a) i) A metric Space is a pair (x, d) Consisting of a set x (of points or Elements) together with a metric 'd'.

ii) The d must be a real Valued function for any two points $x, y \in X$ which has following four properties

$$d(x,y) \geq 0$$
$$d(x,y) = 0 \Rightarrow x = y$$
$$d(x,y) = d(y,x)$$
$$d(x,y) \leq d(x,z) + d(z,y)$$

Ans 'A'

2a) (i) Are hilbert and banach Spaces are two Sides of a Coin

(ii) $\|x\| = \sqrt{\langle x, x\rangle}$, $d(x,y) = \|x-y\| = \sqrt{\langle x-y, x-y\rangle}$

$$\|x\| = \sqrt{\int_0^b x^2(t)\,dt} \cdot \sqrt{\langle x, x\rangle}$$

$$\langle x, y\rangle = \int_0^b x(t)\,y(t)\,dt$$

Ans:- 'A'

3a) (i) gateau Variation is Similar to directional derivative.

(ii) $$\delta J(x,h) = \lim_{\varepsilon \to 0} \frac{J(x+\varepsilon h) - J(x)}{\varepsilon}$$ h, x are Vectors of Vector Space X.

$$\nabla_{\bar{h}} f(\bar{x}) = \lim_{\varepsilon \to 0} \frac{f(\bar{x}+\varepsilon \bar{h}) - f(\bar{x})}{\varepsilon}$$

Ans 'A'

4a) (i) Lemma 1, lemma 2, lemma 3, lemma 4 are proved in an indirect manner. They are helpful to get rid of h in frechet differential and gateaux Variation.

(ii) lemma 2 and 4 Can be obtained from lemma 1 by Integration by Parts.

'13' An

13751 M. Vikrauth Reddy

normed Vector Space is a Vector Space which a norm is defined. This should has the following properties

$\|x\| > 0 \quad x \in X$

$\|x\| = 0 \quad x = \odot$

$\|\alpha x\| = |\alpha| \, \|x\|$

$\|x+y\| \geq \|x\| + \|y\|$ (X)

false (Ans)

This option doesn't meet the requirement.

$X \rightarrow$ a real Valued f^n from $X \rightarrow R$

$f : X \rightarrow R$ whose value at $x \in X$ is denoted by $f(x) = \|x\| \in R$

---

ii) A linear function is a function which Satisfies the conditions as

$J(x+y) = J(x) + J(y)$, $J(\alpha x) = \alpha J(x)$

Ans:- (True)

(iii) Does Sobolev Space deals with Energy Spaces

Ans:- (True)

$$W^{r,q}(\Omega) = \left\{ v \in L'(\Omega) : \|v\|_{W^{r,q}(\Omega)} < \infty \right\}$$

$$\|v\|_{W^{r,q}(\Omega)} = \left(\sum_{|\alpha| \leq r} \|D^{\alpha}_{v}\|^{q}_{L^{q}(\Omega)} \right)^{1/q} \quad 1 \leq q \leq \infty$$

is sobolev form.

---

iv) Does gateau Condition $dJ(x,h) = 0$ Provides Min. or Max of given functional

Ans:- true

(v) J is Said to be frechet differential at x with increment h if and only if $dJ(x,h)$ is linear, Continuous functional of h

Ans:- (True)

6) We can call T as the frechet derivative of f at x_0, and denoted it by $Df(x_0)$

$$T : V \to W \quad \text{(bounded linear operator)}$$

$$\lim_{x \to x_0} \frac{\| f(x) - f(x_0) - T(x - x_0) \|_W}{\| x - x_0 \|_V} = 0$$

Ans True.

ANKIT SHARMA

14074

Assignment - 3

Q.1) Choose the correct alternative among the following:

A. Above statements are correct and assertion follows from the reasoning.

B. Above statements are correct but assertion does not follow from the reasoning.

C. Statement in the assertion is correct but not that in the reasoning.

D. Statement in the assertion is incorrect but that in the reasoning is correct.

① Assertion - Hilbert spaces are Banach spaces.

Reasoning - Inner product spaces are normed vector spaces.

Ans - A.

② A - Ratio of two definite integrals is a linear functional.

R - A functional is said to be linear if $\forall\ x, y \in X$, $\alpha, \beta \in K$

$$J(\alpha x + \beta y) = \alpha J(x) + \beta J(y)$$

X - vector space

K - scalar field.

Ans - D

③ A - Gateaux variation of a functional J at x exists.

R - Functional J is a Frechet differentiable at x.

Ans - B

④ A - Function of functions is a functional.

R - Functional is an operator which maps a function space X to a scalar field element K

Q.2) True or False

① Each real number set in N dimensions $(R, R^2, \dots R^N)$ can have number of metrics.

Ans - True

② An union of two subspaces is a subspace if the uniting subspaces have equal dimension.

Ans - False ∵ Additive closure will be violated.

③ Scalar field is essential to define a vector space.

Ans - True

9/2/2017 HOMEWORK-3 RAGHAV KRISHNA, M.Tech (Res), CPDM

1. a) i, If a functional J is Fréchet differentiable at x, then the Gâteaux variation J at x exists and equal to Fréchet differential.

ii, Using property $dJ(x;\varepsilon h) = \varepsilon\, dJ(x;h)$ and condition $\varepsilon \to 0$ ain $E(x,\varepsilon h)\|h\||\varepsilon| = J(x+\varepsilon h) - J(x) - \varepsilon dJ(x,h)$ has provided the proof as required.

A) → Correct answer.

b). i, If $F(x)$ is continuous in $[a,b]$ and if $\int_a^b F(x)h(x)dx = 0$ for every function $h(x) \in C^0(a,b)$ such that $h(a) = h(b) = 0$, then $F(x) = 0$ for all $x \in [a,b]$

ii, Assumption: $h(x) = (x - x_1)(x_2 - x)$ for $x \in [x_1, x_2]$ and zero outside

considering

$$\int_a^b F(x)\,h(x)dx = \int_a^{x_1} F(x)\,h(x)\,dx + \int_{x_1}^{x_2} F(x)\,h(x)\,dx + \int_{x_2}^{b} F(x)\,h(x)\,dx$$

$$= 0 + \int_{x_1}^{x_2} F(x)h(x)dx + 0$$

$$\to \int_{x_1}^{x_2} F(x)(x - x_1)(x_2 - x)dx > 0$$

∴ $F(x)$ cannot be non-zero at domain $[a,b]$.

A → correct answer

c) i, In Gâteaux variation, we can consider any h and showing the variation is ~~less th~~ equal to zero gives the proof that it is the extreme value.

ii, Similiar to functions' first differential, the extremem value can be proved.

Answer: D) $\rightarrow$ i, Gives only in the pertubed value in h direction.

True or False:

a) Function F being Fréchet differentiable at x, doesn't guarentee the existance of Gâteaux variation at x $\Rightarrow$ False.

b) In finite variable optimization, there exists a derivative for transformation from vector space to co-efficient field $\rightarrow$ True.

c) Given a operator A, set formed from domain ~~A~~ and range A are unrelated $\rightarrow$ False $\rightarrow$ They are image of one another.

Assingment - III

Variational methods in Optimizatn

Question on Lectr 7–10

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SR. No. 13897

Questions:-

I) Assertion & Reasoning type:

1) Assertion:- Euclidian space is a Hilbert Space

Reasoning:- It is a complete normed vector space satisfying the inner product properties.

2) Assertion:- The Rational Line (Line of all Rational Nos.) Q, is a Banach Space.

Reasoning:- Deleting nos. from real line $\mathbb{R}$ renders it incomplete.

3) Assertion:- The inner product in a Hilbert Space is bilinear in its ~~operatn~~ first argument & conjugate linear in 2^{nd}.

Reasoning:- $\langle x+y, z\rangle = \langle x,z\rangle + \langle y,z\rangle$ $\quad x,y,z \in H$

$\langle \alpha x, y\rangle = \alpha\langle x,y\rangle$ $\quad \alpha \in F$ (underlying field)

$\langle x,y\rangle = \overline{\langle y,x\rangle}$

4) Assertion:- In a metric space (X,d)

$x,y \in X$

$d(x,y) \geq 0$

Reasoning:- $d(x,y) = d(y,x)$

$d(x,y) \leq d(x,z) + d(z,y)$

$d(x,x) = 0$

5) Assertion:- The space $C[a,b]$ of all continuous fns in $J=[a,b]$ with norm defined as $\|x\| = \max_{t\in J} |x(t)|$ is a Hilbert space.

Reasoning:- It is a complete normed vector space satisfying parallelogram equality.

6) Assertion:- For a real inner product space, the inner product is rediscoverable from the norm.

Reasoning:- $\langle x,y\rangle = \frac{1}{4}\left(\|x+y\|^2 - \|x-y\|^2\right)$.

Answers

1) A) Statement is correct & follows from reasoning

2) for an Eucledian Space $\mathbb{R}^n$ $\langle x, y \rangle$ with $x = (\xi_1, \xi_2 \dots, \xi_n)$

& $y = (\eta_1, \eta_2 \dots \eta_n)$, $\langle x, y \rangle = \sum_i \xi_i \eta_i$

& $\|x\| = \left(\sum_i \xi_i^2\right)^{1/2}$ $= \sqrt{\langle x, x \rangle}$

& $d(x,y) = \langle x-y, x-y \rangle = \sqrt{\sum_i (\xi_i - \eta_i)^2}$

Moreover this space is complete.

2) A) Statement is true & follows from assertion

Rational line is followed by deleting irrational nos from Real line & there may Cauchy sequences in $\mathbb{R}$ with norm $d(x,y) = |x-y|$ which might converge to irrational nos. So Rational line is not complete.

3) A) Statement is true & follows from assertion.

$\langle \alpha u + \beta v, y \rangle = \langle \alpha u, y \rangle + \langle \beta v, y \rangle = \alpha \langle u, y \rangle + \beta \langle v, y \rangle$

∴ Linear in 1st ~~operator~~ argument

$\langle x, \alpha u + \beta v \rangle = \overline{\langle \beta v + \alpha u, x \rangle} = \overline{\langle \beta v, x \rangle} + \overline{\langle \alpha u, x \rangle} = \bar{\beta} \langle x, v \rangle + \bar{\alpha} \langle x, u \rangle$

∴ Conjugate linear in 2nd argument.

4) A) Statement is true & follows from from assertion.

$d(x,y) \leq d(x,z) + d(z,y)$

$\Rightarrow$ $d(x,x) \leq 2d(x,z)$ as $d(x,z) = d(z,x)$.

as $d(x,x) = 0$

$\Rightarrow$ $d(x,z) \geq 0$ (proved)

5) D) Statement is incorrect but reasoning is correct.

$\|x\| = \max_{t \in J} |x(t)|$ doesn't satisfy the identity $\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$

parallelogram equality.

but it is a complete normed space (Banach space).

6) A) Statement is true & reasoning is correct.

for a real inner prod space.

$$\|x+y\|^2 = \langle x+y, x+y\rangle = \langle x,x\rangle + \langle y,y\rangle + 2\langle x,y\rangle$$

$$\|x-y\|^2 = \langle x-y, x-y\rangle = \langle x,x\rangle + \langle y,y\rangle - 2\langle x,y\rangle$$

$$\Rightarrow \frac{1}{4}\left(\|x+y\|^2 - \|x-y\|^2\right) = \frac{1}{4}\left(4\langle x,y\rangle\right)$$

$$= \langle x+y\rangle \text{ (proved)}$$

True/False type:

7) The Hilbert space is an infinite dimensional space.

Ans: True.

~~8) For a real inner product space, the inner p~~

8) We can treat a variational calculus problem as a multivariable optimization with infinite no. of variables.

Ans: True

9) All Hilbert spaces are Banach spaces ~~but the~~ not vice versa.

Ans: False (the norm ~~the norm~~ should satisfy parallelogram equality).

10) In deriving Euler-Lagrange equations there is an inherent assumptn that ~~only~~ not only the varied path, $\hat{y}(x)$ is close to the extremal $y(x)$ in its neighborhood, but also their derivatives ~~$\hat{y}(x)$ are~~ ~~close to y~~ $\hat{y}'(x)$ are close to $y'(x)$

Ans: True (it is called weak variatn)

11) The satisfactn of the E-L eqns not only give us an extremizing fn but also tell us about ~~the~~ whether its minimizing or maximizing fn locally. (only tells us about a local extremizing fn).

Ans: False

12) The ~~Lesbe~~ Lebesgue Space, $L^q(\Omega) = \{v : v$ is defined on Ω & $\|v\|_{L^q(\Omega)} < \infty$

$$\text{where } \|v\|_{L^q(\Omega)} = \left(\int_\Omega |v(t)|^q dt\right)^{1/q} \qquad 1 \leq q < \infty$$

is a Hilbert space only when $q = 2$

Ans: True (for other values of q, ~~par~~ norm won't satisfy parallelogram equality)

Karan Kaul 13397 M.E. (Mech)

1. Assertion :

If we equate gateaux variation to zero, we get the necessary condition for the local minima of the functional

Reasoning:
Gateaux variation ensures the existence of the limit

$$\lim_{\varepsilon \to 0} \frac{J(x^* + \varepsilon h) - J(x^*)}{\varepsilon}$$

2. A. In calculus of variations, the space in which we search for a minimising function is a banach space

R. In iterative numerical optimisation involving functionals, we search from an initial guess and generate a sequence of functions which ultimately should converge to a limit within the space we are concerned with.

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3. A. A definite integral is a linear functional

R. A Linear functional is one for which $J(\alpha X) = \alpha J(x)$ for all $\alpha \in K$, $x \in X$ and $J(x+y) = J(x) + J(y)$ for $x, y \in X$

Choose the correct alternative among the following

A) The above statements are correct and assertion follows from the reasoning.

b) Above statements are correct but assertion doesn't follow from the reasoning.

c) statement in assertion is ~~true~~ correct but not that in the reasoning

d) Statement in the assertion in the assertion is incorrect but that in the reasoning is correct

True or False:

1) A vector space cannot have more than one norm defined for it.

2) A Banach space can have the limit of its converging sequence outside itself

3) Exponential of a Function is a functional.

Answer Key:

Assertion and Reasoning

1	(a)
2	(a)
3	(a)

True and False

1	F
2	F
3	F

True or false

Raman
14231

1) Lebesgue space is Banach space

2) Banach space has a norm defined for it.

3) A Norm is essential to define a vector space

29 Choose the correct alternative among following

a) The above statement are correct and assertion follows from the reasoning.

b) Above statements are correct but assertion doesn't follow from reasoning.

c) Statement in assertion is correct but not that in reasoning

d) Statement in assertion ~~is~~ the assertion is incorrect but that in the reasoning is correct.

10 a) Gateaux variation is not directly related to continuity of a functional

b) R $\longrightarrow$ It is essential to define a norm to judge continuity of a functional

b) Hilbert spaces are Banach spaces

R→ Inner product spaces are normed vector spaces

c) A function of function is a functional

R→ functional maps function to a scaler feild

Answers

1) T

2) T

3) F

a) a

b) a

c) d

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Course: PhD

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(2)

Q No. 1

Let X be a function space, which is a vector space, equipped with an inner product. If every sequence in X converges and hence has a limit, then X is called Hilbert space

Ans: False

Reason: Limit should be in X

Q No. 2

Lebesgue space and Sobolev spaces are special/particular cases of Banach space

Ans: True

Q No. 3

If functional J is Frechet differentiable at u, then it is guaranteed Gateaux variation of J at u exists and both are equal.

Ans: True

Q No. 4

Variational derivative (interpreted as partial derivative equivalent for Calculus of variations) of a functional J, denoted by $\frac{\delta J}{\delta y}$, will be corresponding expression in Euler Lagrange equation and hence should be zero.

Ans: True

① Q. No. 1

Assertion : Every inner product give rise to a norm that Can be used to measure the magnitude or length of elements of underlying vector space

Reasoning : Every norm that is used in analysis and applications arises from an inner product.

Answer : B

Q No. 2

Assertion : Design space considered in structural optimization problems belong to Sobolev space

Reasoning : Energy quantities commonly arising in structural optimization problems, like kinetic energy, strain energy and potential energy live in Sobolev space

Answer : A

Q No. 3

Assertion : Notwithstanding Gateaux variation is very useful in minimization of a functional, there is a necessity of another differential named Frechet differential.

Reasoning : Continuity of a functional is related to the concept of norm which is not used in Gateaux variation

Answer : A

Q No. 1

Let X be a function space, which is a vector space, equipped with an inner product. If every sequence in X converges and hence has a limit, then X is called Hilbert space

Ans: False

Reason: Limit should be in X

Q No. 2

Lebesgue space and Sobolev spaces are special/particular cases of Banach space

Ans: True

Q No. 3

If functional J is Frechet differentiable at u, then it is guaranteed Gateaux variation of J at u exists and both are equal.

Ans: True

Q No. 4

Variational derivative (interpreted as partial derivative equivalent for Calculus of variations) of a functional J, denoted by $\frac{\delta J}{\delta y}$, will be corresponding expression in Euler Lagrange equation and hence should be zero.

Ans: True

Q No. 5

In Functional analysis, metric gives the concept of distance, norm gives how close vectors are and inner product gives the notion of geometrical aspects like angles.

Answer: True

8|2|2017

Assignment #3

Variational Methods and Structural optimization

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PhD (Mechanical)

1 a) Assertion: $(V, \oplus, \odot)$, the set of all $n \times n$ matrices with real entries defined as

$$A \oplus B = 3A + 3B \quad \text{and} \quad c \odot A = cA^T, \quad c \in \mathbb{R}$$

is a vector space.

Reasoning: All the axioms of the vector space are satisfied.

(A) The statements in assertion and reasoning are correct and the assertion follows the reasoning.

(B) The statements in assertion and reasoning are correct but the assertion does not follow from the reasoning.

(C) The statement in the assertion is correct but not that in the reasoning.

(D) The statement in the assertion and reasoning are incorrect.

Answer (D):

Checking for the axioms.

1. $A \oplus B = 3A + 3B \quad \forall A, B \in V$ ✓

$(3A + 3B) \in V$

2. $A \oplus B = B \oplus A$ commutative law holds. ✓

3. $(A \oplus B) \oplus C = (3A + 3B) \oplus C = 9A + 9B + 3C$

$A \oplus (B \oplus C) = A \oplus (3B + 3C) = 3A + 9B + 9C$

$\therefore$ Associative law fails. ✗

4. Additive identity $A \oplus \theta = \theta \oplus A = A \quad \forall A \in V$ does not exist. ×

5. Therefore additive inverse doesnot exist. ×

6. $\forall \alpha \in \mathbb{R}, A \in V, \alpha \odot A = \alpha A^T \in V$ ✓

7. $\forall \alpha, \beta \in \mathbb{R}, A \in V$

$(\alpha+\beta) \odot A = (\alpha+\beta) A^T \in V$

$\alpha \odot A + \beta \odot A = \alpha A^T + \beta A^T \in V$ ✓

8. ~~$(\alpha+\beta) \odot A = (\alpha+\beta) A^T$~~

$\forall \alpha \in \mathbb{R}, \& A, B \in V$

$\alpha \odot (A \oplus B) = \alpha (3A+3B)^T$ ✓

$(\alpha \odot A) \oplus (\alpha \odot \beta) = 3(\alpha A^T) + 3(\alpha B^T)$

9. $(\alpha\beta) \odot A = \alpha\beta A^T$ ×

$\alpha \odot (\beta \odot A) = \alpha (\beta A^T)^T = \alpha \beta A$

10. There exists no multiplicative identity ×
such that $1 \odot A = A$ and $0 \odot A = \theta$

Therefore it is not a vector space.

1 b) <u>Assertion</u>: Sobolev space is a more appropriate space for the study of solution to partial differential equations.

<u>Reasoning</u>: Differentiation is permitted.

(A) The statements in assertion and reasoning are correct and assertion follows the reasoning.

(B) The statements in assertion and reasoning are correct but assertion doesnot follow from the reasoning.

(C) Statement in the assertion is correct but not that in the reasoning.

(D) The statement in the assertion is incorrect but that in the reasoning is correct.

<u>answer</u> (A)

1 c) <u>Assertion</u>: Gâteaux variation can be used to minimize a functional.

<u>Reasoning</u>: Gâteaux variation ensures continuity of a functional.

(A) The statements in assertion and reasoning are correct and the assertion follows the reasoning.

(B) The statements in assertion and reasoning are correct but the assertion doesnot follow the reasoning.

(C) The statement in the assertion is correct but not that in the reasoning.

(D) The statement in the assertion is incorrect but that in the reasoning is correct.

<u>Answer</u> (C)

Gâteaux variation doesnot ensure continuity. Because it doesnot talk about norm. So we cannot determine how close two functions are.

P.T.O

(12) True/false type Questions

a) $C^1[a,b]$ $a, b \in K$; $\|x\| = \max_{a \le t \le b} |\dot{x}(t)|$

is a normed function space. (True / false)

Answer: false ; Norm itself is not defined.

(b) Every convergent sequence with a limit L is a Cauchy sequence (True / false)

Answer: True.

Any real number $\varepsilon > 0$, beyond some fixed point, every term of the sequence is within $\frac{\varepsilon}{2}$ of L, so any two terms of the sequence are within a distance ε of each other.

c) The mode shapes (eigen vectors) in a two degree of freedom system belong to Hilbert space. (True / false)

True: Mode shapes are a complete set and orthogonal.

d) The set $[1, \infty)$ is a compact set. (True / false)

False: Set is closed but not bounded.

(e) $J = \int_{x_1}^{x_2} \sqrt{1 + (y')^2}\, dx$ is a linear functional (true / false)

False: $J(\alpha y' + \beta z') \neq \alpha J(y') + \beta J(z')$

α, β are constants.

ASSIGNMENT - 3

Structural Optimization & Variational Calculus

Submitted to:
Prof. G. K. Ananthasuresh
Mech. Engg. deptt.

Submitted by:
Vivek Dixit
Sr. No. 13460 (Mtech, ME)

Q.1 Assertion & Reasoning

① Assertion: Gateaux variation equal to zero ensure extremum of a functional.

Reason: $\delta J(x^*; h) = 0 \ \forall \ h \in X$ (normed vector space) is necessary condn for m̸x̸.of extremisation.

Ans. Assertion → wrong Reason: Right.

→ For ensuring of ~~extr~~ extremum of a functional, Gateaux variation should be defined / exist.

→ for Gateaux variation to exist, functional should be continuous which is checked by frechet differential.

→ Thus if a functional is frechet differentiable at 'x' then Gateaux variation of J at x exists and equal to frechet Differential.

② Assertion: Directional derivative of a functional in direction of vector $\bar{h}$ is same as Gateaux variation.

Reason: Directional derivative of a function f in direction of unit vector $\bar{h}$ given by

$$DD_h f(\bar{x}) = \lim_{\varepsilon \to 0} \frac{f(\bar{x} + \varepsilon \bar{h}) - f(\bar{x})}{\varepsilon}$$

Ans. Both Assertion & Reason are correct and Reason explains the assertion.

→ we can write Directional derivative as $\left.\frac{d}{d\varepsilon} f(x + \varepsilon h)\right|_{\varepsilon = 0}$

which is actually the operationally used definition of Gateaux variation.

(3) Assertion: All sequences in Hilbert space converge.

Reason: Hilbert space is the subset of Banach space.

Ans. Assertion & Reason both are correct.

Because Banach space is complete normed vector space and every Cauchy sequence from x (normed vector space) has a limit in x.

True & false

① frechet differential is linear in Nature (T) True.

Ans. $dJ(x; a_1h_1 + a_2h_2) = a_1\, dJ(x, h_1) + a_2\, dJ(x; h_2)$

② Union of Any no. of open sets is again an Open set.
(True)

③.

ASSIGNMENT- 3

Submitted to

Prof. G. K Ananthasuresh
MECH DEPT.

Submitted by:

Ankit Kumar
ME (M.TECH)
SR-13634

Q.1)

a) Assertion :- All Banach spaces are Hilbert spaces

Reasoning :- The inner product spaces are normed vector spaces.

→ Ans:) Assertion is false, reasoning is true.
All complete normed vector spaces are banach spaces but not all are inner product spaces.
The converse, hilbert spaces are banach space is true.

b) Assertion. Sequences in Banach spaces are converging in nature

Reasoning : Sequence in complete normed vector space are of Cauchy sequence.

⇒ Ans.) Assertion is true & reasoning is also true as cauchy sequence is converging and Banach space is a complete normed vector space.

c) <u>Assertion</u>: Gateaux variation of J at x is equal to Frechet differential if J is frechet differential at x.

$$\delta J(x;h) = dJ(x;h)$$

<u>Reasoning</u>: ~~Fretch~~ ~~Frech~~ Frechet differentiable is linear in nature and given by

$$\lim_{\|h\| \to 0} \frac{J(x+h) - J(x) - dJ(x;h)}{\|h\|} = 0$$

→ Assertion is true and reasoning is true and also it is correct explanation as linearity of frechet differentiable is important for it to be equal to gateaux variation

$$dJ(x;\varepsilon h) = \varepsilon dJ(x;h) \quad [\text{linearity}]$$

$$\therefore J(x+\varepsilon h) - J(x) - \varepsilon dJ(x;h) = E(x,\varepsilon h)\,\|h\|\,|\varepsilon|$$

$$\frac{J(x+\varepsilon h) - J(x)}{\varepsilon} = dJ(x;h)\ E(x,\varepsilon h)\|h\|\frac{|\varepsilon|}{\varepsilon}$$

① lim $\varepsilon \to 0$

$$\delta J = \lim_{\varepsilon \to 0} \frac{J(x+\varepsilon h) - J(x)}{\varepsilon} = dJ(x,h)$$

Q.2)

i) Interfection of any two closed set is a closed set

True

ii) Grateaux variation ~~alone~~ equal to zero alone ensure that extremum of functional exist.

False

⟹ Grateaux variation doesn't show any continuity of functional at that extremum.

~~iii) D Inner dot product space~~

iii) sequences in inner dot product space are converging in nature.

True

CALCULUS OF VARIATIONS & TOPOLOGY OPTIMIZATION

HOMEWORK No 3

Date: 14th Feb 2017

SHARATH S

Mech

05-07-00-11-12-16-1-14019

Assertion & Reasoning

1) ASSERTION: $C([a,b])$, the space of all complex-valued continuous functions is a Hilbert space

REASONING: inner product on $C([a,b])$

$$(h,k) = \int_a^b \overline{h(x)}\, k(x)\, dx \quad \rightarrow (1)$$

The inner product space defined in (1) is <u>not complete</u>. Hence, space of all complex valued function is not a Hilbert space

> Assertion is wrong, Reasoning is correct

2) ASSERTION: The set M of rational numbers (of the form a/b where a & b are integers) is not complete

REASONING: M is not complete with respect to the norm $\|x\| = \sqrt{x^2}$.

ex: $\sqrt{2}$ converges to 1·4142

- The sequence 1, 1·4, 1·41, 1.414 belongs to M but $\sqrt{2}$ itself does not belong to M

> Both assertion & reasoning are correct

3) ASSERTION: $\int_a^b \left(\sqrt{1+y'^2}\right) dx$ is a "local" functional
$\hookrightarrow (1)$

REASONING: If the curve $y=y(x)$ is divided into parts and functional (1) evaluated for each part, then their sum is equal to the functional of the whole curve

> Assertion & reasoning are correct

True or False statements

1) If x^* is a maximum, then $J(x^*+\epsilon h) - J(x^*) \leq 0$

TRUE

2) In the function $J(y) = \int_{x_1}^{x_2} F(x, y(x), y'(x))\,dx$

where $y'(x) = \frac{dy}{dx}$, 'x' is a member of the normed vector space X

FALSE

3) Functional is defined as an operator whose range is a complex number set

TRUE

ME256 | Assignment 03

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Assertion Reason Type

1	A: Inner Products always give rise to norms, but given a norm there may or may not be an inner product associated with it. R: A norm must satisfy triangle inequality
	Answer: B **Explanation**: An inner product always gives rise to norm given by, $\|x\| = \sqrt{\langle x, x\rangle}$. But for a given norm there is an associated inner product only if the parallelogram equality hold for every u and v in an inner product space. Parallelogram equality: $2\|u\|^2 + 2\|v\|^2 = \|u+v\|^2 + \|u-v\|^2$ If a norm satisfies the parallelogram equality, then it must come from an inner product. Thus, the parallelogram equality characterizes those norms that arise from an inner product. So both the statements are independently true but R is not correct explanation for A.
2	A. In structural optimization we usually consider function spaces in which norm are defined (normed vector space). R. Continuity of a functional is defined using norm.
	Answer: A **Explanation**: We desire our function to be continuous (upto some order, C0/C1/C2..) and to check continuity we need norms because norms helps us in measuring closeness between the elements in the function space. The definition of norm changes depending on the order upto which you want the functional to be continuous. So, Both statements are true and R is correct explanation for A.
3	A. Hilbert space is also a Banach space. R. Hilbert space is a complete normed vector space and Banach space is complete inner product space.
	Answer: D **Explanation**: An inner product always gives rise to norm given by, $\|x\| = \sqrt{\langle x, x\rangle}$ and since Hilbert space is complete inner product space we can be sure that it has got at least one norm defined for all the elements. And since norm is defined we can conclude that this is also a banach space. Banach space is complete normed space. So, Reason is wrong but assertion is true.
4	A. PE is equal to sum of strain energy and work potential of external forces. In this Work potential is taken negative. R. Work potential can be interpreted as lost work potential which could have been stored as PE but was lost due to displacement work done by the force.
	Answer: A **Explanation**: Both statements are true and R is correct explanation for A.
5	A. Euclidian space is also Hilbert space. R. Hilbert space is complete inner product space.
	Answer: A **Explanation**: Dot product in Euclidian space satisfies all the properties of the inner product. So, Both statements are true and R is correct explanation for A.

True/False

1. Gateaux variation provides necessary and sufficient condition for minimum of functional. **(F)**
2. For solving variational problem Euler reduced the problem of finding extrema of a functional to the problem of finding extrema of a function of n variables. **(T)**
3. Gateaux variation (and thus the extremum obtained by equating it to zero) depends on the arbitrary function h. **(F)**
4. Hilbert space is an infinite dimension space. **(T)**
5. Functionals can be regarded as the functions of infinitely many variables **(T)**