Lecture 19a

More than a dozen problems in optimizing a bar

ME 256 at the Indian Institute of Science, Bengaluru

Variational Methods and Structural Optimization

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Outline of the lecture

Some problems in optimizing the cross-section profile of a bar under axial loading.

What we will learn:

How to apply the concepts and ideas learned so far to solve problems in calculus of variations with particular examples of axially loaded bars.

Problem 1

$$\underset{A(x)}{Min}\; MC = \int_0^L p\,u\,dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$Data: L, p(x), E, V^*$$

Minimize the mean compliance (a measure of stiffness) for given volume of material under given loading.

Problem 2

$$\underset{A(x)}{Min}\ SE = \int_0^L \frac{1}{2} EAu'^2 dx$$

Subject to

$$\lambda(x): \qquad (EAu')' + p = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$Data: L, p(x), E, V^*$$

Minimize the strain enegry (a measure of stiffness) for given volume of material under given loading.

Problem 3

$$\underset{A(x)}{Min}\ V = \int_0^L A\,dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L \frac{1}{2} EAu'^2 dx - SE^* \le 0$$

$$Data: L, p(x), E, SE^*$$

Minimize volume of material to be used with an upper bound constraint on strain energy under given loading.

Problem 4

$$\underset{A(x)}{Min}\, V = \int_0^L A\, dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L p\, u\, dx - MC^* \leq 0$$

$$Data: L,\, p(x),\, E,\, MC^*$$

Minimize volume of material to be used with an upper bound constraint on mean compliance under given loading.

Problem 5

$$\underset{A(x)}{Min}\ SE = \int_0^L \frac{P^2}{2AE}\,dx$$

Subject to

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$Data: L, P(x), E, V^*$$

Minimize the strain energy, expressed in terms of internal forces in a bar, subject to the volume constraint.

Problem 6

$$\underset{A(x)}{Max}\ \underset{u(x)}{Min}\ PE = \int_0^L \left(\frac{1}{2} EAu'^2 - pu \right) dx$$

Subject to

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$Data: L, p(x), E, V^*$$

A min-max formulation for the stiffest bar for given volume of material based on the Clayperon's theorem.

Problem 7

$$\underset{A(x)}{Min}\ MC = \int_0^L p\,u\,dx$$

Subject to

$$\Gamma: \qquad \int_0^L \left(EAu'v' - pv \right) dx = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$Data: L, p(x), E, V^*$$

Minimize the mean compliance for given volume of material with the governing equation in the weak form.

Problem 8

$$\underset{A(x)}{Min}\ MC = \int_0^L p\,u\,dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$\mu_u(x): \qquad A - A_u \le 0$$

$$\mu_l(x): \qquad A_l - A \le 0$$

$$Data: L,\ p(x), E, V^*, A_l, A_u$$

Minimize the mean compliance for given volume and with upper and lower bounds on the area of cross-section.

Problem 9

$$\underset{A(x)}{Min}\ V = \int_0^L A\,dx$$

Subject to

$$\Lambda: \qquad \int_0^L \frac{PP_d}{AE}\,dx - \Delta^* = 0$$

$$Data: L, P(x), P_d(x), E, \Delta^*$$

Minimize the volume of a statically determinate bar with a deflection constraint at a point somewhere on its axis. Internal force, P, due to some applied load, and that due to a unit virtual load at the point of interest are given.

Problem 10

$$\underset{A(x)}{Min}\ V = \int_0^L A\,dx$$

$$Subject\ to$$

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\lambda_d(x): \qquad \left(EAu_d'\right)' + p_d = 0$$

$$\Lambda: \qquad \int_0^L EAu'u_d'\,dx - \Delta^* = 0$$

$$Data: L,\ p(x),\ p_d(x), E, \Delta^*$$

Minimize the volume of a bar (indeterminate or determinate) for a deflection constraint at a point.

Problem 11

$$\underset{A(x)}{Min}\ V = \int_0^L A\,dx$$

Subject to

$$\lambda(x): \qquad (EAu')' + p = 0$$

$$\lambda_d(x): \qquad (EAu_d')' + p_d = 0$$

$$\Lambda: \qquad \int_0^L EAu'u_d'\,dx - \Delta^* = 0$$

$$\Gamma: \qquad \int_0^L \frac{1}{2} EAu'^2\,dx - SE^* = 0$$

$$Data: L, p(x), p_d(x), E, \Delta^*, SE^*$$

Minimize the volume of a bar (indeterminate or determinate) for a deflection constraint at a point and strain energy constraint.

Problem 12

$$\underset{A(x)}{Min}\ MC = \int_0^L p\,u\,dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$\mu_u(x): \qquad u - u_u \le 0$$

$$\mu_l(x): \qquad u_l - u \le 0$$

$$Data: L, p(x), E, V^*, u_l, u_u$$

Minimize the mean compliance of a bar for given volume and upper and lower bounds on the displacement.

Problem 13

$$\underset{A(x)}{Min}\ MC = \int_0^L p\,u\,dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \le 0$$

$$\mu_t(x): \qquad Eu' - \sigma_t \le 0$$

$$\mu_c(x): \qquad \sigma_c - Eu' \le 0$$

$$Data: L, p(x), E, V^*, \sigma_t, \sigma_c$$

Minimize the mean compliance of a bar for given volume with stress constraints.

Problem 14

$$\underset{p(x)}{Max}\ MC = \int_0^L p\,u\,dx$$

Subject to

$$\lambda(x): \qquad \left(EAu'\right)' + p = 0$$

$$\Lambda: \qquad \int_0^L p\,dx - W^* \le 0$$

$$Data: L, A(x), E, W^*$$

Find the world load distribution for a bar of given geometry. Notice the total load is specified.

Problem 15

$$\underset{A(x)}{Min}\ MSC = \int_0^L p u^2\, dx$$

Subject to

$$\Gamma: \qquad \int_0^L \left(EAu'v' - pv\right) dx = 0$$

$$\Lambda: \qquad \int_0^L A\,dx - V^* \leq 0$$

$$Data: L,\ p(x), E, V^*$$

Minimize a general objective function for given volume.

The end note

Practice problems in calculus of variations

- Axially deforming bar is the simplest structural optimization problem.
- Mean compliance and strain energy are measures of stiffness.
  Volume of material used is a cost-measure.
  Objective function and functional constraint can be interchanged without affecting the nature of the solution.
- Equilibrium equation can be posed in strong or weak form without changing the nature of the solution.
- Constraints can also be imposed on profile of the area of cross-section.
- Constraints on displacements and strains (stresses) can be imposed.

Thanks