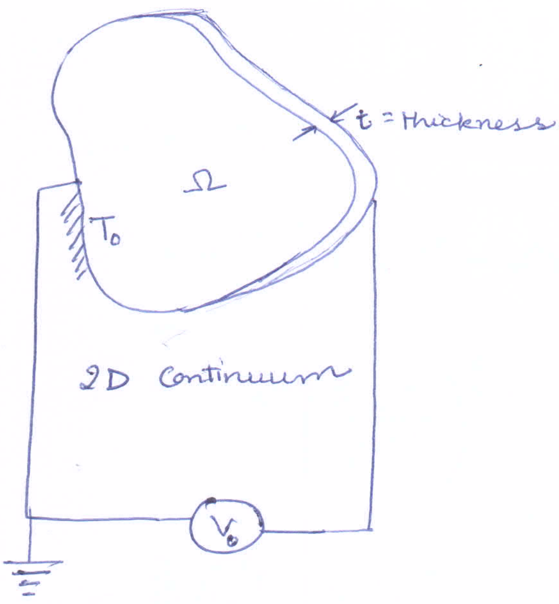
Recap

* Implementation details of the optimality criteria method
  - Handling the bounds on $A(x)$
  - finding Λ in the inner loop



Electrical conductivity

$$\nabla\cdot(K_e \nabla V) = 0. \quad \text{--- (1)}$$

$$\nabla\cdot(K_m \nabla \phi) + \underbrace{(K_e \nabla V)\cdot\nabla V}_{\text{Joule heating}} = 0$$

thermal conductivity

$\phi = T - T_0$

↳ ambient temp

Phonons (equivalent of e^- as (in electrical) in ~~therp~~ thermo)

↳ not a physical thing

So if we have FEM for (1), we get ∇V, then ϕ then Temp.

Now we will like to measure thermal stresses. as deformation are restricted.

$$\nabla\cdot(\bar{D}\bar{\varepsilon} - \bar{D}\,\bar{\varepsilon}_m) = 0$$

$$\text{where} \quad \bar{\varepsilon}_m = \alpha\phi\begin{bmatrix}1\\1\\0\end{bmatrix}$$

thermal expansion coeff

$$\bar{\varepsilon} = \frac{\nabla\bar{u} + \nabla\bar{u}^T}{2} = \begin{bmatrix}\frac{du_x}{dx}\\ \frac{du_y}{dy}\\ \frac{1}{2}\left(\frac{du_x}{dy} + \frac{du_y}{dx}\right)\end{bmatrix}$$

$$\underset{0 \le \rho(\bar{x}) \le 1}{\text{Min SE}} \int_{\Omega} \frac{1}{2} \bar{\varepsilon}^T \underbrace{\bar{\bar{D}} \bar{\varepsilon}}_{\text{Stress.}} \, d\Omega$$

subject to

$$\Gamma_u: \quad \int_{\Omega} \bar{\varepsilon}^T \bar{D} \bar{\varepsilon}_v t \, d\Omega - \int_{\Omega} \bar{\varepsilon}_m^T \bar{D} \bar{\varepsilon}_v \, t \, d\Omega = 0$$

$$\bar{\varepsilon}_v = \begin{bmatrix} \frac{dV_x}{dx} \\ \frac{dV_y}{dy} \\ \frac{1}{2}\left(\frac{dV_x}{dy} + \frac{dV_y}{dx}\right) \end{bmatrix}$$

$\underline{v = \delta u}$ (Variation of u)

$$\Gamma_\phi: \quad \int_{\Omega} \nabla^T \phi \, K_{\phi m} \nabla \phi_v t \, d\Omega + \int_{\Omega} \nabla V^T K_e \nabla V \phi_v \, t \, d\Omega = 0$$

See how to get weak form from strong form.

$$\Gamma_v: \quad \int \nabla^T V \, K_e \nabla V_v \, t \, d\Omega = 0$$

$$\Delta: \quad \int_{\Omega} \rho \, d\Omega - V^* \le 0$$

$\rho(\bar{x}) = \rho(x, y)$

↳ indicator function / fictitious density

↳ It indicates the presence & absence of the material.

$D = \rho^\eta D_0 \qquad \eta \ge 3$

↳ this will help to push the value of ρ close of zero toward zero and those close to 1 towards 1.

$K_m = \rho^\eta K_{m0}$

$K_e = \rho^\eta K_{e0}$

$\alpha = \rho^\eta \alpha_0$

Data: Ω, $\bar{D}_0$, K_{m0}, K_{e0}, α_0, T_0, V^*, t

7 diff eq^n,

Unknown fn; ρ; ϕ_v, ϕ, u, v, V_v, V