

Dynamic loading

$$\min_{A(x)} \int_0^T \int_0^L P(x, t) u(x, t) dx$$

subject to

$$L(x, t): \quad \underbrace{(EAu')'}_{EA'u' + EAu''} + P(x, t) + \rho A(x) \ddot{u} + \underbrace{b\dot{u}}_{\text{Damping}} = 0$$

$$\Delta \quad \int_0^L A dx - V^* \leq 0$$

Data: $E, P(x, t), \rho, b, V^*, L, T$

$$\delta_u L = 0$$

$$P - (kEA)' - \dot{h}b + (kEA)'' + (\dot{h}PA)' = 0$$

$$P - (kEA)' - \dot{h}b + (kEA)'' + \dot{h}PA = 0$$

$$P + (k'EA)' - \dot{b}h + \dot{h}PA = 0$$

Adjoint Eq

So in order match adjoint eqⁿ with Gov eqⁿ then

integrating G.E

$$\int_z^0$$

$$\boxed{z = -T}$$

It's like if we discretize w.r.t to time then we will have

$$\text{Adj } 1 \quad \text{Adj } 2 \quad \dots \quad \text{Adj } 10$$

So these adjoint equations need to be solved in reverse order.